Control System Toolbox

For Use with MATLAB®

Computation

Visualization

Programming

Using the Control System Toolbox

Version 1

How to Contact The MathWorks:

508-647-7000 Phone

508-647-7001 Fax

The MathWorks, Inc.
3 Apple Hill Drive
Natick, MA 01760-2098 Mail

`http://www.mathworks.com`	Web
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Using the Control System Toolbox

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Contents

3 Operations on LTI Models

4 Model Analysis Tools

5 Arrays of LTI Models

9 Customizing Response Plot Properties

10 Design Case Studies

11 Reliable Computations

12 GUI Reference

13 SISO Design Tool Reference

14 LTI Viewer Reference

15 Right-Click Menus for Response Plots

16 Function Reference

Index

1

Introduction

For convenience, this document gathers together nearly all the online doc for the Control System Toolbox in PDF format for printing. This document does not include *Getting Started with the Control System Toolbox*, which is available in both printed and PDF format.

2

LTI Models

The Control System Toolbox offers extensive tools to manipulate and analyze linear time-invariant (LTI) models. It supports both continuous- and discrete-time systems. Systems can be single-input/single-output (SISO) or multiple-input/multiple-output (MIMO). In addition, you can store several LTI models in an array under a single variable name. See Chapter 4, "Arrays of LTI Models" for information on LTI arrays.

This section introduces key concepts about the MATLAB representation of LTI models, including LTI objects, precedence rules for operations, and an analogy between LTI systems and matrices. In addition, it summarizes the basic commands you can use on LTI objects.

LTI Models

You can specify LTI models as:

- Transfer functions (TF), for example,

$$P(s) = \frac{s+2}{s^2+s+10}$$

- Zero-pole-gain models (ZPK), for example,

$$H(z) = \begin{bmatrix} \frac{2(z-0.5)}{z(z+0.1)} & \frac{(z^2+z+1)}{(z+0.2)(z+0.1)} \end{bmatrix}$$

- State-space models (SS), for example,

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

  where A, B, C, and D are matrices of appropriate dimensions, x is the state vector, and u and y are the input and output vectors.

- Frequency response data (FRD) models

  FRD models consist of sampled measurements of a system's frequency response. For example, you can store experimentally collected frequency response data in an FRD.

Using LTI Models in the Control System Toolbox

You can manipulate TF, SS, and ZPK models using the arithmetic and model interconnection operations described in Chapter 3, "Operations on LTI Models" and analyze them using the model analysis functions, such as `bode` and `step`. FRD models can be manipulated and analyzed in much the same way you analyze the other model types, but analysis is restricted to frequency-domain methods.

Using a variety of design techniques, you can design compensators for systems specified with TF, ZPK, SS, and FRD models. These techniques include root locus analysis, pole placement, LQG optimal control, and frequency domain loop-shaping. For FRD models, you can either:

- Obtain an identified TF, SS, or ZPK model using system identification techniques.
- Use frequency-domain analysis techniques.

Other Uses of FRD Models

FRD models are unique model types available in the Control System Toolbox collection of LTI model types, in that they don't have a parametric representation. In addition to the standard operations you may perform on FRD models, you can also use them to:

- Perform frequency-domain analysis on systems with nonlinearities using describing functions.
- Validate identified models against experimental frequency response data.

LTI Objects

Depending on the type of model you use, the data for your model may consist of a simple numerator/denominator pair for SISO transfer functions, four matrices for state-space models, and multiple sets of zeros and poles for MIMO zero-pole-gain models or frequency and response vectors for FRD models. For convenience, the Control System Toolbox provides customized data structures (*LTI objects*) for each type of model. These are called the TF, ZPK, SS, and FRD objects. These four LTI objects encapsulate the model data and enable you to manipulate LTI systems as single entities rather than collections of data vectors or matrices.

Creating an LTI Object: An Example

An LTI object of the type TF, ZPK, SS, or FRD is created whenever you invoke the corresponding constructor function, `tf`, `zpk`, `ss`, or `frd`. For example,

```
P = tf([1 2],[1 1 10])
```

creates a TF object, `P`, that stores the numerator and denominator coefficients of the transfer function

$$P(s) = \frac{s+2}{s^2+s+10}$$

See "Creating LTI Models" on page 2-8 for methods for creating all of the LTI object types.

LTI Properties and Methods

The LTI object implementation relies on MATLAB object-oriented programming capabilities. Objects are MATLAB structures with an additional flag indicating their class (TF, ZPK, SS, or FRD for LTI objects) and have pre-defined fields called *object properties*. For LTI objects, these properties include the model data, sample time, delay times, input or output names, and input or output groups (see "LTI Properties" on page 2-25 for details). The functions that operate on a particular object are called the object *methods*. These may include customized versions of simple operations such as addition or multiplication. For example,

```
P = tf([1 2],[1 1 10])
Q = 2 + P
```

performs transfer function addition.

$$Q(s) = 2 + P(s) = \frac{2s^2+3s+22}{s^2+s+10}$$

The object-specific versions of such standard operations are called *overloaded* operations. For more details on objects, methods, and object-oriented programming, see Chapter 14, "Classes and Objects" in *Using MATLAB*. For details on operations on LTI objects, see Chapter 3, "Operations on LTI Models."

Precedence Rules

Operations like addition and commands like `feedback` operate on more than one LTI model at a time. If these LTI models are represented as LTI objects of different types (for example, the first operand is TF and the second operand is SS), it is not obvious what type (for example, TF or SS) the resulting model should be. Such type conflicts are resolved by *precedence rules*. Specifically, TF, ZPK, SS, and FRD objects are ranked according to the precedence hierarchy.

FRD > SS > ZPK > TF

Thus ZPK takes precedence over TF, SS takes precedence over both TF and ZPK, and FRD takes precedence over all three. In other words, any operation involving two or more LTI models produces:

- An FRD object if at least one operand is an FRD object
- An SS object if no operand is an FRD object and at least one operand is an SS object
- A ZPK object if no operand is an FRD or SS object and at least one is an ZPK object
- A TF object only if all operands are TF objects

Operations on systems of different types work as follows: the resulting type is determined by the precedence rules, and all operands are first converted to this type before performing the operation.

Viewing LTI Systems As Matrices

In the frequency domain, an LTI system is represented by the linear input/output map

$$y = Hu$$

This map is characterized by its transfer matrix H, a function of either the Laplace or Z-transform variable. The transfer matrix H maps inputs to outputs, so there are as many columns as inputs and as many rows as outputs.

If you think of LTI systems in terms of (transfer) matrices, certain basic operations on LTI systems are naturally expressed with a matrix-like syntax. For example, the parallel connection of two LTI systems `sys1` and `sys2` can be expressed as

```
sys = sys1 + sys2
```

because parallel connection amounts to adding the transfer matrices. Similarly, subsystems of a given LTI model sys can be extracted using matrix-like subscripting. For instance,

```
sys(3,1:2)
```

provides the I/O relation between the first two inputs (column indices) and the third output (row index), which is consistent with

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H(1,1) & H(2,1) \\ H(2,1) & H(2,2) \\ H(3,1) & H(3,2) \end{bmatrix} \begin{bmatrix} u_1 & u_2 \end{bmatrix}$$

for $y = Hu$.

Command Summary

The next two tables give an overview of the main commands you can apply to LTI models.

Table 2-1: Creating LTI Models

Command	Description
`drss`	Generate random discrete state-space model.
`dss`	Create descriptor state-space model.
`filt`	Create discrete filter with DSP convention.
`frd`	Create an FRD model.
`frdata`	Retrieve FRD model data.
`get`	Query LTI model properties.
`set`	Set LTI model properties.
`rss`	Generate random continuous state-space model.
`ss`	Create a state-space model.

Table 2-1: Creating LTI Models (Continued)

Command	Description
`ssdata`, `dssdata`	Retrieve state-space data (respectively, descriptor state-space data).
`tf`	Create a transfer function.
`tfdata`	Retrieve transfer function data.
`zpk`	Create a zero-pole-gain model.
`zpkdata`	Retrieve zero-pole-gain data.

Table 2-2: Converting LTI Models

Command	Description
`c2d`	Continuous- to discrete-time conversion.
`d2c`	Discrete- to continuous-time conversion.
`d2d`	Resampling of discrete-time models.
`frd`	Conversion to an FRD model.
`pade`	Padé approximation of input delays.
`ss`	Conversion to state space.
`tf`	Conversion to transfer function.
`zpk`	Conversion to zero-pole-gain.

Creating LTI Models

The functions `tf`, `zpk`, `ss`, and `frd` create transfer function models, zero-pole-gain models, state-space models, and frequency response data models, respectively. These functions take the model data as input and produce TF, ZPK, SS, or FRD objects that store this data in a single MATLAB variable. This section shows how to create continuous or discrete, SISO or MIMO LTI models with `tf`, `zpk`, `ss`, and `frd`.

Note You can only specify TF, ZPK, and SS models for systems whose transfer matrices have real-valued coefficients.

Transfer Function Models

This section explains how to specify continuous-time SISO and MIMO transfer function models. The specification of discrete-time transfer function models is a simple extension of the continuous-time case (see "Discrete-Time Models" on page 2-19). In this section you can also read about how to specify transfer functions consisting of pure gains.

SISO Transfer Function Models

A continuous-time SISO transfer function

$$h(s) = \frac{n(s)}{d(s)}$$

is characterized by its numerator $n(s)$ and denominator $d(s)$, both polynomials of the Laplace variable s.

There are two ways to specify SISO transfer functions:

- Using the `tf` command
- As rational expressions in the Laplace variable s

To specify a SISO transfer function model $h(s) = n(s)/d(s)$ using the `tf` command, type

```
h = tf(num,den)
```

where num and den are row vectors listing the coefficients of the polynomials $n(s)$ and $d(s)$, respectively, when these polynomials are ordered in *descending* powers of s. The resulting variable h is a TF object containing the numerator and denominator data.

For example, you can create the transfer function $h(s) = s/(s^2 + 2s + 10)$ by typing

```
h = tf([1 0],[1 2 10])
```

MATLAB responds with

```
Transfer function:
      s
--------------
s^2 + 2 s + 10
```

Note the customized display used for TF objects.

You can also specify transfer functions as rational expressions in the Laplace variable s by:

1 Defining the variable s as a special TF model

```
s = tf('s');
```

2 Entering your transfer function as a rational expression in s

For example, once s is defined with tf as in **1**,

```
H = s/(s^2 + 2*s +10);
```

produces the same transfer function as

```
h = tf([1 0],[1 2 10]);
```

Note You need only define the variable s as a TF model once. All of the subsequent models you create using rational expressions of s are specified as TF objects, unless you convert the variable s to ZPK. See "Model Conversion" on page 2-40 for more information.

MIMO Transfer Function Models

MIMO transfer functions are two-dimensional arrays of elementary SISO transfer functions. There are several ways to specify MIMO transfer function models, including:

- Concatenation of SISO transfer function models
- Using `tf` with cell array arguments

Consider the rational transfer matrix

$$H(s) = \begin{bmatrix} \dfrac{s-1}{s+1} \\ \dfrac{s+2}{s^2+4s+5} \end{bmatrix}$$

You can specify $H(s)$ by concatenation of its SISO entries. For instance,

```
h11 = tf([1 -1],[1 1]);
h21 = tf([1 2],[1 4 5]);
```

or, equivalently,

```
s = tf('s')
h11 = (s-1)/(s+1);
h21 = (s+2)/(s^2+4*s+5);
```

can be concatenated to form $H(s)$.

```
H = [h11; h21]
```

This syntax mimics standard matrix concatenation and tends to be easier and more readable for MIMO systems with many inputs and/or outputs. See "Model Interconnection Functions" on page 3-16 for more details on concatenation operations for LTI systems.

Alternatively, to define MIMO transfer functions using `tf`, you need two cell arrays (say, `N` and `D`) to represent the sets of numerator and denominator polynomials, respectively. See Chapter 13, "Structures and Cell Arrays" in *Using MATLAB* for more details on cell arrays.

For example, for the rational transfer matrix $H(s)$, the two cell arrays `N` and `D` should contain the row-vector representations of the polynomial entries of

$$N(s) = \begin{bmatrix} s-1 \\ s+2 \end{bmatrix} \qquad D(s) = \begin{bmatrix} s+1 \\ s^2+4s+5 \end{bmatrix}$$

You can specify this MIMO transfer matrix $H(s)$ by typing

```
N = {[1 –1];[1 2]}; % cell array for N(s)
D = {[1 1];[1 4 5]}; % cell array for D(s)
H = tf(N,D)
```

MATLAB responds with

```
Transfer function from input to output...
      s – 1
 #1:  -----
      s + 1

          s + 2
 #2:  -------------
      s^2 + 4 s + 5
```

Notice that both `N` and `D` have the same dimensions as H. For a general MIMO transfer matrix $H(s)$, the cell array entries `N{i,j}` and `D{i,j}` should be row-vector representations of the numerator and denominator of $H_{ij}(s)$, the ijth entry of the transfer matrix $H(s)$.

Pure Gains

You can use `tf` with only one argument to specify simple gains or gain matrices as TF objects. For example,

```
G = tf([1 0;2 1])
```

produces the gain matrix

$$G = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

while

```
E = tf
```

creates an empty transfer function.

Zero-Pole-Gain Models

This section explains how to specify continuous-time SISO and MIMO zero-pole-gain models. The specification for discrete-time zero-pole-gain models is a simple extension of the continuous-time case. See "Discrete-Time Models" on page 2-19.

SISO Zero-Pole-Gain Models

Continuous-time SISO zero-pole-gain models are of the form

$$h(s) = k\frac{(s-z_1)\ldots(s-z_m)}{(s-p_1)\ldots(s-p_n)}$$

where k is a real-valued scalar (the *gain*), and $z_1,\ldots,z_m$ and $p_1,\ldots,p_n$ are the real or complex conjugate pairs of zeros and poles of the transfer function $h(s)$. This model is closely related to the transfer function representation: the zeros are simply the numerator roots, and the poles, the denominator roots.

There are two ways to specify SISO zero-pole-gain models:

- Using the `zpk` command
- As rational expressions in the Laplace variable *s*

The syntax to specify ZPK models directly using `zpk` is

```
h = zpk(z,p,k)
```

where `z` and `p` are the vectors of zeros and poles, and `k` is the gain. This produces a ZPK object `h` that encapsulates the `z`, `p`, and `k` data. For example, typing

```
h = zpk(0, [1-i 1+i 2], -2)
```

produces

```
Zero/pole/gain:
        -2 s
--------------------
(s-2) (s^2 - 2s + 2)
```

You can also specify zero-pole-gain models as rational expressions in the Laplace variable *s* by:

1. Defining the variable s as a ZPK model

   ```
   s = zpk('s')
   ```

2. Entering the transfer function as a rational expression in s.

For example, once s is defined with zpk,

```
H = –2s/((s – 2)*(s^2 + 2*s + 2));
```

returns the same ZPK model as

```
h = zpk([0], [2 –1–i –1+i ], –2);
```

Note You need only define the ZPK variable s once. All subsequent rational expressions of s will be ZPK models, unless you convert the variable s to TF. See "Model Conversion" on page 2-40 for more information on conversion to other model types.

MIMO Zero-Pole-Gain Models

Just as with TF models, you can also specify a MIMO ZPK model by concatenation of its SISO entries (see "Model Interconnection Functions" on page 3-16).

You can also use the command zpk to specify MIMO ZPK models. The syntax to create a *p*-by-*m* MIMO zero-pole-gain model using zpk is

```
H = zpk(Z,P,K)
```

where

- Z is the *p*-by-*m* cell array of zeros (Z{i,j} = zeros of $H_{ij}(s)$)
- P is the *p*-by-*m* cell array of poles (P{i,j} = poles of $H_{ij}(s)$)
- K is the *p*-by-*m* matrix of gains (K(i,j) = gain of $H_{ij}(s)$)

For example, typing

```
Z = {[],–5;[1–i 1+i] []};
```

```
P = {0,[–1 –1];[1 2 3],[]};
```

```
K = [-1  3;2  0];

H = zpk(Z,P,K)
```

creates the two-input/two-output zero-pole-gain model

$$H(s) = \begin{bmatrix} \frac{-1}{s} & \frac{3(s+5)}{(s+1)^2} \\ \frac{2(s^2-2s+2)}{(s-1)(s-2)(s-3)} & 0 \end{bmatrix}$$

Notice that you use `[]` as a place-holder in `Z` (or `P`) when the corresponding entry of $H(s)$ has no zeros (or poles).

State-Space Models

State-space models rely on linear differential or difference equations to describe the system dynamics. Continuous-time models are of the form

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

where x is the state vector and u and y are the input and output vectors. Such models may arise from the equations of physics, from state-space identification, or by state-space realization of the system transfer function.

Use the command `ss` to create state-space models

```
sys = ss(A,B,C,D)
```

For a model with `Nx` states, `Ny` outputs, and `Nu` inputs

- `A` is an `Nx`-by-`Nx` real-valued matrix.
- `B` is an `Nx`-by-`Nu` real-valued matrix.
- `C` is an `Ny`-by-`Nx` real-valued matrix.
- `D` is an `Ny`-by-`Nu` real-valued matrix.

This produces an SS object `sys` that stores the state-space matrices A, B, C, and D. For models with a zero D matrix, you can use `D = 0` (zero) as a shorthand for a zero matrix of the appropriate dimensions.

As an illustration, consider the following simple model of an electric motor.

$$\frac{d^2\theta}{dt^2} + 2\frac{d\theta}{dt} + 5\theta = 3I$$

where θ is the angular displacement of the rotor and I the driving current. The relation between the input current $u = I$ and the angular velocity $y = d\theta/dt$ is described by the state-space equations

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx$$

where

$$x = \begin{bmatrix} \theta \\ \frac{d\theta}{dt} \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

This model is specified by typing

```
sys = ss([0 1;-5 -2],[0;3],[0 1],0)
```

to which MATLAB responds

```
a =
                    x1           x2
           x1            0      1.00000
           x2     -5.00000     -2.00000

b =
                    u1
           x1            0
           x2      3.00000

c =
                    x1           x2
           y1            0      1.00000

d =
                    u1
           y1            0
```

In addition to the *A*, *B*, *C*, and *D* matrices, the display of state-space models includes state names, input names, and output names. Default names (here, `x1`, `x2`, `u1`, and `y1`) are displayed whenever you leave these unspecified. See "LTI Properties" on page 2-25 for more information on how to specify state, input, or output names.

Descriptor State-Space Models

Descriptor state-space (DSS) models are a generalization of the standard state-space models discussed above. They are of the form

$$E\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

The Control System Toolbox supports only descriptor systems with a nonsingular E matrix. While such models have an equivalent explicit form

$$\frac{dx}{dt} = (E^{-1}A)x + (E^{-1}B)u$$
$$y = Cx + Du$$

it is often desirable to work with the descriptor form when the E matrix is poorly conditioned with respect to inversion.

The function `dss` is the counterpart of `ss` for descriptor state-space models. Specifically,

```
sys = dss(A,B,C,D,E)
```

creates a continuous-time DSS model with matrix data `A,B,C,D,E`. For example, consider the dynamical model

$$J\frac{d\omega}{dt} + F\omega = T$$
$$y = \omega$$

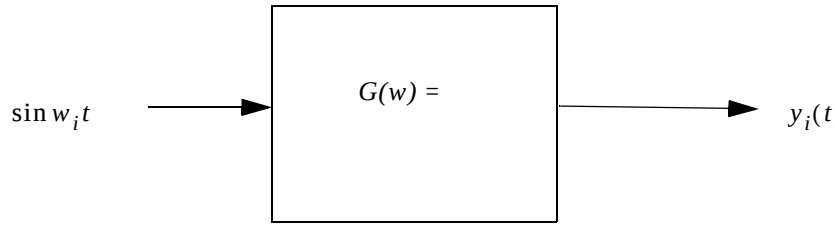
with vector ω of angular velocities. If the inertia matrix J is poorly conditioned with respect to inversion, you can specify this system as a descriptor model by

```
sys = dss(-F,eye(n),eye(n),0,J)   % n = length of vector ω
```

Frequency Response Data (FRD) Models

In some instances, you may only have sampled frequency response data, rather than a transfer function or state-space model for the system you want to analyze or control. For information on frequency response analysis of linear systems, see Chapter 8 of [1].

For example, suppose the frequency response function for the SISO system you want to model is *G(w)*. Suppose, in addition, that you perform an experiment to evaluate *G(w)* at a fixed set of frequencies, $w_1, w_2, \ldots, w_n$. You can do this by driving the system with a sequence of sinusoids at each of these frequencies, as depicted below.



Here w_i is the input frequency of each sinusoid, $i = 1 \ldots n$, and $G(w) = |G(w)| \exp(j \angle G(w))$. The steady state output response of this system satisfies

$$y_i(t) = |G(w_i)| \sin(w_i t + \angle G(w_i)); \quad i = 1 \ldots n$$

A *frequency response data* (FRD) *object* is a model form you can use to store frequency response data (complex frequency response, along with a corresponding vector of frequency points) that you obtain either through simulations or experimentally. In this example, the frequency response data is obtained from the set of response pairs: $\{(G(w_i), w_i)\}$, $i = 1 \ldots n$.

Once you store your data in an FRD model, you can treat it as an LTI model, and manipulate an FRD model in most of the same ways you manipulate TF, SS, and ZPK models.

The basic syntax for creating a SISO FRD model is

```
sys = frd(response,frequencies,units)
```

where

- `frequencies` is a real vector of length `Nf`.

- `response` is a vector of length `Nf` of complex frequency response values for these frequencies.
- `units` is an optional string for the units of frequency: either `'rad/s'` (default) or `'Hz'`

For example, the MAT-file `LTIexamples.mat` contains a frequency vector `freq`, and a corresponding complex frequency response data vector `respG`. To load this frequency-domain data and construct an FRD model, type

```
load LTIexamples
sys = frd(respG,freq)

Continuous-time frequency response with 1 output and 1 input
at 5 frequency points.

From input 1 to:
Frequency(rad/s)        output 1
----------------        --------
        1        -0.812505 -0.000312i
        2        -0.092593 -0.462963i
        4        -0.075781 -0.001625i
        5         -0.043735 -0.000390i
```

The syntax for creating a MIMO FRD model is the same as for the SISO case, except that `response` is a *p*-by-*m*-by-*Nf* multidimensional array, where *p* is the number of outputs, *m* is the number of inputs, and *Nf* is the number of frequency data points (the length of `frequency`).

The following table summarizes the complex-valued response data format for FRD models.

Table 2-3: Data Format for the Argument response in FRD Models

Model Form	Response Data Format
SISO model	Vector of length `Nf` for which `response(i)` is the frequency response at the frequency `frequency(i)`

Table 2-3: Data Format for the Argument response in FRD Models (Continued)

Model Form	Response Data Format
MIMO model with `Ny` outputs and `Nu` inputs	`Ny`-by-`Nu`-by-`Nf` multidimensional array for which `response(i,j,k)` specifies the frequency response from input `j` to output `i` at frequency `frequency(k)`
`S1`-by-...-by-`Sn` array of models with `Ny` outputs and `Nu` inputs	`Ny`-by-`Nu`-by-`S1`-by-...-by-`Sn` multidimensional array, for which `response(i,j,k,:)` specifies the array of frequency response data from input `j` to output `i` at frequency `frequency(k)`

Discrete-Time Models

Creating discrete-time models is very much like creating continuous-time models, except that you must also specify a sampling period or *sample time* for discrete-time models. The sample time value should be scalar and expressed in seconds. You can also use the value –1 to leave the sample time unspecified.

To specify discrete-time LTI models using `tf`, `zpk`, `ss`, or `frd`, simply append the desired sample time value `Ts` to the list of inputs.

```
sys1 = tf(num,den,Ts)
sys2 = zpk(z,p,k,Ts)
sys3 = ss(a,b,c,d,Ts)
sys4 = frd(response,frequency,Ts)
```

For example,

```
h = tf([1 -1],[1 -0.5],0.1)
```

creates the discrete-time transfer function $h(z) = (z-1)/(z-0.5)$ with sample time 0.1 seconds, and

```
sys = ss(A,B,C,D,0.5)
```

specifies the discrete-time state-space model

$$x[n+1] = Ax[n] + Bu[n]$$
$$y[n] = Cx[n] + Du[n]$$

with sampling period 0.5 second. The vectors $x[n]$, $u[n]$, $y[n]$ denote the values of the state, input, and output vectors at the nth sample.

By convention, the sample time of continuous-time models is `Ts = 0`. Setting `Ts = –1` leaves the sample time of a discrete-time model unspecified. For example,

```
h = tf([1 –0.2],[1 0.3],–1)
```

produces

```
Transfer function:
z – 0.2
-------
z + 0.3

Sampling time: unspecified
```

Note Do not simply omit `Ts` in this case. This would make `h` a continuous-time transfer function.

If you forget to specify the sample time when creating your model, you can still set it to the correct value by reassigning the LTI property `Ts`. See "Sample Time" on page 2-33 for more information on setting this property.

Discrete-Time TF and ZPK Models

You can specify discrete-time TF and ZPK models using `tf` and `zpk` as indicated above. Alternatively, it is often convenient to specify such models by:

1. Defining the variable *z* as a particular discrete-time TF or ZPK model with the appropriate sample time
2. Entering your TF or ZPK model directly as a rational expression in *z*.

This approach parallels the procedure for specifying continuous-time TF or ZPK models using rational expressions. This procedure is described in "SISO Transfer Function Models" on page 2-8 and "SISO Zero-Pole-Gain Models" on page 2-12.

For example,

```
z = tf('z', 0.1);
H = (z+2)/(z^2 + 0.6*z + 0.9);
```

creates the same TF model as

```
H = tf([1 2], [1 0.6 0.9], 0.1);
```

Similarly,

```
z = zpk('z', 0.1);
H = [z/(z+0.1)/(z+0.2) ; (z^2+0.2*z+0.1)/(z^2+0.2*z+0.01)]
```

produces the single-input, two-output ZPK model

```
Zero/pole/gain from input to output...
             z
 #1:  ---------------
      (z+0.1) (z+0.2)

      (z^2 + 0.2z + 0.1)
 #2:  ------------------
          (z+0.1)^2

Sampling time: 0.1
```

Note that:

- The syntax `z = tf('z')` is equivalent to `z = tf('z',-1)` and leaves the sample time unspecified. The same applies to `z = zpk('z')`.
- Once you have defined `z` as indicated above, any rational expressions in `z` creates a discrete-time model of the same type and with the same sample time as `z`.

Discrete Transfer Functions in DSP Format

In digital signal processing (DSP), it is customary to write discrete transfer functions as rational expressions in z^{-1} and to order the numerator and denominator coefficients in *ascending powers of* z^{-1}. For example, the numerator and denominator of

$$H(z^{-1}) = \frac{1 + 0.5z^{-1}}{1 + 2z^{-1} + 3z^{-2}}$$

would be specified as the row vectors `[1 0.5]` and `[1 2 3]`, respectively. When the numerator and denominator have different degrees, this convention

clashes with the "*descending powers of z*" convention assumed by `tf` (see "Transfer Function Models" on page 2-8, or `tf`). For example,

```
h = tf([1 0.5],[1 2 3])
```

produces the transfer function

$$\frac{z + 0.5}{z^2 + 2z + 3}$$

which differs from $H(z^{-1})$ by a factor z.

To avoid such convention clashes, the Control System Toolbox offers a separate function `filt` dedicated to the DSP-like specification of transfer functions. Its syntax is

```
h = filt(num,den)
```

for discrete transfer functions with unspecified sample time, and

```
h = filt(num,den,Ts)
```

to further specify the sample time `Ts`. This function creates TF objects just like `tf`, but expects `num` and `den` to list the numerator and denominator coefficients in *ascending powers of* z^{-1}. For example, typing

```
h = filt([1 0.5],[1 2 3])
```

produces

```
Transfer function:
   1 + 0.5 z^-1
------------------
1 + 2 z^-1 + 3 z^-2

Sampling time: unspecified
```

You can also use `filt` to specify MIMO transfer functions in z^{-1}. Just as for `tf`, the input arguments `num` and `den` are then cell arrays of row vectors of appropriate dimensions (see "Transfer Function Models" on page 2-8 for details). Note that each row vector should comply with the "ascending powers of z^{-1}" convention.

Data Retrieval

The functions `tf`, `zpk`, `ss`, and `frd` pack the model data and sample time in a single LTI object. Conversely, the following commands provide convenient data retrieval for any type of TF, SS, or ZPK model `sys`, or FRD model `sysfr`.

```
[num,den,Ts] = tfdata(sys)     % Ts = sample time
[z,p,k,Ts] = zpkdata(sys)
[a,b,c,d,Ts] = ssdata(sys)
[a,b,c,d,e,Ts] = dssdata(sys)
[response,frequency,Ts] = frdata(sysfr)
```

Note that:

- `sys` can be any type of LTI object, *except* an FRD model
- `sysfr`, the input argument to `frdata`, can only be an FRD model

You can use any variable names you want in the output argument list of any of these functions. The ones listed here correspond to the model property names described in Tables 2-2 – 2.5.

The output arguments `num` and `den` assigned to `tfdata`, and `z` and `p` assigned to `zpkdata`, are cell arrays, even in the SISO case. These cell arrays have as many rows as outputs, as many columns as inputs, and their *ij*th entry specifies the transfer function from the *j*th input to the *i*th output. For example,

```
H = [tf([1 -1],[1 2 10]) , tf(1,[1 0])]
```

creates the one-output/two-input transfer function

$$H(s) = \left[\begin{array}{cc} \dfrac{s-1}{s^2+2s+10} & \dfrac{1}{s} \end{array}\right]$$

Typing

```
[num,den] = tfdata(H);
num{1,1}, den{1,1}
```

displays the coefficients of the numerator and denominator of the first input channel.

```
ans =
     0     1    -1
ans =
```

```
     1     2    10
```

Note that the same result is obtained using

```
H.num{1,1}, H.den{1,1}
```

See “Direct Property Referencing” on page 2-31 for more information about this syntax.

To obtain the numerator and denominator of SISO systems directly as row vectors, use the syntax

```
[num,den,Ts] = tfdata(sys,'v')
```

For example, typing

```
sys = tf([1 3],[1 2 5]);
[num,den] = tfdata(sys,'v')
```

produces

```
num =

     0     1     3

den =

     1     2     5
```

Similarly,

```
[z,p,k,Ts] = zpkdata(sys,'v')
```

returns the zeros, `z`, and the poles, `p`, as *vectors* for SISO systems.

LTI Properties

The previous section shows how to create LTI objects that encapsulate the model data and sample time. You also have the option to attribute additional information, such as the input names or notes on the model history, to LTI objects. This section gives a complete overview of the *LTI properties*, i.e., the various pieces of information that can be attached to the TF, ZPK, SS, and FRD objects. Type `help ltiprops` for online help on available LTI properties.

From a data structure standpoint, the LTI properties are the various fields in the TF, ZPK, SS, and FRD objects. These fields have names (the *property names*) and are assigned values (the *property values*). We distinguish between *generic properties*, common to all four types of LTI objects, and *model-specific properties* that pertain only to one particular type of model.

Generic Properties

The generic properties are those shared by all four types of LTI models (TF, ZPK, SS, and FRD objects). They are listed in the table below.

Table 2-4: Generic LTI Properties

Property Name	Description	Property Value
`ioDelay`	I/O delay(s)	Matrix
`InputDelay`	Input delay(s)	Vector
`InputGroup`	Input channel groups	Cell array
`InputName`	Input channel names	Cell vector of strings
`Notes`	Notes on the model history	Text
`OutputDelay`	Output delay(s)	Vector
`OutputGroup`	Output channel groups	Cell array
`OutputName`	Output channel names	Cell vector of strings
`Ts`	Sample time	Scalar
`Userdata`	Additional data	Arbitrary

The sample time property `Ts` keeps track of the sample time (in seconds) of discrete-time systems. By convention, `Ts` is `0` (zero) for continuous-time systems, and `Ts` is `–1` for discrete-time systems with unspecified sample time. `Ts` is always a scalar, even for MIMO systems.

The `InputDelay`, `OutputDelay`, and `ioDelay` properties allow you to specify time delays in the input or output channels, or for each input/output pair. Their default value is zero (no delay). See “Time Delays” on page 2-43 for details on modeling delays.

The `InputName` and `OutputName` properties enable you to give names to the *individual* input and output channels. The value of each of these properties is a cell vector of strings with as many cells as inputs or outputs. For example, the `OutputName` property is set to

```
{ 'temperature' ; 'pressure' }
```

for a system with two outputs labeled `temperature` and `pressure`. The default value is a cell of empty strings.

Using the `InputGroup` and `OutputGroup` properties of LTI objects, you can create different groups of input or output channels, and assign names to the groups. For example, you may want to designate the first four inputs of a five-input model as `controls`, and the last input as `noise`. See “Input Groups and Output Groups” on page 2-36 for more information.

Finally, `Notes` and `Userdata` are available to store additional information on the model. The `Notes` property is dedicated to any text you want to supply with your model, while the `Userdata` property can accommodate arbitrary user-supplied data. They are both empty by default.

For more detailed information on how to use LTI properties, see “Additional Insight into LTI Properties” on page 2-32.

Model-Specific Properties

The remaining LTI properties are specific to one of the four model types (TF, ZPK, SS, or FRD). For single LTI models, these are summarized in the

following four tables. The property values differ for LTI arrays. See `set` for more information on these values.

Table 2-5: TF-Specific Properties

Property Name	Description	Property Value
`den`	Denominator(s)	Real cell array of row vectors
`num`	Numerator(s)	Real cell array of row vectors
`Variable`	Transfer function variable	String `'s'`, `'p'`, `'z'`, `'q'`, or `'z^-1'`

Table 2-6: ZPK-Specific Properties

Property Name	Description	Property Value
`k`	Gains	Two-dimensional real matrix
`p`	Poles	Cell array of column vectors
`Variable`	Transfer function variable	String `'s'`, `'p'`, `'z'`, `'q'`, or `'z^-1'`
`z`	Zeros	Cell array of column vectors

Table 2-7: SS-Specific Properties

Property Name	Description	Property Value
`a`	State matrix A	2-D real matrix
`b`	Input-to-state matrix B	2-D real matrix
`c`	State-to-output matrix C	2-D real matrix
`d`	Feedthrough matrix D	2-D real matrix
`e`	Descriptor E matrix	2-D real matrix
`Nx`	Number of states	Scalar integer
`StateName`	State names	Cell vector of strings

Table 2-8: FRD-Specific Properties

Property Name	Description	Property Value
`Frequency`	Frequency data points	Real-valued vector
`ResponseData`	Frequency response	Complex-valued multidimensional array
`Units`	Units for frequency	String `'rad/s'` or `'Hz'`

Most of these properties are dedicated to storing the model data. Note that the E matrix is set to `[]` (the empty matrix) for standard state-space models, a storage-efficient shorthand for the true value $E = I$.

The `Variable` property is only an attribute of TF and ZPK objects. This property defines the frequency variable of transfer functions. The default values are `'s'` (Laplace variable s) in continuous time and `'z'` (Z-transform variable z) in discrete time. Alternative choices include `'p'` (equivalent to s) and `'q'` or `'z^-1'` for the reciprocal $q = z^{-1}$ of the z variable. The influence of the variable choice is mostly limited to the display of TF or ZPK models. One exception is the specification of discrete-time transfer functions with `tf` (see `tf` for details).

Note that `tf` produces the same result as `filt` when the `Variable` property is set to `'z^-1'` or `'q'`.

Finally, the `StateName` property is analogous to the `InputName` and `OutputName` properties and keeps track of the state names in state-space models.

Setting LTI Properties

There are three ways to specify LTI property values:

- You can set properties when creating LTI models with `tf`, `zpk`, `ss`, or `frd`.
- You can set or modify the properties of an existing LTI model with `set`.
- You can also set property values using structure-like assignments.

This section discusses the first two options. See “Direct Property Referencing” on page 2-31 for details on the third option.

The function `set` for LTI objects follows the same syntax as its Handle Graphics counterpart. Specifically, each property is updated by a pair of arguments

PropertyName,`PropertyValue`

where

- *PropertyName* is a string specifying the property name. You can type the property name without regard for the case (upper or lower) of the letters in the name. Actually, you need only type any abbreviation of the property name that uniquely identifies the property. For example, `'user'` is sufficient to refer to the `Userdata` property.
- `PropertyValue` is the value to assign to the property (see `set` for details on admissible property values).

As an illustration, consider the following simple SISO model for a heating system with an input delay of 0.3 seconds, an input called “energy,” and an output called “temperature.”

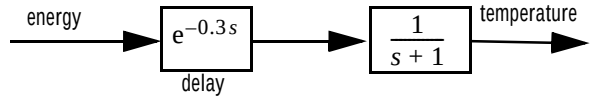


Figure 2-1: A Simple Heater Model

You can use a TF object to represent this delay system, and specify the time delay, the input and output names, and the model history by setting the corresponding LTI properties. You can either set these properties directly when you create the LTI model with `tf`, or by using the `set` command.

For example, you can specify the delay directly when you create the model, and then use the `set` command to assign `InputName`, `OutputName`, and `Notes` to `sys`.

```
sys = tf(1,[1 1],'Inputdelay',0.3);
set(sys,'inputname','energy','outputname','temperature',...
        'notes','A simple heater model')
```

Finally, you can also use the set command to obtain a listing of all settable properties for a given LTI model type, along with valid values for these properties. For the transfer function sys created above

```
set(sys)
```

produces

```
num:  Ny-by-Nu cell of row vectors (Nu = no. of inputs)
den:  Ny-by-Nu cell of row vectors (Ny = no. of outputs)
Variable:  [ 's' | 'p' | 'z' | 'z^-1' | 'q' ]
Ts:  scalar
InputDelay:  Nu-by-1 vector
OutputDelay:  Ny-by-1 vector
ioDelay:  Ny-by-Nu array (I/O delays)
InputName:  Nu-by-1 cell array of strings
OutputName:  Ny-by-1 cell array of strings
InputGroup:  M-by-2 cell array if M input groups
OutputGroup:  P-by-2 cell array if P output groups
Notes:  array or cell array of strings
UserData:  arbitrary
```

Accessing Property Values Using get

You access the property values of an LTI model sys with get. The syntax is

```
PropertyValue = get(sys,PropertyName)
```

where the string *PropertyName* is either the full property name, or any abbreviation with enough characters to identify the property uniquely. For example, typing

```
h = tf(100,[1 5 100],'inputname','voltage',...
                        'outputn','current',...
                        'notes','A simple circuit')
get(h,'notes')
```

produces

```
ans =

    'A simple circuit'
```

To display all of the properties of an LTI model sys (and their values), use the syntax get(sys). In this example,

```
get(h)
```

produces

```
num = {[0 0 100]}
den = {[1 5 100]}
Variable = 's'
Ts = 0
InputDelay = 0
OutputDelay = 0
ioDelay = 0
InputName = {'voltage'}
OutputName = {'current'}
InputGroup = {0x2 cell}
OutputGroup = {0x2 cell}
Notes = {'A simple circuit'}
UserData = []
```

Notice that default (output) values have been assigned to any LTI properties in this list that you have not specified.

Finally, you can also access property values using direct structure-like referencing. This topic is explained in the next section.

Direct Property Referencing

An alternative way to query/modify property values is by structure-like referencing. Recall that LTI objects are basic MATLAB structures except for the additional flag that marks them as TF, ZPK, SS, or FRD objects (see “LTI Objects” on page 2-3). The field names for LTI objects are the property names, so you can retrieve or modify property values with the structure-like syntax.

```
PropertyValue = sys.PropertyName% gets property value
sys.PropertyName = PropertyValue% sets property value
```

These commands are respectively equivalent to

```
PropertyValue = get(sys,'PropertyName')
set(sys,'PropertyName',PropertyValue)
```

For example, type

```
sys = ss(1,2,3,4,'InputName','u');
sys.a
```

and you get the value of the property “a” for the state-space model sys.

```
ans =
     1
```

Similarly,

```
sys.a = –1;
```

resets the state transition matrix for sys to –1.

Unlike standard MATLAB structures, you do not need to type the entire field name or use upper-case characters. You only need to type the minimum number of characters sufficient to identify the property name uniquely. Thus either of the commands

```
sys.InputName
sys.inputn
```

produces

```
ans =

    'u'
```

Any valid syntax for structures extends to LTI objects. For example, given the TF model $h(p) = 1/p$

```
h = tf(1,[1,0],'variable','p');
```

you can reset the numerator to $p + 2$ by typing

```
h.num{1} = [1 2];
```

or equivalently, with

```
h.num{1}(2) = 2;
```

Additional Insight into LTI Properties

By reading this section, you can learn more about using the `Ts`, `InputName`, `OutputName`, `InputGroup`, and `OutputGroup` LTI properties through a set of examples. For basic information on `Notes` and `Userdata`, see “Generic

Properties" on page 2-25. For detailed information on the use of `InputDelay`, `OutputDelay`, and `ioDelay`, see "Time Delays" on page 2-43.

Sample Time

The sample time property `Ts` is used to specify the sampling period (in seconds) for either discrete-time or discretized continuous-time LTI models. Suppose you want to specify

$$H(z) = \frac{z}{2z^2 + z + 1}$$

as a discrete-time transfer function model with a sampling period of 0.5 seconds. To do this, type

```
h = tf([1 0],[2 1 1],0.5);
```

This sets the `Ts` property to the value 0.5, as is confirmed by

```
h.Ts

ans =
    0.5000
```

For continuous-time models, the sample time property `Ts` is 0 by convention. For example, type

```
h = tf(1,[1 0]);
get(h,'Ts')

ans =
     0
```

To leave the sample time of a discrete-time LTI model unspecified, set `Ts` to –1. For example,

```
h = tf(1,[1 -1],-1)
```

produces

```
Transfer function:
  1
-----
z - 1
```

```
Sampling time: unspecified
```

The same result is obtained by using the `Variable` property.

```
h = tf(1,[1 -1],'var','z')
```

In operations that combine several discrete-time models, all *specified* sample times must be identical, and the resulting discrete-time model inherits this common sample time. The sample time of the resultant model is unspecified if all operands have unspecified sample times. With this inheritance rule for `Ts`, the following two models are equivalent.

```
tf(0.1,[1 -1],0.1) + tf(1,[1 0.5],-1)
```

and

```
tf(0.1,[1 -1],0.1) + tf(1,[1 0.5],0.1)
```

Note that

```
tf(0.1,[1 -1],0.1) + tf(1,[1 0.5],0.5)
```

returns an error message.

```
??? Error using ==> lti/plus
In SYS1+SYS2, both models must have the same sample time.
```

Caution: Resetting the sample time of a continuous-time LTI model `sys` from zero to a nonzero value does *not* discretize the original model `sys`. The command

```
set(sys,'Ts',0.1)
```

only affects the `Ts` property and does not alter the remaining model data. Use `c2d` and `d2c` to perform continuous-to-discrete and discrete-to-continuous conversions. For example, use

```
sysd = c2d(sys,0.1)
```

---

to discretize a continuous system `sys` at a 10Hz sampling rate.

Use `d2d` to change the sample time of a discrete-time system and resample it.

Input Names and Output Names

You can use the `InputName` and `OutputName` properties (in short, I/O names) to assign names to any or all of the input and output channels in your LTI model.

For example, you can create a SISO model with input `thrust`, output `velocity`, and transfer function $H(p) = 1/(p+10)$ by typing

```
h = tf(1,[1 10]);
set(h,'inputname','thrust','outputname','velocity',...
                           'variable','p')
```

Equivalently, you can set these properties directly by typing

```
h = tf(1,[1 10],'inputname','thrust',...
                'outputname','velocity',...
                'variable','p')
```

This produces

```
Transfer function from input "thrust" to output "velocity":
  1
------
p + 10
```

Note how the display reflects the input and output names and the variable selection.

In the MIMO case, use cell vectors of strings to specify input or output channel names. For example, type

```
num = {3 , [1 2]};
den = {[1 10] , [1 0]};
H = tf(num,den);        % H(s) has one output and two inputs

set(H,'inputname',{'temperature' ; 'pressure'})
```

The specified input names appear in the display of `H`.

```
Transfer function from input "temperature" to output:
  3
------
s + 10

Transfer function from input "pressure" to output:
```

```
s + 2
-----
  s
```

To leave certain names undefined, use the empty string `''` as in

```
H = tf(num,den,'inputname',{ 'temperature' ; '' })
```

Input Groups and Output Groups

In many applications, you may want to create several (distinct or intersecting) groups of input or output channels and name these groups. For example, you may want to label one set of input channels as `noise` and another set as `controls`.

To see how input and output groups (I/O groups) work:

1. Create a random state-space model with one state, three inputs, and three outputs.
2. Assign the first two inputs to a group named `controls`, the first output to a group named `temperature`, and the last two outputs to a group named `measurements`.

To do this, type

```
h = rss(1,3,3);
set(h, 'InputGroup',{[1 2] 'controls'})
set(h, 'OutputGroup', {[1] 'temperature'; [2 3] 'measurements'})
h
```

and MATLAB returns a state-space model of the following form.

```
a =
                    x1
           x1  -0.64884


b =
                    u1          u2          u3
           x1   0.12533           0           0
```

```
c =
                    x1
          y1      1.1909
          y2      1.1892
          y3           0


d =
                    u1          u2          u3
          y1     0.32729           0     -0.1364
          y2           0           0           0
          y3           0      2.1832           0

I/O Groups:
   Group Name     I/O    Channel(s)
    controls       I        1,2
  temperature      O         1
  measurements     O        2,3

Continuous-time model.
```

Notice that the middle column of the I/O group listing indicates whether the group is an input group (`I`) or an output group (`O`).

In general, to specify M input groups (or output groups), you need an M-by-2 cell array organized as follows.

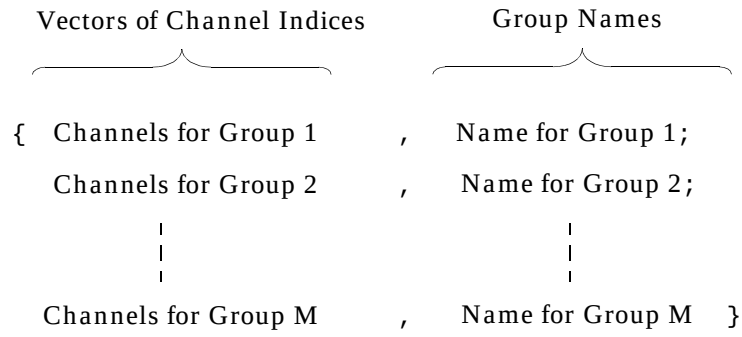


Figure 2-2: Two Column Cell Array

When you specify the cell array for input (or output) groups, keep in mind:

- Each row of this cell array designates a different input (output) group.
- You can add input (or output) groups by appending rows to the cell array.
- You can choose not to assign any of the group names when you assign the groups, and leave off the second column of this array. In that case,
  - Empty strings are assigned to the group names by default.
  - If you append rows to a cell array with no group names assigned, you have to assign empty strings (`''`) to the group names.

For example,

```
h.InputGroup = [h.InputGroup; {[3] 'disturbance'}];
```

adds another input group called `disturbance` to `h`.

You can use regular cell array syntax for accessing or modifying I/O group components. For example, to delete the first output group, `temperature`, type

```
h.OutputGroup(1,:) = []

ans =
    [1x2 double]    'measurements'
```

Similarly, you can add or delete channels from an existing input or output group. Recalling that input group channels are stored in the first column of the corresponding cell array, to add channel three to the input group `controls`, type

```
h.inputgroup{1,1} = [h.inputgroup{1,1} 3]
```

or, equivalently,

```
h.inputgroup{1,1} = [1 2 3]
```

Model Conversion

There are four LTI model types you can use with the Control System Toolbox: TF, ZPK, SS, and FRD. This section shows how to convert models from one type to the other.

Explicit Conversion

Model conversions are performed by `tf`, `ss`, `zpk`, and `frd`. Given any TF, SS, or ZPK model `sys`, the syntax for conversion to another model type is

```
sys = tf(sys)                  % Conversion to TF
sys = zpk(sys)                 % Conversion to ZPK
sys = ss(sys)                  % Conversion to SS
sys = frd(sys,frequency)       % Conversion to FRD
```

Notice that FRD models can't be converted to the other model types. In addition, you must also include a vector of frequencies (`frequency`) as an input argument when converting to an FRD model.

For example, you can convert the state-space model

```
sys = ss(-2,1,1,3)
```

to a zero-pole-gain model by typing

```
zpk(sys)
```

to which MATLAB responds

```
Zero/pole/gain:
3 (s+2.333)
-----------
   (s+2)
```

Note that the transfer function of a state-space model with data (A, B, C, D) is

$$H(s) = D + C(sI - A)^{-1}B$$

for continuous-time models, and

$$H(z) = D + C(zI - A)^{-1}B$$

for discrete-time models.

Automatic Conversion

Some algorithms operate only on one type of LTI model. For example, the algorithm for zero-order-hold discretization with `c2d` can only be performed on state-space models. Similarly, commands like `tfdata` expect one particular type of LTI models (TF). For convenience, such commands automatically convert LTI models to the appropriate or required model type. For example, in

```
sys = ss(0,1,1,0)
[num,den] = tfdata(sys)
```

`tfdata` first converts the state-space model `sys` to an equivalent transfer function in order to return numerator and denominator data.

Note that conversions to state-space models are not uniquely defined. For this reason, automatic conversions to state space are disabled when the result depends on the choice of state coordinates, for example, in commands like `initial` or `kalman`.

Caution About Model Conversions

When manipulating or converting LTI models, keep in mind that:

- The three LTI model types TF, ZPK, and SS, are not equally well-suited for numerical computations. In particular, the accuracy of computations using high-order transfer functions is often poor. Therefore, it is often preferable to work with the state-space representation. In addition, it is often beneficial to balance and scale state-space models using `ssbal`. You get this type of balancing automatically when you convert any TF or ZPK model to state space using `ss`.
- Conversions to the transfer function representation using `tf` may incur a loss of accuracy. As a result, the transfer function poles may noticeably differ from the poles of the original zero-pole-gain or state-space model.
- Conversions to state space are not uniquely defined in the SISO case, nor are they guaranteed to produce a minimal realization in the MIMO case. For a given state-space model `sys`,

```
ss(tf(sys))
```

may return a model with different state-space matrices, or even a different number of states in the MIMO case. Therefore, if possible, it is best to avoid converting back and forth between state-space and other model types.

Time Delays

Using the `ioDelay`, `InputDelay`, and `OutputDelay` properties of LTI objects, you can specify delays in both continuous- and discrete-time LTI models. With these properties, you can, for example, represent:

- LTI models with independent delays for each input/output pair. For example, the continuous-time model with transfer function

$$H(s) = \begin{bmatrix} e^{-0.1s}\,\dfrac{2}{s} & e^{-0.3s}\,\dfrac{s+1}{s+10} \\ 10 & e^{-0.2s}\,\dfrac{s-1}{s+5} \end{bmatrix}$$

- State-space models with delayed inputs and/or delayed outputs. For example,

$$\dot{x}(t) = Ax(t) + Bu(t-\tau)$$
$$y(t) = Cx(t-\theta) + Du(t-(\theta+\tau))$$

  where τ is the time delay between the input $u(t)$ and the state vector $x(t)$, and θ is the time delay between $x(t)$ and the output $y(t)$.

You can assign the delay properties `ioDelay`, `InputDelay`, and `OutputDelay` either when first creating your model with the `tf`, `zpk`, `ss`, or `frd` constructors, or later with the `set` command (see "LTI Properties and Methods" on page 2-4 for details).

Supported Functionality

Most analysis commands support time delays, including:

- All time and frequency response commands
- Conversions between model types
- Continuous-to-discrete conversions (`c2d`)
- Horizontal and vertical concatenation
- Series, parallel, and feedback interconnections of discrete-time models with delays

- Interconnections of continuous-time delay systems as long as the resulting transfer function from input j to output i is of the form $\exp(-s\tau_{ij})\ h_{ij}(s)$ where $h_{ij}(s)$ is a rational function of s
- Padé approximation of time delays (`pade`)

Specifying Input/Output Delays

Using the `ioDelay` property, you can specify frequency-domain models with independent delays in each entry of the transfer function. In continuous time, such models have a transfer function of the form

$$H(s) = \begin{bmatrix} e^{-s\tau_{11}}h_{11}(s) & \dots & e^{-s\tau_{1m}}h_{1m}(s) \\ \vdots & & \vdots \\ e^{-s\tau_{p1}}h_{p1}(s) & \dots & e^{-s\tau_{pm}}h_{pm}(s) \end{bmatrix} = [\exp(-s\tau_{ij})\ h_{ij}(s)]$$

where the h_{ij}'s are rational functions of s, and τ_{ij} is the time delay between input j and output i. See "Specifying Delays in Discrete-Time Models" on page 2-50 for details on the discrete-time counterpart. We collectively refer to the scalars τ_{ij} as the *I/O delays*.

The syntax to create $H(s)$ above is

```
H = tf(num,den,'ioDelay',Tau)
```

or

```
H = zpk(z,p,k,'ioDelay',Tau)
```

where

- `num`, `den` (respectively, `z`, `p`, `k`) specify the rational part $[h_{ij}(s)]$ of the transfer function $H(s)$
- `Tau` is the matrix of time delays for each I/O pair. That is, `Tau(i,j)` specifies the I/O delay τ_{ij} in seconds. Note that `Tau` and $H(s)$ should have the same row and column dimensions.

You can also use the `ioDelay` property in conjunction with state-space models, as in

```
sys = ss(A,B,C,D,'ioDelay',Tau)
```

This creates the LTI model with the following transfer function.

$$H(s) = \left[\exp(-s\tau_{ij})\ r_{ij}(s)\right]$$

Here $r_{ij}(s)$ is the (i, j) entry of

$$R(s) = D + C(sI - A)^{-1}B$$

Note State-space models with I/O delays have only a frequency-domain interpretation. They cannot, in general, be described by state-space equations with delayed inputs and outputs.

Distillation Column Example

This example is adapted from [2] and illustrates the use of I/O delays in process modeling. The process of interest is the distillation column depicted by the figure below. This column is used to separate a mix of methanol and water (the *feed*) into *bottom products* (mostly water) and a methanol-saturated *distillate*.

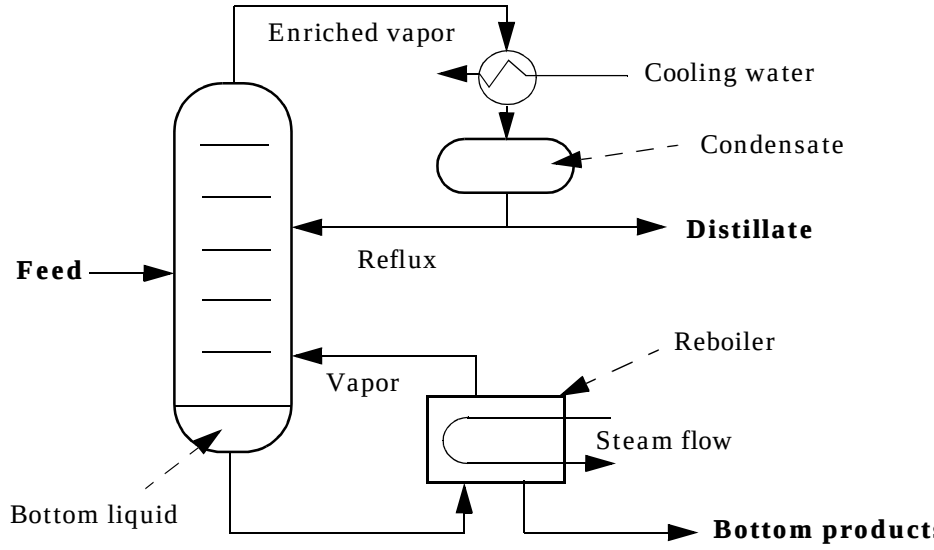


Figure 2-3: Distillation Column

Schematically, the distillation process functions as follows:

- Steam flows into the reboiler and vaporizes the bottom liquid. This vapor is reinjected into the column and mixes with the feed
- Methanol, being more volatile than water, tends to concentrate in the vapor moving upward. Meanwhile, water tends to flow downward and accumulate as the bottom liquid
- The vapor exiting at the top of the column is condensed by a flow of cooling water. Part of this condensed vapor is extracted as the distillate, and the rest of the condensate (the *reflux*) is sent back to the column.
- Part of the bottom liquid is collected from the reboiler as bottom products (waste).

The regulated output variables are:

- Percentage X_D of methanol in the distillate
- Percentage X_B of methanol in the bottom products.

The goal is to maximize X_D by adjusting the reflux flow rate R and the steam flow rate S in the reboiler.

To obtain a linearized model around the steady-state operating conditions, the transient responses to pulses in steam and reflux flow are fitted by first-order plus delay models. The resulting transfer function model is

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-1s}}{16.7e+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix}$$

Note the different time delays for each input/output pair.

You can specify this MIMO transfer function by typing

```
H = tf({12.8 -18.9;6.6 -19.4},...
       {[16.7 1] [21 1];[10.9 1] [14.4 1]},...
       'iodelay',[1 3;7 3],...
       'inputname',{'R' , 'S'},...
       'outputname',{'Xd' , 'Xb'})
```

The resulting TF model is displayed as

```
Transfer function from input "R" to output...
                    12.8
 Xd:  exp(-1*s) * ----------
                  16.7 s + 1

                     6.6
 Xb:  exp(-7*s) * ----------
                  10.9 s + 1

Transfer function from input "S" to output...
                   -18.9
 Xd:  exp(-3*s) * --------
                  21 s + 1

                    -19.4
 Xb:  exp(-3*s) * ----------
                  14.4 s + 1
```

Specifying Delays on the Inputs or Outputs

While ideal for frequency-domain models with I/O delays, the `ioDelay` property is inadequate to capture delayed inputs or outputs in state-space models. For example, the two models

$$(M_1)\ \begin{cases} \dot{x}(t) = -x(t) + u(t-0.1) \\ y(t) = x(t) \end{cases} \qquad (M_2)\ \begin{cases} \dot{z}(t) = -z(t) + u(t) \\ y(t) = z(t-0.1) \end{cases}$$

share the same transfer function

$$h(s) = \frac{e^{-0.1s}}{s+1}$$

As a result, they cannot be distinguished using the `ioDelay` property (the I/O delay value is 0.1 seconds in both cases). Yet, these two models have different state trajectories since $x(t)$ and $z(t)$ are related by

$$z(t) = x(t-0.1)$$

Note that the 0.1 second delay is on the *input* in the first model, and on the *output* in the second model.

InputDelay and OutputDelay Properties

When the state trajectory is of interest, you should use the `InputDelay` and `OutputDelay` properties to distinguish between delays on the inputs and delays on the outputs in state-space models. For example, you can accurately specify the two models above by

```
M1 = ss(-1,1,1,0,'inputdelay',0.1)
M2 = ss(-1,1,1,0,'outputdelay',0.1)
```

In the MIMO case, you can specify a different delay for each input (or output) channel by assigning a vector value to `InputDelay` (or `OutputDelay`). For example,

```
sys = ss(A,[B1 B2],[C1;C2],[D11 D12;D21 D22])
sys.inputdelay = [0.1 0]
sys.outputdelay = [0.2 0.3]
```

creates the two-input, two-output model

$$\dot{x}(t) = Ax(t) + B_1 u_1(t-0.1) + B_2 u_2(t)$$
$$y_1(t+0.2) = C_1 x(t) + D_{11} u_1(t-0.1) + D_{12} u_2(t)$$
$$y_2(t+0.3) = C_2 x(t) + D_{21} u_1(t-0.1) + D_{22} u_2(t)$$

You can also use the `InputDelay` and `OutputDelay` properties to conveniently specify input or output delays in TF, ZPK, or FRD models. For example, you can create the transfer function

$$H(s) = \begin{bmatrix} \frac{1}{s} \\ \frac{2}{s+1} \end{bmatrix} e^{-0.1s}$$

by typing

```
s = tf('s');
H = [1/s ; 2/(s+1)];   % rational part
H.inputdelay = 0.1
```

The resulting model is displayed as

```
Transfer function from input to output...
                    1
 #1:  exp(-0.1*s) * -
                    s

                      2
 #2:  exp(-0.1*s) * -----
                    s + 1
```

By comparison, to produce an equivalent transfer function using the `ioDelay` property, you would need to type

```
H = [1/s ; 2/(s+1)];
H.iodelay = [0.1 ; 0.1];
```

Notice that the 0.1 second delay is repeated twice in the I/O delay matrix. More generally, for a TF, ZPK, or FRD model with input delays $[\alpha_1, ..., \alpha_m]$ and output delays $[\beta_1, ..., \beta_p]$, the equivalent I/O delay matrix is

$$\begin{bmatrix} \alpha_1+\beta_1 & \alpha_2+\beta_1 & \dots & \alpha_m+\beta_1 \\ \alpha_1+\beta_2 & \alpha_2+\beta_2 & & \alpha_m+\beta_2 \\ \vdots & \vdots & & \vdots \\ \alpha_1+\beta_p & \alpha_2+\beta_p & \dots & \alpha_m+\beta_p \end{bmatrix}$$

Specifying Delays in Discrete-Time Models

You can also use the `ioDelay`, `InputDelay`, and `OutputDelay` properties to specify delays in discrete-time LTI models. You specify time delays in discrete-time models with integer multiples of the sampling period. The integer `k` you supply for the time delay of a discrete-time model specifies a time delay of k sampling periods. Such a delay contributes a factor z^{-k} to the transfer function.

For example,

```
h = tf(1,[1 0.5 0.2],0.1,'inputdelay',3)
```

produces the discrete-time transfer function

```
Transfer function:
                1
z^(-3) * -----------------
         z^2 + 0.5 z + 0.2

Sampling time: 0.1
```

Notice the `z^(-3)` factor reflecting the three-sampling-period delay on the input.

Mapping Discrete-Time Delays to Poles at the Origin

Since discrete-time delays are equivalent to additional poles at $z = 0$, they can be easily absorbed into the transfer function denominator or the state-space equations. For example, the transfer function of the delayed integrator

$$y[k+1] = y[k] + u[k-2]$$

is

$$H(z) = \frac{z^{-2}}{z-1}$$

You can specify this model either as the first-order transfer function $1/(z-1)$ with a delay of two sampling periods on the input

```
Ts = 1;    % sampling period
H1 = tf(1,[1 -1],Ts,'inputdelay',2)
```

or directly as a third-order transfer function:

```
H2 = tf(1,[1 -1 0 0],Ts)    % 1/(z^3-z^2)
```

While these two models are mathematically equivalent, `H1` is a more efficient representation both in terms of storage and subsequent computations.

When necessary, you can map all discrete-time delays to poles at the origin using the command `delay2z`. For example,

```
H2 = delay2z(H1)
```

absorbs the input delay in `H1` into the transfer function denominator to produce the third-order transfer function

```
Transfer function:
    1
---------
z^3 - z^2

Sampling time: 1
```

Note that

```
H2.inputdelay
```

now returns `0` (zero).

Retrieving Information About Delays

There are several ways to retrieve time delay information from a given LTI model `sys`:

- Use property display commands to inspect the values of the `ioDelay`, `InputDelay`, and `OutputDelay` properties. For example,

```
sys.iodelay
get(sys,'inputdelay')
```

- Use the helper function `hasdelay` to determine if `sys` has any delay at all. The syntax is

  ```
  hasdelay(sys)
  ```

  which returns `1` (true) if `sys` has any delay, and `0` (false) otherwise
- Use the function `totaldelay` to determine the total delay between each input and each output (cumulative contribution of the `ioDelay`, `InputDelay`, and `OutputDelay` properties). Type `help totaldelay` or see the Reference pages for details.

Padé Approximation of Time Delays

The function `pade` computes rational approximations of time delays in continuous-time LTI models. The syntax is

```
sysx = pade(sys,n)
```

where `sys` is a continuous-time model with delays, and the integer `n` specifies the Padé approximation order. The resulting LTI model `sysx` is of the same type as `sys`, but is delay free.

For models with multiple delays or a mix of input, output, and I/O delays, you can use the syntax

```
sysx = pade(sys,ni,no,nio)
```

where the vectors `ni` and `no`, and the matrix `nio` specify independent approximation orders for each input, output, and I/O delay, respectively. Set `ni=[]` if there are no input delays, and similarly for `no` and `nio`.

For example, consider the “Distillation Column Example” on page 2-45. The two-input, two-output transfer function in this example is

$$H(s) = \begin{bmatrix} \dfrac{12.8e^{-1s}}{16.7e+1} & \dfrac{-18.9e^{-3s}}{21.0s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}$$

To compute a Padé approximation of $H(s)$ using:

- A first-order approximation for the 1 second and 3 second delays
- A second-order approximation for the 7 second delay,

type

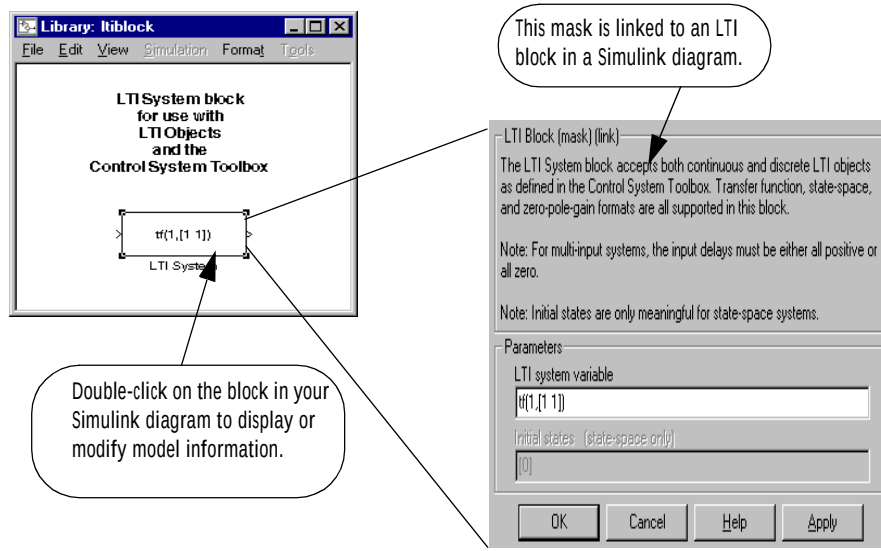
```
pade(H,[],[],[1 1;2 1])
```

where `H` is the TF representation of $H(s)$ defined in the distillation column example. This command produces a rational transfer function.

```
Transfer function from input "R" to output...
        -12.8 s + 25.6
 Xd:  ---------------------
      16.7 s^2 + 34.4 s + 2

             6.6 s^2 – 5.657 s + 1.616
 Xb:  ---------------------------------------
      10.9 s^3 + 10.34 s^2 + 3.527 s + 0.2449

Transfer function from input "S" to output...
          18.9 s – 12.6
 Xd:  ----------------------
      21 s^2 + 15 s + 0.6667

            19.4 s – 12.93
 Xb:  -------------------------
      14.4 s^2 + 10.6 s + 0.6667
```

Simulink Block for LTI Systems

You can incorporate LTI objects into Simulink diagrams using the LTI System block shown below.



The LTI System block can be accessed either by typing

```
ltiblock
```

at the MATLAB prompt or by selecting **Control System Toolbox** from the **Blocksets and Toolboxes** section of the main Simulink library.

The LTI System block consists of the dialog box shown on the right in the figure above. In the editable text box labeled **LTI system variable**, enter either the variable name of an LTI object located in the MATLAB workspace (for example, `sys`) or a MATLAB expression that evaluates to an LTI object (for example, `tf(1,[1 1])`). The LTI System block accepts both continuous and discrete LTI objects in either transfer function, zero-pole-gain, or state-space form. Simulink converts the model to its state-space equivalent prior to initializing the simulation.

Use the editable text box labeled **Initial states** to enter an initial state vector for state-space models. The concept of “initial state” is not well-defined for

transfer functions or zero-pole-gain models, as it depends on the choice of state coordinates used by the realization algorithm. As a result, you cannot enter nonzero initial states when you supply TF or ZPK models to LTI blocks in a Simulink diagram.

Note:

- For MIMO systems, the input delays stored in the LTI object must be either all positive or all zero.
- LTI blocks in a Simulink diagram cannot be used for FRD models or LTI arrays.

References

[1] Dorf, R.C. and R.H. Bishop, *Modern Control Systems*, Addison-Wesley, Menlo Park, CA, 1998.

[2] Wood, R.K. and M.W. Berry, "Terminal Composition Control of a Binary Distillation Column," *Chemical Engineering Science*, 28 (1973), pp. 1707-1717.

3

Operations on LTI Models

You can perform basic matrix operations such as addition, multiplication, or concatenation on LTI models. Such operations are "overloaded," which means that they use the same syntax as they do for matrices, but are adapted so as to apply to the LTI model context. These overloaded operations and their interpretation in this context are discussed in this chapter. You can read about discretization methods in this chapter as well. The following topics and operations on LTI models are covered in this chapter:

- Precedence and Property Inheritance
- Extracting and Modifying Subsystems
- Arithmetic Operations
- Model Interconnection Functions
- Continuous/Discrete-Time Conversions of LTI Models
- Resampling of Discrete-Time Models

These operations can be applied to LTI models of different types. As a result, before discussing operations on LTI models, we discuss model type precedence and how LTI model properties are inherited when models are combined using these operations. To read about how you can apply these operations to arrays of LTI models, see "Operations on LTI Arrays" on page 4-24. To read about the available functions with which you can analyze LTI models, see Chapter 5, "Model Analysis Tools,"

Precedence and Property Inheritance

You can apply operations to LTI models of different types. The resulting type is then determined by the rules discussed in “Precedence Rules” on page 2-5. For example, if `sys1` is a transfer function and `sys2` is a state-space model, then the result of their addition

```
sys = sys1 + sys2
```

is a state-space model, since state-space models have precedence over transfer function models.

To supersede the precedence rules and force the result of an operation to be a given type, for example, a transfer function (TF), you can either:

- Convert all operands to TF *before* performing the operation
- Convert the result to TF *after* performing the operation

Suppose, in the above example, you want to compute the transfer function of sys. You can either use *a priori* conversion of the second operand

```
sys = sys1 + tf(sys2);
```

or *a posteriori* conversion of the result

```
sys = tf(sys1 + sys2)
```

Note These alternatives are not equivalent numerically; computations are carried out on transfer functions in the first case, and on state-space models in the second case.

Another issue is property inheritance, that is, how the operand property values are passed on to the result of the operation. While inheritance is partly operation-dependent, some general rules are summarized below:

- In operations combining discrete-time LTI models, all models must have identical or unspecified (`sys.Ts = -1`) sample times. Models resulting from such operations inherit the specified sample time, if there is one.
- Most operations ignore the `Notes` and `Userdata` properties.

- In general, when two LTI models `sys1` and `sys2` are combined using operations such as `+`, `*`, `[,]`, `[;]`, `append`, and `feedback`, the resulting model inherits its I/O names and I/O groups from `sys1` and `sys2`. However, conflicting I/O names or I/O groups are not inherited. For example, the `InputName` property for `sys1 + sys2` is left unspecified if `sys1` and `sys2` have different `InputName` property values.
- A model resulting from operations on TF or ZPK models inherits its `Variable` property value from the operands. Conflicts are resolved according the following rules:
  - For continuous-time models, `'p'` has precedence over `'s'`.
  - For discrete-time models, `'z^-1'` has precedence over `'q'` and `'z'`, while `'q'` has precedence over `'z'`.

Extracting and Modifying Subsystems

Subsystems relate subsets of the inputs and outputs of a system. The transfer matrix of a subsystem is a submatrix of the system transfer matrix. For example, if sys is a system with two inputs, three outputs, and I/O relation

$$y = Hu$$

then $H(3, 1)$ gives the relation between the first input and third output.

$$y_3 = H(3,1)\ u_1$$

Accordingly, use matrix-like subindexing to extract this subsystem.

```
SubSys = sys(3,1)
```

The resulting subsystem SubSys is an LTI model of the same type as sys, with its sample time, time delay, I/O name, and I/O group property values inherited from sys.

For example, if sys has an input group named controls consisting of channels one, two, and three, then SubSys also has an input group named controls with the first channel of SubSys assigned to it.

If sys is a state-space model with matrices a, b, c, d, the subsystem sys(3,1) is a state-space model with data a, b(:,1), c(3,:), d(3,1). Note the following rules when extracting subsystems:

- In the expression sys(3,1), the first index selects the output channel while the second index selects the input channel.
- When extracting a subsystem from a given state-space model, the resulting state-space model may not be minimal. Use the command sminreal to eliminate unnecessary states in the subsystem.

You can use similar syntax to modify the LTI model sys. For example,

```
sys(3,1) = NewSubSys
```

redefines the I/O relation between the first input and third output, provided NewSubSys is a SISO LTI model.

The following rules apply when modifying LTI models:

- sys, the LTI model that has had a portion reassigned, retains its original model type (TF, ZPK, SS, or FRD) regardless of the model type of `NewSubSys`.
- Subsystem assignment does not reassign any I/O names or I/O group names of `NewSubSys` that are already assigned to `NewSubSys`.
- Reassigning parts of a MIMO state-space model generally increases its order.
- If `NewSubSys` is an FRD model, then sys must also be an FRD model. Furthermore, their frequencies must match.

Other standard matrix subindexing extends to LTI objects as well. For example,

```
sys(3,1:2)
```

extracts the subsystem mapping the first two inputs to the third output.

```
sys(:,1)
```

selects the first input and all outputs, and

```
sys([1 3],:)
```

extracts a subsystem with the same inputs, but only the first and third outputs.

For example, consider the two-input/two-output transfer function

$$T(s) = \begin{bmatrix} \dfrac{1}{s+0.1} & 0 \\ \dfrac{s-1}{s^2+2s+2} & \dfrac{1}{s} \end{bmatrix}$$

To extract the transfer function $T_{11}(s)$ from the first input to the first output, type

```
T(1,1)

Transfer function:
   1
-------
s + 0.1
```

Next reassign $T_{11}(s)$ to $1/(s+0.5)$ and modify the second input channel of `T` by typing

```
T(1,1) = tf(1,[1 0.5]);
T(:,2) = [ 1 ; tf(0.4,[1 0]) ]

Transfer function from input 1 to output...
         1
 #1:  -------
      s + 0.5

          s - 1
 #2:  -------------
      s^2 + 2 s + 2

Transfer function from input 2 to output...
 #1:  1

      0.4
 #2:  ---
       s
```

Referencing FRD Models Through Frequencies

You can extract subsystems from FRD models, as you do with other LTI model types, by indexing into input and output (I/O) dimensions. You can also extract subsystems by indexing into the frequencies of an FRD model.

To index into the frequencies of an FRD model, use the string *'Frequency'* (or any abbreviation, such as, *'freq'*, as long as it does not conflict with existing I/O channel or group names) as a keyword. There are two ways you can specify FRD models using frequencies:

- Using integers to index into the frequency vector of the FRD model
- Using a Boolean (logical) expression to specify desired frequency points in an FRD model

For example, if `sys` is an FRD model with five frequencies, (e.g., `sys.Frequency=[1 1.1 1.2 1.3 1.4]`), then you can create a new FRD model `sys2` by indexing into the frequencies of `sys` as follows.

```
sys2 = sys('frequency', 2:3);
```

```
sys2.Frequency

ans =
    1.1000
    1.2000
```

displays the second and third entries in the frequency vector.

Similarly, you can use logical indexing into the frequencies.

```
sys2 = sys('frequency',sys.Frequency >1.0 & sys.Frequency <1.15);
sys2.freq

ans =
    1.1000
```

You can also combine model extraction through frequencies with indexing into the I/O dimensions. For example, if `sys` is an FRD model with two inputs, two outputs, and frequency vector `[2.1 4.2 5.3]`, with `sys.Units` specified in rad/s, then

```
sys2 = sys(1,2,'freq',1)
```

specifies `sys2` as a SISO FRD model, with one frequency data point, 2.1 rad/s.

Referencing Channels by Name

You can also extract subsystems using I/O group or channel names. For example, if `sys` has an input group named `noise`, consisting of channels two, four, and five, then

```
sys(1,'noise')
```

is equivalent to

```
sys(1,[2 4 5])
```

Similarly, if `pressure` is the name assigned to an output channel of the LTI model `sys`, then

```
sys('pressure',1) = tf(1, [1 1])
```

reassigns the subsystem from the first input of `sys` to the output labeled `pressure`.

You can reference a set of channels by input or output name by using a cell array of strings for the names. For example, if sys has one output channel named pressure and one named temperature, then these two output channels can be referenced using

```
sys({'pressure','temperature'})
```

Resizing LTI Systems

Resizing a system consists of adding or deleting inputs and/or outputs. To delete the first two inputs, simply type

```
sys(:,1:2) = []
```

In deletions, at least one of the row/column indexes should be the colon (:) selector.

To perform input/output augmentation, you can proceed by concatenation or subassignment. Given a system sys with a single input, you can add a second input using

```
sys = [sys,h];
```

or, equivalently, using

```
sys(:,2) = h;
```

where h is any LTI model with one input, and the same number of outputs as sys. There is an important difference between these two options: while concatenation obeys the precedence rules (see page 2-5), subsystem assignment does not alter the model type. So, if sys and h are TF and SS objects, respectively, the first statement produces a state-space model, and the second statement produces a transfer function.

For state-space models, both concatenation and subsystem assignment increase the model order because they assume that sys and h have independent states. If you intend to keep the same state matrix and merely update the input-to-state or state-to-output relations, use set instead and modify the corresponding state-space data directly. For example,

```
sys = ss(a,b1,c,d1)
set(sys,'b',[b1 b2],'d',[d1 d2])
```

adds a second input to the state-space model sys by appending the *B* and *D* matrices. You should *simultaneously* modify both matrices with a single set command. Indeed, the statements

```
sys.b = [b1 b2]
```

and

```
set(sys,'b',[b1 b2])
```

cause an error because they create invalid intermediate models in which the *B* and *D* matrices have inconsistent column dimensions.

Arithmetic Operations

You can apply almost all arithmetic operations to LTI models, including those shown below.

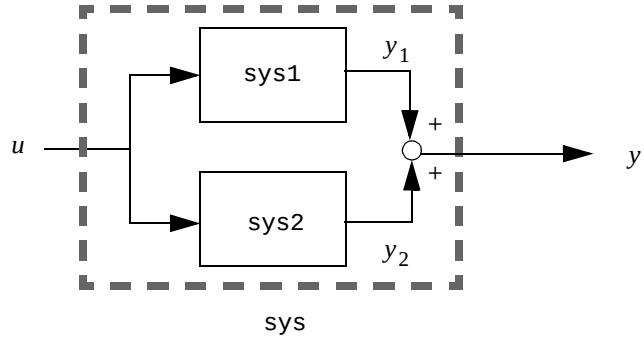
Operation	Description
`+`	Addition
`–`	Subtraction
`*`	Multiplication
`/`	Right matrix divide
`\`	Left matrix divide
`inv`	Matrix inversion
`'`	Pertransposition
`.'`	Transposition
`^`	Powers of an LTI model (as in `s^2`)

Addition and Subtraction

Adding LTI models is equivalent to connecting them in parallel. Specifically, the LTI model

```
sys = sys1 + sys2
```

represents the parallel interconnection shown below.



If `sys1` and `sys2` are two state-space models with data A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2, the state-space data associated with `sys1 + sys2` is

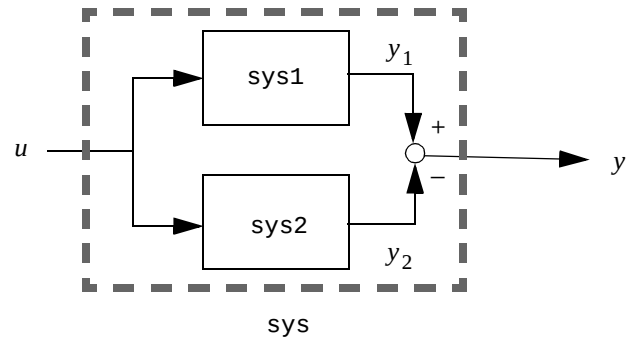
$$\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \qquad \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \qquad \begin{bmatrix} C_1 & C_2 \end{bmatrix}, \qquad D_1 + D_2$$

Scalar addition is also supported and behaves as follows: if `sys1` is MIMO and `sys2` is SISO, `sys1 + sys2` produces a system with the same dimensions as `sys1` whose *ij*th entry is `sys1(i,j) + sys2`.

Similarly, the subtraction of two LTI models

```
sys = sys1 - sys2
```

is depicted by the following block diagram.

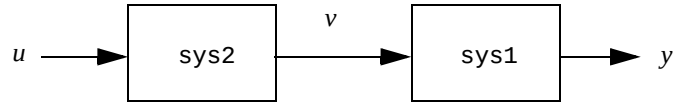


Multiplication

Multiplication of two LTI models connects them in series. Specifically,

```
sys = sys1 * sys2
```

returns an LTI model sys for the series interconnection shown below.



Notice the reverse orders of sys1 and sys2 in the multiplication and block diagram. This is consistent with the way transfer matrices are combined in a series connection: if sys1 and sys2 have transfer matrices H_1 and H_2, then

$$y = H_1 v = H_1(H_2 u) = (H_1 \times H_2)\, u$$

For state-space models sys1 and sys2 with data A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2, the state-space data associated with sys1*sys2 is

$$\begin{bmatrix} A_1 & B_1C_2 \\ 0 & A_2 \end{bmatrix}, \qquad \begin{bmatrix} B_1D_2 \\ B_2 \end{bmatrix}, \qquad \begin{bmatrix} C_1 & D_1C_2 \end{bmatrix}, \qquad D_1D_2$$

Finally, if sys1 is MIMO and sys2 is SISO, then sys1*sys2 or sys2*sys1 is interpreted as an entry-by-entry scalar multiplication and produces a system with the same dimensions as sys1, whose *ij*th entry is sys1(i,j)*sys2.

Inversion and Related Operations

Inversion of LTI models amounts to inverting the following input/output relationship.

$$y = H\, u \qquad \rightarrow \qquad u = H^{-1} y$$

This operation is defined only for square systems (that is, systems with as many inputs as outputs) and is performed using

```
inv(sys)
```

The resulting inverse model is of the same type as sys. Related operations include:

- Left division `sys1\sys2`, which is equivalent to `inv(sys1)*sys2`
- Right division `sys1/sys2`, which is equivalent to `sys1*inv(sys2)`

For a state-space model `sys` with data A, B, C, D, `inv(sys)` is defined only when D is a square invertible matrix, in which case its state-space data is

$$A - BD^{-1}C, \qquad BD^{-1}, \qquad -D^{-1}C, \qquad D^{-1}$$

Transposition

You can transpose an LTI model `sys` using

```
sys.'
```

This is a literal operation with the following effect:

- For TF models (with input arguments, `num` and `den`), the cell arrays `num` and `den` are transposed.
- For ZPK models (with input arguments, `z`, `p`, and `k`), the cell arrays, `z` and `p`, and the matrix `k` are transposed.
- For SS models (with model data A, B, C, D), transposition produces the state-space model A^T, C^T, B^T, D^T.
- For FRD models (with complex frequency response matrix `Response`), the matrix of frequency response data at each frequency is transposed.

Pertransposition

For a continuous-time system with transfer function $H(s)$, the *pertransposed* system has the transfer function

$$G(s) = [H(-s)]^T$$

The discrete-time counterpart is

$$G(z) = [H(z^{-1})]^T$$

Pertransposition of an LTI model `sys` is performed using

```
sys'
```

You can use pertransposition to obtain the Hermitian (conjugate) transpose of the frequency response of a given system. The frequency response of the

pertranspose of $H(s)$, $G(s) = [H(-s)]^T$, is the Hermitian transpose of the frequency response of $H(s)$: $G(jw) = H(jw)^H$.

To obtain the Hermitian transpose of the frequency response of a system `sys` over a frequency range specified by the vector `w`, type

```
freqresp(sys', w);
```

Model Interconnection Functions

The Control System Toolbox provides a number of functions to help with the model building process. These include model interconnection functions to perform I/O concatenation ([,], [;], and append), general parallel and series connections (parallel and series), and feedback connections (feedback and lft). These functions are useful to model open- and closed-loop systems.

Interconnection Operator	Description
[,]	Concatenates horizontally
[;]	Concatenates vertically
append	Appends models in a block diagonal configuration
augstate	Augments the output by appending states
connect	Forms an SS model from a block diagonal LTI object for an arbitrary interconnection matrix
feedback	Forms the feedback interconnection of two models
lft	Produces the LFT interconnection (Redheffer Star product) of two models
parallel	Forms the generalized parallel connection of two models
series	Forms the generalized series connection of two models

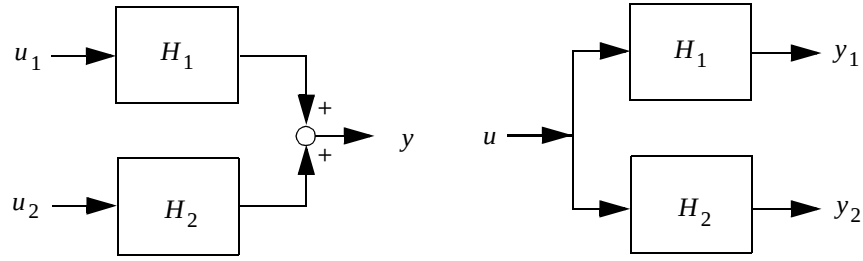
Concatenation of LTI Models

LTI model concatenation is done in a manner similar to the way you concatenate matrices in MATLAB, using

```
sys = [sys1 , sys2]% horizontal concatenation
sys = [sys1 ; sys2]% vertical concatenation
```

```
sys = append(sys1,sys2)% block diagonal appending
```

In I/O terms, horizontal and vertical concatenation have the following block-diagram interpretations (with H_1 and H_2 denoting the transfer matrices of sys1 and sys2).



$$y = \begin{bmatrix} H_1 , H_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Horizontal Concatenation

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} u$$

Vertical Concatenation

You can use concatenation as an easy way to create MIMO transfer functions or zero-pole-gain models. For example,

```
H = [ tf(1,[1 0])  1 ; 0  tf([1 -1],[1 1]) ]
```

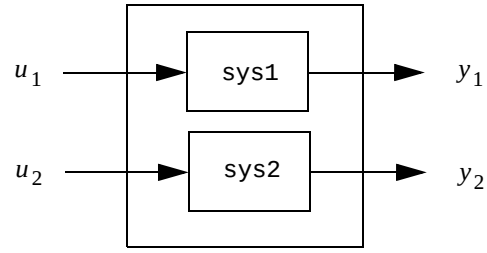
specifies

$$H(s) = \begin{bmatrix} \frac{1}{s} & 1 \\ 0 & \frac{s-1}{s+1} \end{bmatrix}$$

Use

```
append(sys1,sys2)
```

to specify the block-decoupled LTI model interconnection.



Appended Models

$$\begin{bmatrix} sys1 & 0 \\ 0 & sys2 \end{bmatrix}$$

Transfer Function

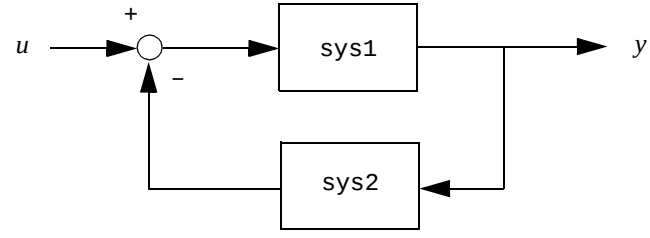
See append for more information on this function.

Feedback and Other Interconnection Functions

The following LTI model interconnection functions are useful for specifying closed- and open-loop model configurations:

- `feedback` puts two LTI models with compatible dimensions in a feedback configuration.
- `series` connects two LTI models in series.
- `parallel` connects two LTI models in parallel.
- `lft` performs the Redheffer star product on two LTI models.
- `connect` works with `append` to apply an arbitrary interconnection scheme to a set of LTI models.

For example, if `sys1` has *m* inputs and *p* outputs, while `sys2` has *p* inputs and *m* outputs, then the negative feedback configuration of these two LTI models



is realized with

```
feedback(sys1,sys2)
```

This specifies the LTI model with m inputs and p outputs whose I/O map is

$$(I + sys1 \cdot sys2)^{-1} sys1$$

See the reference pages online for more information on `feedback`, `series`, `parallel`, `lft`, and `connect`.

Continuous/Discrete Conversions of LTI Models

The function `c2d` discretizes continuous-time TF, SS, or ZPK models. Conversely, `d2c` converts discrete-time TF, SS, or ZPK models to continuous time. Several discretization/interpolation methods are supported, including zero-order hold (ZOH), first-order hold (FOH), Tustin approximation with or without frequency prewarping, and matched poles and zeros.

The syntax

```
sysd = c2d(sysc,Ts);     % Ts = sampling period in seconds
sysc = d2c(sysd);
```

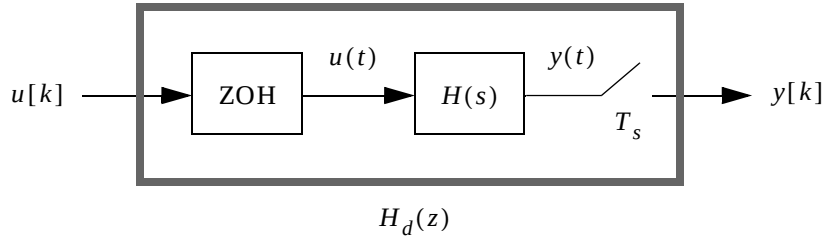
performs ZOH conversions by default. To use alternative conversion schemes, specify the desired method as an extra string input:

```
sysd = c2d(sysc,Ts,'foh');% use first-order hold
sysc = d2c(sysd,'tustin');% use Tustin approximation
```

The conversion methods and their limitations are discussed next.

Zero-Order Hold

Zero-order hold (ZOH) devices convert sampled signals to continuous-time signals for analyzing sampled continuous-time systems. The zero-order-hold discretization $H_d(z)$ of a continuous-time LTI model $H(s)$ is depicted in the following block diagram.



The ZOH device generates a continuous input signal $u(t)$ by holding each sample value $u[k]$ constant over one sample period.

$$u(t) = u[k], \qquad kT_s \le t \le (k+1)T_s$$

The signal $u(t)$ is then fed to the continuous system $H(s)$, and the resulting output $y(t)$ is sampled every T_s seconds to produce $y[k]$.

Conversely, given a discrete system $H_d(z)$, the d2c conversion produces a continuous system $H(s)$ whose ZOH discretization coincides with $H_d(z)$. This inverse operation has the following limitations:

- d2c cannot operate on LTI models with poles at $z = 0$ when the ZOH is used.
- Negative real poles in the z domain are mapped to *pairs* of complex poles in the s domain. As a result, the d2c conversion of a discrete system with negative real poles produces a continuous system with higher order.

The next example illustrates the behavior of d2c with real negative poles. Consider the following discrete-time ZPK model.

```
hd = zpk([],-0.5,1,0.1)

Zero/pole/gain:
   1
-------
(z+0.5)

Sampling time: 0.1
```

Use d2c to convert this model to continuous-time

```
hc = d2c(hd)
```

and you get a second-order model.

```
Zero/pole/gain:
   4.621 (s+149.3)
----------------------
(s^2 + 13.86s + 1035)
```

Discretize the model again

```
c2d(hc,0.1)
```

and you get back the original discrete-time system (up to canceling the pole/zero pair at z=–0.5):

```
Zero/pole/gain:
 (z+0.5)
```

```
---------
(z+0.5)^2

Sampling time: 0.1
```

First-Order Hold

First-order hold (FOH) differs from ZOH by the underlying hold mechanism. To turn the input samples $u[k]$ into a continuous input $u(t)$, FOH uses linear interpolation between samples.

$$u(t) = u[k] + \frac{t - kT_s}{T_s}(u[k+1] - u[k]), \qquad kT_s \le t \le (k+1)T_s$$

This method is generally more accurate than ZOH for systems driven by smooth inputs. Due to causality constraints, this option is only available for `c2d` conversions, and not `d2c` conversions.

Note This FOH method differs from standard causal FOH and is more appropriately called *triangle approximation* (see [2], p. 151). It is also known as *ramp-invariant* approximation because it is distortion-free for ramp inputs.

Tustin Approximation

The Tustin or bilinear approximation uses the approximation

$$z = e^{sT_s} \approx \frac{1 + sT_s/2}{1 - sT_s/2}$$

to relate s-domain and z-domain transfer functions. In `c2d` conversions, the discretization $H_d(z)$ of a continuous transfer function $H(s)$ is derived by

$$H_d(z) = H(s'), \text{ where } \qquad s' = \frac{2}{T_s}\frac{z-1}{z+1}$$

Similarly, the `d2c` conversion relies on the inverse correspondence

$$H(s) = H_d(z'), \text{ where } \qquad z' = \frac{1 + sT_s/2}{1 - sT_s/2}$$

Tustin with Frequency Prewarping

This variation of the Tustin approximation uses the correspondence

$$H_d(z) = H(s'), \qquad s' = \frac{\omega}{\tan(\omega T_s/2)} \frac{z-1}{z+1}$$

This change of variable ensures the matching of the continuous- and discrete-time frequency responses at the frequency ω.

$$H(j\omega) = H_d(e^{j\omega T_s})$$

Matched Poles and Zeros

The matched pole-zero method applies only to SISO systems. The continuous and discretized systems have matching DC gains and their poles and zeros correspond in the transformation

$$z = e^{sT_s}$$

See [2], p. 147 for more details.

Discretization of Systems with Delays

You can also use `c2d` to discretize SISO or MIMO continuous-time models with time delays. If `Ts` is the sampling period used for discretization:

- A delay of `tau` seconds in the continuous-time model is mapped to a delay of `k` sampling periods in the discretized model, where `k = fix(tau/Ts)`.
- The residual *fractional delay* `tau – k*Ts` is absorbed into the coefficients of the discretized model (for the zero-order-hold and first-order-hold methods only).

For example, to discretize the transfer function

$$H(s) = e^{-0.25s} \frac{10}{s^2 + 3s + 10}$$

using zero-order hold on the input, and a 10 Hz sampling rate, type

```
h = tf(10,[1 3 10],'inputdelay',0.25);
hd = c2d(h,0.1)
```

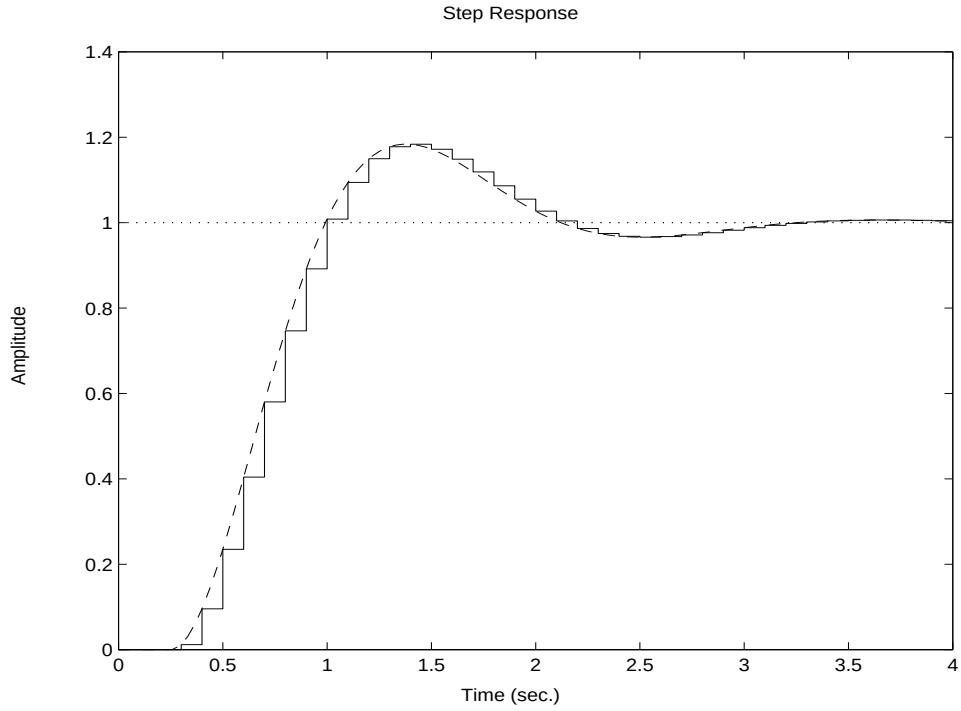
This produces the discrete-time transfer function

```
Transfer function:
         0.01187 z^2 + 0.06408 z + 0.009721
z^(-2) * ----------------------------------
             z^3 - 1.655 z^2 + 0.7408 z

Sampling time: 0.1
```

Here the input delay in $H(s)$ amounts to 2.5 times the sampling period of 0.1 seconds. Accordingly, the discretized model `hd` inherits an input delay of two sampling periods, as confirmed by the value of `hd.inputdelay`. The residual half-period delay is factored into the coefficients of `hd` by the discretization algorithm.

The step responses of the continuous and discretized models are compared in the figure below. This plot was produced by the command

```
step(h,'--',hd,'-')
```



Note The Tustin and matched pole/zero methods are accurate only for delays that are integer multiples of the sampling period. It is therefore preferable to use the `zoh` and `foh` discretization methods for models with delays.

Resampling of Discrete-Time Models

You can resample a discrete-time TF, SS, or ZPK model `sys1` by typing

```
sys2 = d2d(sys1,Ts)
```

The new sampling period `Ts` does not have to be an integer multiple of the original sampling period. For example, typing

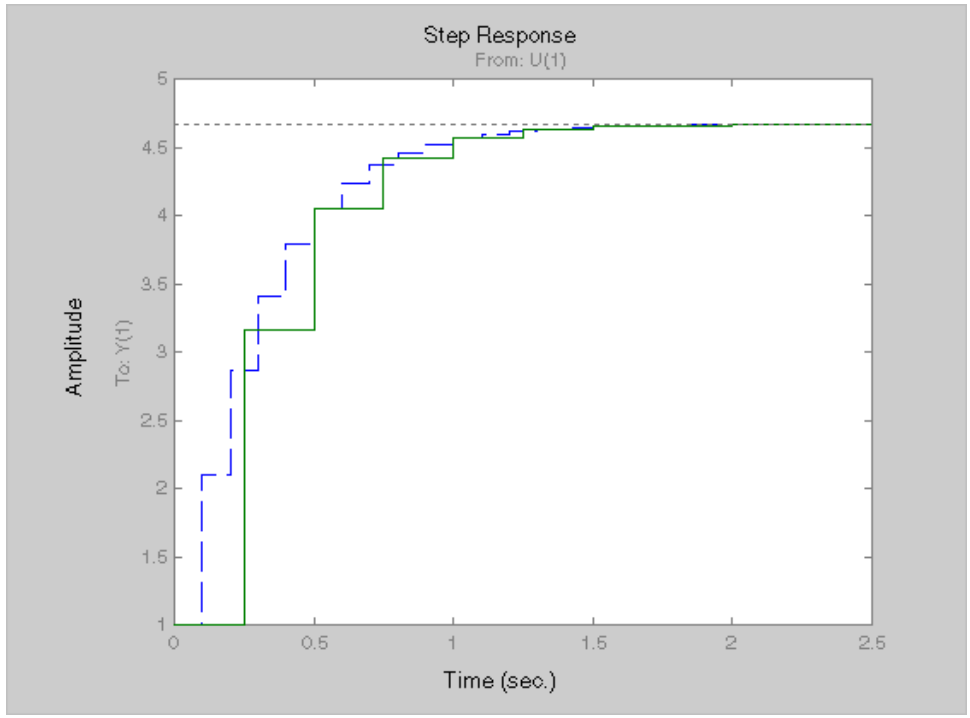
```
h1 = tf([1 0.4],[1 -0.7],0.1);
h2 = d2d(h1,0.25);
```

resamples `h1` at the sampling period of 0.25 seconds, rather than 0.1 seconds.

You can compare the step responses of `h1` and `h2` by typing

```
step(h1,'--',h2,'-')
```

The resulting plot is shown on the figure below (`h1` is the dashed line).



References

[1] Åström, K.J. and B. Wittenmark, *Computer-Controlled Systems: Theory and Design*, Prentice-Hall, 1990, pp. 48–52.

[2] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.

4

Model Analysis Tools

General Model Characteristics

General model characteristics include the model type, I/O dimensions, and continuous or discrete nature. Related commands are listed in the table below. These commands operate on continuous- or discrete-time LTI models or arrays of LTI models of any type.

General Model Characteristics Commands	
`class`	Display model type (`'tf'`, `'zpk'`, `'ss'`, or `'frd'`).
`hasdelay`	Test true if LTI model has any type of delay.
`isa`	Test true if LTI model is of specified class.
`isct`	Test true for continuous-time models.
`isdt`	Test true for discrete-time models.
`isempty`	Test true for empty LTI models.
`isproper`	Test true for proper LTI models.
`issiso`	Test true for SISO models.
`ndims`	Display the number of model/array dimensions.
`reshape`	Change the shape of an LTI array.
`size`	Output/input/array dimensions. Used with special syntax, `size` also returns the number of state dimensions for state-space models, and the number of frequencies in an FRD model.

This example illustrates the use of some of these commands. See the related reference pages for more details.

```
H = tf({1 [1 -1]},{[1 0.1] [1 2 10]})

Transfer function from input 1 to output:
   1
-------
s + 0.1
```

```
Transfer function from input 2 to output:
    s – 1
--------------
s^2 + 2 s + 10


class(H)

ans =
tf

size(H)
Transfer function with 2 input(s) and 1 output(s).

[ny,nu] = size(H)% Note: ny = number of outputs

ny =
     1

nu =
     2

isct(H)% Is this system continuous?

ans =
     1

isdt(H)% Is this system discrete?

ans =
     0
```

Model Dynamics

The Control System Toolbox offers commands to determine the system poles, zeros, DC gain, norms, etc. You can apply these commands to single LTI models or LTI arrays. The following table gives an overview of these commands.

Model Dynamics	
`covar`	Covariance of response to white noise.
`damp`	Natural frequency and damping of system poles.
`dcgain`	Low-frequency (DC) gain.
`dsort`	Sort discrete-time poles by magnitude.
`esort`	Sort continuous-time poles by real part.
`norm`	Norms of LTI systems (H_2 and L_∞).
`pole`, `eig`	System poles.
`pzmap`	Pole/zero map.
`zero`	System transmission zeros.

With the exception of L_∞ `norm`, these commands are not supported for FRD models.

Here is an example of model analysis using some of these commands.

```
h = tf([4 8.4 30.8 60],[1 4.12 17.4 30.8 60])

Transfer function:
     4 s^3 + 8.4 s^2 + 30.8 s + 60
---------------------------------------
s^4 + 4.12 s^3 + 17.4 s^2 + 30.8 s + 60

pole(h)

ans =
  -1.7971 + 2.2137i
```

```
  -1.7971 - 2.2137i
  -0.2629 + 2.7039i
  -0.2629 - 2.7039i

zero(h)
ans =
    -0.0500 + 2.7382i
    -0.0500 - 2.7382i
    -2.0000

dcgain(h)

ans =
     1

[ninf,fpeak] = norm(h,inf)% peak gain of freq. response

ninf =
1.3402     % peak gain

fpeak =
1.8537     % frequency where gain peaks
```

These functions also operate on LTI arrays and return arrays. For example, the poles of a three dimensional LTI array sysarray are obtained as follows.

```
sysarray = tf(rss(2,1,1,3))

Model sysarray(:,:,1,1)
=======================
Transfer function:
    -0.6201 s - 1.905
  ---------------------
  s^2 + 5.672 s + 7.405

Model sysarray(:,:,2,1)
=======================
Transfer function:
  0.4282 s^2 + 0.3706 s + 0.04264
  -------------------------------
      s^2 + 1.056 s + 0.1719
```

```
Model sysarray(:,:,3,1)
=======================
Transfer function:
    0.621 s + 0.7567
  ---------------------
  s^2 + 2.942 s + 2.113

3x1 array of continuous-time transfer functions.

pole(sysarray)
ans(:,:,1) =
   -3.6337
   -2.0379
ans(:,:,2) =
   -0.8549
   -0.2011
ans(:,:,3) =
   -1.6968
   -1.2452
```

State-Space Realizations

The following functions are useful to analyze, perform state coordinate transformations on, and derive canonical state-space realizations for single state-space LTI models or LTI arrays of state-space models.

State-Space Realizations	
`canon`	Canonical state-space realizations.
`ctrb`	Controllability matrix.
`ctrbf`	Controllability staircase form.
`gram`	Controllability and observability gramians.
`obsv`	Observability matrix.
`obsvf`	Observability staircase form.
`ss2ss`	State coordinate transformation.
`ssbal`	Diagonal balancing of state-space realizations.

The function `ssbal` uses a simple *diagonal* similarity transformation

$$(A, B, C) \rightarrow (T^{-1}AT, T^{-1}B, CT)$$

to balance the state-space data (A, B, C) . This is accomplished by reducing the norm of the matrix.

$$\begin{bmatrix} T^{-1}AT & T^{-1}B \\ CT & 0 \end{bmatrix}$$

Such balancing usually improves the numerical conditioning of subsequent state-space computations. Note that conversions to state-space using `ss` produce balanced realizations of transfer functions and zero-pole-gain models.

By contrast, the canonical realizations produced by `canon`, `ctrbf`, or `obsvf` are often badly scaled, sensitive to perturbations of the data, and

poorly suited for state-space computations. Consequently, it is wise to use them only for analysis purposes and not in control design algorithms.

5

Arrays of LTI Models

Introduction

In many applications, it is useful to consider collections of linear, time invariant (LTI) models. For example, you may want to consider a model with a single parameter that varies, such as

```
sys1 = tf(1, [1 1 1]);
sys2 = tf(1, [1 1 2]);
sys3 = tf(1, [1 1 3]);
```

and so on. A convenient way to store and analyze a collection like this is to use LTI arrays. Continuing this example, you can create this LTI array and store all three transfer functions in one variable.

```
sys_ltia = (sys1, sys2, sys3);
```

You can use the LTI array `sys_ltia` just like you would use, for example, sys1.

You can use LTI arrays to collect a set of LTI models into a single MATLAB variable. You then use this variable to manipulate or analyze the entire collection of models in a vectorized fashion. You access the individual models in the collection through indexing rather than by individual model names.

LTI arrays extend the concept of single LTI models in a similar way to how multidimensional arrays extend two-dimensional matrices in MATLAB (see Chapter 12, “Multidimensional Arrays” in *Using MATLAB*).

When to Collect a Set of Models in an LTI Array

You can use LTI arrays to represent:

- A set of LTI models arising from the linearization of a nonlinear system at several operating points
- A collection of transfer functions that depend on one or more parameters
- A set of LTI models arising from several system identification experiments applied to one plant
- A set of gain-scheduled LTI controllers
- A list of LTI models you want to collect together under the same name

Restrictions for LTI Models Collected in an Array

For each model in an LTI array, the following properties must be the same:

- The number of inputs and outputs
- The sample time, for discrete-time models
- The I/O names and I/O groups

Note You cannot specify Simulink LTI blocks with LTI arrays.

Where to Find Information on LTI Arrays

The next two sections give examples that illustrate the concept of an LTI array, its dimensions, and size. To read about how to build an LTI array, go to "Building LTI Arrays" on page 4-12. The remainder of the chapter is devoted to indexing and operations on LTI Arrays. You can also apply the analysis functions in the Control System Toolbox to LTI arrays. See Chapter 5, "Model Analysis Tools," for more information on these functions. You can also view response plots of LTI arrays with the LTI Viewer.

The Concept of an LTI Array

To visualize the concept of an LTI array, consider the set of five transfer function models shown below. In this example, each model has two inputs and two outputs. They differ by parameter variations in the individual model components.

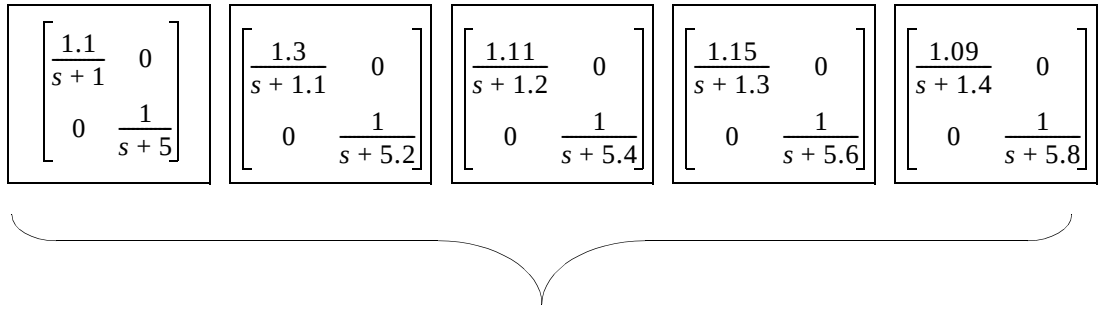


Figure 5-1: Five LTI Models to be Collected in an LTI Array

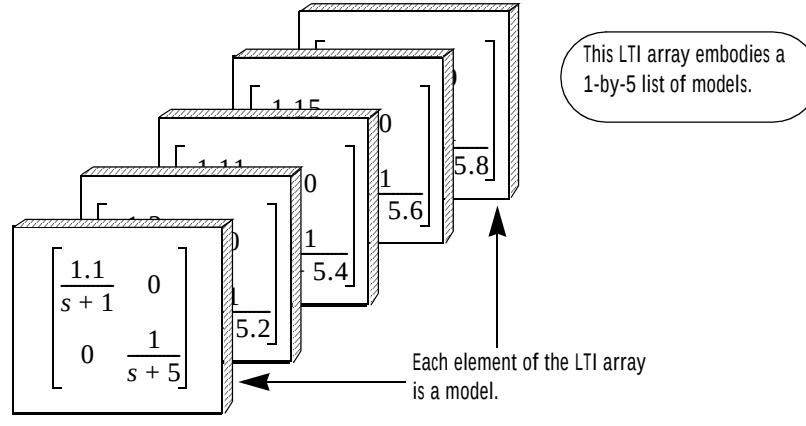
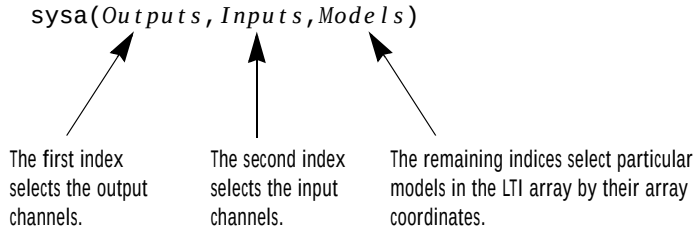


Figure 5-2: An LTI Array Containing These Five Models

Just as you might collect a set of two-by-two matrices in a multidimensional array, you can collect this set of five transfer function models as a list in an LTI array under one variable name, say, sys. Each element of the LTI array is an LTI model.

Individual models in the LTI array sys are accessed via indexing. The general form for the syntax you use to access data in an LTI array is



For example, you can access the third model in sys with sys(:,:,3). The following illustrates how you can use indexing to select models or their components from sys.

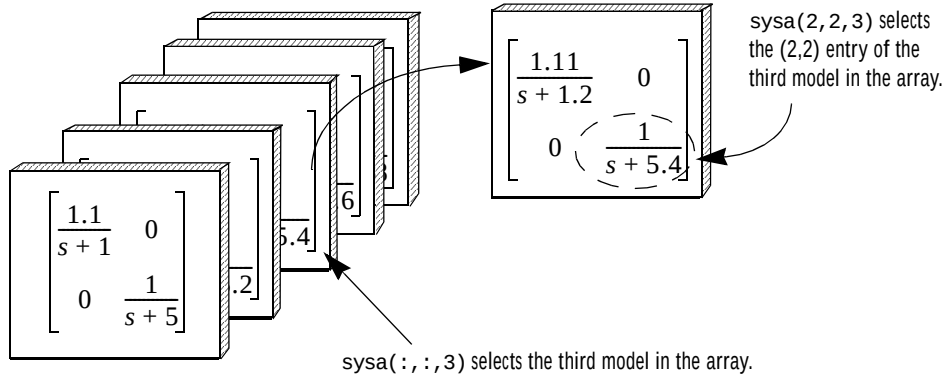


Figure 5-3: Using Indices to Select Models and Their Components

See “Indexing Into LTI Arrays” for more information on indexing.

Higher Dimensional Arrays of LTI Models

You can also collect a set of models in a two-dimensional array. The following diagram illustrates a 2-by-3 array of six, two-output, one-input models called `m2d`.

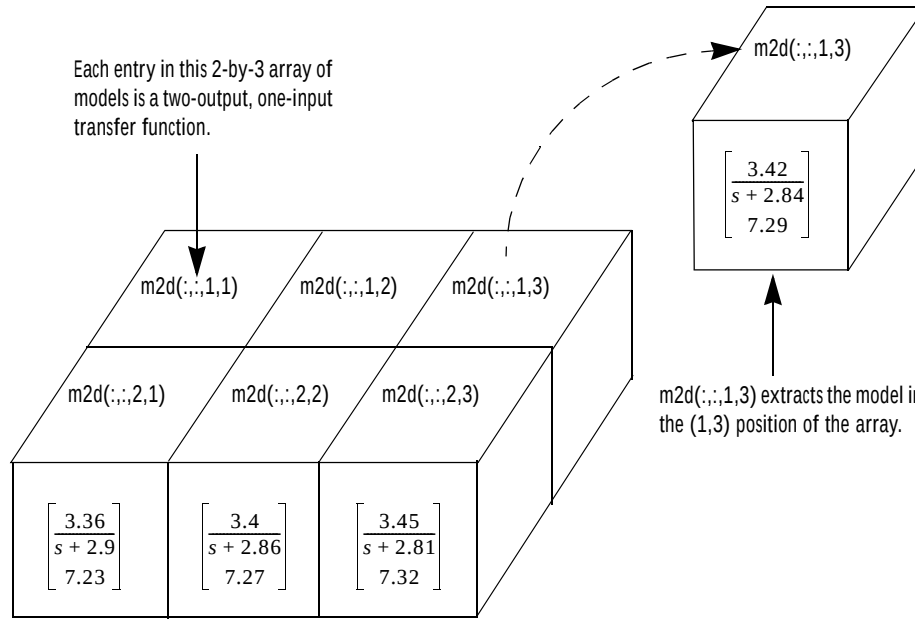


Figure 5-4: m2d: A 2-by-3 Array of Two-Output, One-Input Models

More generally, you can organize models into a 3-D or higher-dimensional array, in much the same way you arrange numerical data into multidimensional arrays (see “Multidimensional Arrays” in *Using MATLAB).*

Dimensions, Size, and Shape of an LTI Array

The dimensions and size of a single LTI model are determined by the output and input channels. An array of LTI models has additional quantities that determine its dimensions, size, and shape.

There are two sets of dimensions associated with LTI arrays:

- The *I/O dimensions*—the output dimension and input dimension common to all models in the LTI array
- The *array dimensions*—the dimensions of the array of models itself

The size of the LTI array is determined by:

- The lengths of the I/O dimensions—the number of outputs (or inputs) common to all models in the LTI array
- The length of each array dimension—the number of models along that array dimension

The next figure illustrates the concepts of dimension and size for the LTI array m2d, a 2-by-3 array of one-input, two-output transfer function models.

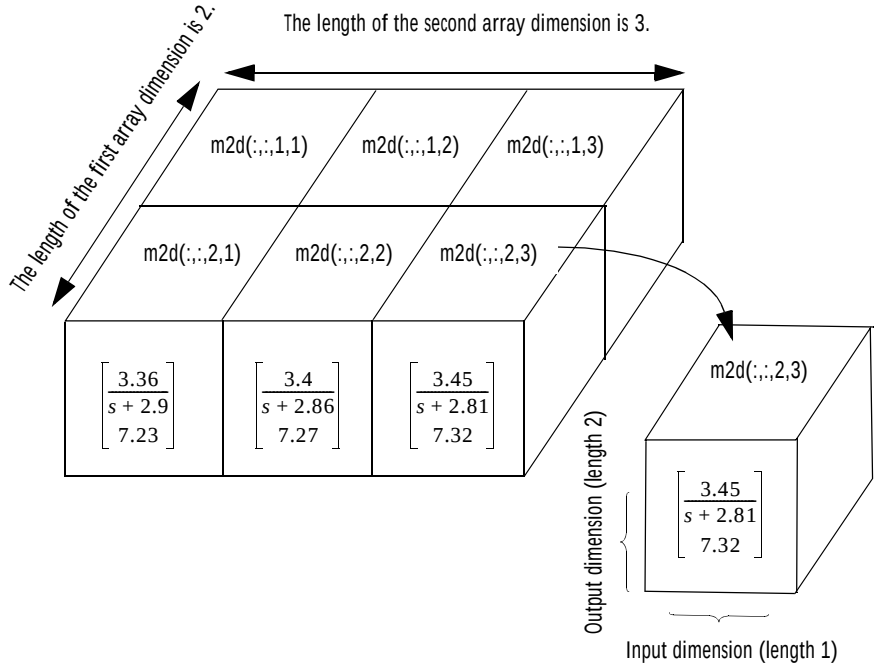


Figure 5-5: Dimensions and Size of m2d, an LTI Array

You can load this sample LTI array into your workspace by typing

```
load LTIexamples
size(m2d)

2x3 array of continuous-time transfer functions
Each transfer function has 2 outputs and 1 input.
```

The I/O dimensions correspond to the row and column dimensions of the transfer matrix. The two I/O dimensions are both of length 1 for SISO models. For MIMO models the lengths of these dimensions are given by the number of outputs and inputs of the model.

Five related quantities are pertinent to understanding the array dimensions:

- N, the number of models in the LTI array
- K, the number of array dimensions
- $S_1 S_2 \ldots S_K$, the list of lengths of the array dimensions
  - S_i is the number of models along the i^{th} dimension.
- S_1–by–S_2–by–…–by–S_K, the configuration of the models in the array
  - The configuration determines the shape of the array.
  - The product of these integers $S_1 \times S_2 \times \ldots \times S_K$ is N.

In the example model `m2d`,:

- The length of the output dimension, the first I/O dimension, is 2, since there are two output channels in each model.
- The length of the input dimension, the second I/O dimension, is 1, since there is only one input channel in each model.
- N, the number of models in the LTI array, is 6.
- K, the number of array dimensions, is 2.
- The array dimension lengths are [2 3].
- The array configuration is 2-by-3.

size and ndims

You can access the dimensions and shape of an LTI array using:

- `size` to determine the lengths of each of the dimensions associated with an LTI array
- `ndims` to determine the total number of dimensions in an LTI array

When applied to an LTI array, `size` returns

```
[Ny Nu S1 S2 ... Sk]
```

where

- `Ny` is the number of outputs common to all models in the LTI array.
- `Nu` is the number of inputs common to all models in the LTI array.
- `S1 S2 ... Sk` are the lengths of the array dimensions of a *k*-dimensional array of models. `Si` is the number of models along the *i*th array dimension.

Note the following when using the `size` function:

- By convention, a single LTI model is treated as a 1-by-1 array of models. For single LTI models, size returns only the I/O dimensions `[Ny Nu]`.
- For LTI arrays, size always returns at least two array dimensions. For example, the size of a 2-by-1 LTI array in `[Ny Nu 2 1]`
- `size` ignores trailing singleton dimensions beyond the second array dimension. For example, `size` returns `[Ny Nu 2 3]` for a 2-by-3-by-1-by-1 LTI array of models with `Ny` outputs and `Nu` inputs.

The function `ndims` returns the total number of dimensions in an LTI array:

- 2, for single LTI models
- 2 + `p`, for LTI arrays, where `p` (greater than 2) is the number of array dimensions

Note that

```
ndims (sys) = length(size(sys))
```

To see how these work on the sample 2-by-3 LTI array `m2d` of two-output, one-input models, type

```
load LTIexamples
s = size(m2d)

s =

     2     1     2     3
```

Notice that `size` returns a vector whose entries correspond to the length of each of the four dimensions of `m2d`: two outputs and one input in a 2-by-3 array of models. Type

```
ndims(m2d)

ans =
     4
```

to see that there are indeed four dimensions attributed to this LTI array.

reshape

Use `reshape` to reorganize the arrangement (array configuration) of the models of an existing LTI array.

For example, to arrange the models in an LTI Array `sys` as a $w_1 \times \ldots \times w_p$ array, type

```
reshape(sys,w1,...,wp)
```

where `w1,...,wp` are any set of integers whose product is *N*, the number of models in `sys`.

You can reshape the LTI array `m2d` into a 3-by-2, a 6-by-1, or a 1-by-6 array using `reshape`. For example, type

```
load LTIexamples
sys = reshape(m2d,6,1);
size(sys)

6x1 array of continuous-time transfer functions
Each transfer function has 2 outputs and 1 inputs.

s = size(sys)

s =
     2     1     6     1
```

Building LTI Arrays

There are several ways to build LTI arrays:

- Using a `for` loop to assign each model in the array
- Using `stack` to concatenate LTI models into an LTI array
- Using `tf`, `zpk`, `ss`, or `frd`

In addition, you can use the command `rss` to generate LTI arrays of random state-space models.

Generating LTI Arrays Using rss

A convenient way to generate arrays of state-space models with the same number of states in each model is to use `rss`. The syntax is

```
rss(N,P,M,sdim1,...,sdimk)
```

where

- `N` is the number of states of each model in the LTI array.
- `P` is the number of outputs of each model in the LTI array.
- `M` is the number of inputs of each model in the LTI array.
- `sdim1,...,sdimk` are the lengths of the array dimensions.

For example, to create a 4-by-2 array of random state-space models with three states, one output, and one input, type

```
sys = rss(3,2,1,4,2);
size(sys)

4x2 array of continuous-time state-space models
Each model has 2 outputs, 1 input, and 3 states.
```

Building LTI Arrays Using for Loops

Consider the following second-order SISO transfer function that depends on two parameters, ζ and ω

$$H(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$$

Suppose, based on measured input and output data, you estimate confidence intervals $[\omega_1,\omega_2]$, and $[\zeta_1,\zeta_2]$ for each of the parameters, ω and ζ. All of the possible combinations of the confidence limits for these model parameter values give rise to a set of four SISO models.

	ω_1	ω_2
ζ_1	$H_{11}(s)=\dfrac{\omega_1^2}{s^2+2\zeta_1\omega_1 s+\omega_1^2}$	$H_{12}(s)=\dfrac{\omega_1^2}{s^2+2\zeta_2\omega_1 s+\omega_1^2}$
ζ_2	$H_{21}(s)=\dfrac{\omega_2^2}{s^2+2\zeta_1\omega_2 s+\omega_2^2}$	$H_{22}(s)=\dfrac{\omega_2^2}{s^2+2\zeta_2\omega_2 s+\omega_2^2}$

Figure 5-6: Four LTI Models Depending on Two Parameters

You can arrange these four models in a 2-by-2 array of SISO transfer functions called `H`.

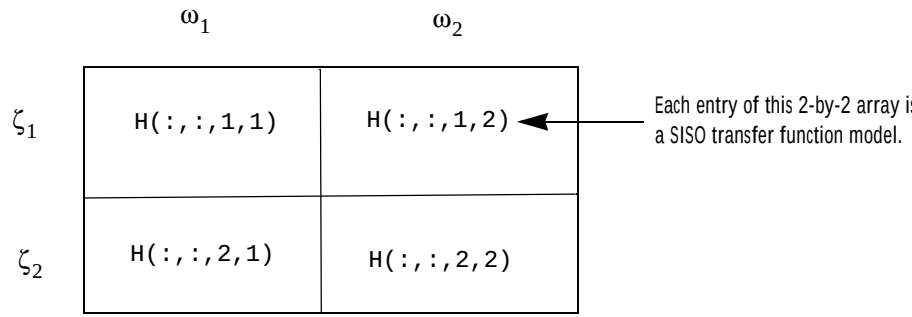


Figure 5-7: The LTI Array H

Here, for $i,j \in \{1,2\}$, `H(:,:,i,j)` represents the transfer function

$$\frac{\omega_j^2}{s^2+2\zeta_i\omega_j s+\omega_j^2}$$

corresponding to the parameter values $\zeta = \zeta_i$ and $\omega = \omega_j$.

The first two colon indices (:) select all I/O channels from the I/O dimensions of H. The third index of H refers to the first array dimension (ζ), while the fourth index is for the second array dimension (ω).

Suppose the limits of the ranges of values for ζ and ω are [0.66,0.76] and [1.2,1.5], respectively. Enter these at the command line.

```
zeta = [0.66,0.75];
w = [1.2,1.5];
```

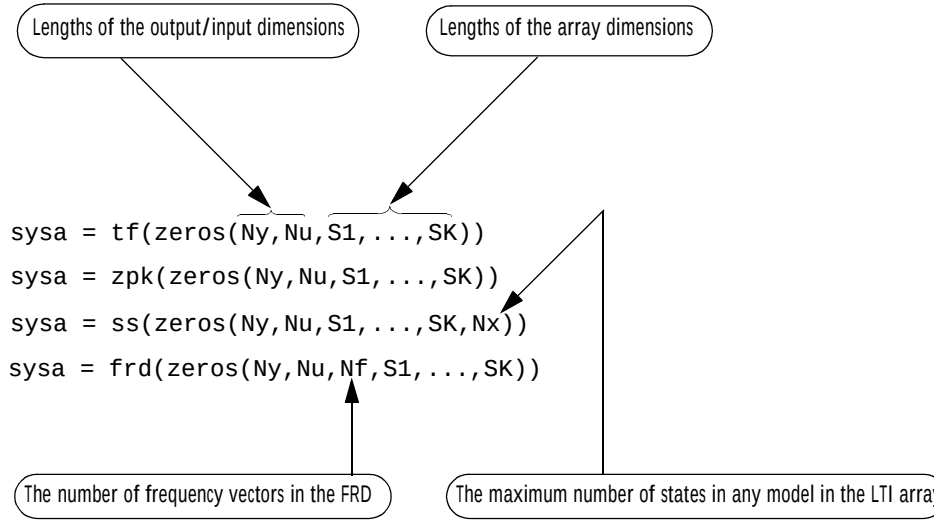
Since the four models have the same parametric structure, it's convenient to use two nested for loops to construct the LTI array.

```
for i = 1:2
  for j = 1:2
    H(:,:,i,j) = tf(w(j)^2,[1 2*zeta(i)*w(j) w(j)^2]);
  end
end
```

H now contains the four models in a 2-by-2 array. For example, to display the transfer function in the (1,2) position of the array, type

```
H(:,:,1,2)

Transfer function:
       2.25
-------------------
s^2 + 1.98 s + 2.25
```

For the purposes of efficient computation, you can initialize an LTI array to zero, and then reassign the entire array to the values you want to specify. The general syntax for zero assignment of LTI arrays is



To initialize `H` in the above example to zero, type

```
H = tf(zeros(1,1,2,2));
```

before you implement the nested `for` loops.

Building LTI Arrays Using the stack Function

Another way to build LTI arrays is using the function `stack`. This function operates on single LTI models as well as LTI arrays. It concatenates a list of LTI arrays or single LTI models only along the array dimension. The general syntax for `stack` is

```
stack(Arraydim,sys1,sys2...)
```

where

- `Arraydim` is the array dimension along which to concatenate the LTI models or arrays.
- `sys1`, `sys2`, ... are the LTI models or LTI arrays to be concatenated.

When you concatenate several models or LTI arrays along the *j*th array dimension, such as in

```
stack(j,sys1,sys2,...,sysn)
```

- The lengths of the I/O dimensions of `sys1,...,sysn` must all match.
- The lengths of all but the *j*th array dimension of `sys1,...,sysn` must match.

For example, if two TF models `sys1` and `sys2` have the same number of inputs and outputs,

```
sys = stack(1,sys1,sys2)
```

concatenates them into a 2-by-1 array of models.

There are two principles that you should keep in mind:

- `stack` only concatenates along an array dimension, not an I/O dimension.
- To concatenate LTI models or LTI arrays along an input or output dimension, use the bracket notation (`[,] [;]`). See "Model Interconnection Functions" for more information on the use of bracket notation to concatenate models. See also "Special Cases for Operations on LTI Arrays" for some examples of this type of concatenation of LTI arrays.

Here's an example of how to build the LTI array `H` using the function `stack`.

```
% Set up the parameter vectors.

zeta = [0.66,0.75];
w = [1.2,1.5];

% Specify the four individual models with those parameters.
%
H11 = tf(w(1)^2,[1 2*zeta(1)*w(1) w(1)^2]);
H12 = tf(w(2)^2,[1 2*zeta(1)*w(2) w(2)^2]);
H21 = tf(w(1)^2,[1 2*zeta(2)*w(1) w(1)^2]);
H22 = tf(w(2)^2,[1 2*zeta(2)*w(2) w(2)^2]);

% Set up the LTI array using stack.

COL1 = stack(1,H11,H21); % The first column of the 2-by-2 array
COL2 = stack(1,H12,H22); % The second column of the 2-by-2 array
H = stack(2, COL1, COL2); % Concatenate the two columns of models.
```

Notice that this result is very different from the single MIMO LTI model returned by

```
H = [H11,H12;H21,H22];
```

Building LTI Arrays Using tf, zpk, ss, and frd

You can also build LTI arrays using the `tf`, `zpk`, `ss`, and `frd` constructors. You do this by using multidimensional arrays in the input arguments for these functions.

Specifying Arrays of TF models tf

For TF models, use

```
sys = tf(num,den)
```

where

- Both `num` and `den` are multidimensional cell arrays the same size as `sys` (see "size and ndims" on page 4-9).
- `sys(i,j,n1,...,nK)` is the (i, j) entry of the transfer matrix for the model located in the $(n_1, \ldots, n_K)$ position of the array.
- `num(i,j,n1,...,nK)` is a row vector representing the numerator polynomial of `sys(i,j,n1,...,nK)`.
- `den(i,j,n1,...,nK)` is a row vector representing denominator polynomial of `sys(i,j,n1,...,nK)`.

See "MIMO Transfer Function Models" on page 2-10 for related information on the specification of single TF models.

Specifying Arrays of ZPK Models Using zpk

For ZPK models, use

```
sys = zpk(zeros,poles,gains)
```

where

- Both `zeros` and `poles` are multidimensional cell arrays whose cell entries contain the vectors of zeros and poles for each I/O pair of each model in the LTI array.

- gains is a multidimensional array containing the scalar gains for each I/O pair of each model in the array.
- The dimensions (and their lengths) of zeros, poles, and gains, determine those of the LTI array, sys.

Specifying Arrays of SS Models Using ss

To specify arrays of SS models, use

```
sys = ss(a,b,c,d)
```

where a, b, c, and d are real-valued multidimensional arrays of appropriate dimensions. All models in the resulting array of SS models have the same number of states, outputs, and inputs.

Note You cannot use the ss constructor to build an array of state-space models with different numbers of states. Use stack to build such LTI arrays.

The Size of LTI Array Data for SS Models

The size of the model data for arrays of state-space models is summarized in the following table.

Data	Size (Data)
a	$\left[N_s N_s S_1 S_2 \ldots S_K\right]$
b	$\left[N_s N_u S_1 S_2 \ldots S_K\right]$
c	$\left[N_y N_s S_1 S_2 \ldots S_K\right]$
d	$\left[N_y N_u S_1 S_2 \ldots S_K\right]$

where

- N_s is the maximum of the number of states in each model in the array.
- N_u is the number of inputs in each model.
- N_y is the number of outputs in each model.
- $S_1, S_2, \ldots, S_K$ are the lengths of the array dimensions.

Specifying Arrays of FRD Models Using frd

To specify a *K*-dimensional array of *p*-output, *m*-input FRD models for which $S_1, S_2, \ldots, S_K$ are the lengths of the array dimensions, use

```
sys = frd(response,frequency,units)
```

where

- `frequency` is a real vector of *n* frequency data points common to all FRD models in the LTI array.
- `response` is a *p*-by-*m*-by-*n*-*by*-S_1-by-…-by-S_K complex-valued multidimensional array.
- `units` is the optional string specifying `'rad/s'` or `'Hz'`.

Note that for specifying an LTI array of SISO FRD models, `response` can also be a multidimensional array of 1-by-*n* matrices whose remaining dimensions determine the array dimensions of the FRD.

Indexing Into LTI Arrays

You can index into LTI arrays in much the same way as you would for multidimensional arrays to:

- Access models
- Extract subsystems
- Reassign parts of an LTI array
- Delete parts of an LTI array

When you index into an LTI array sys, the indices should be organized according to the following format

sys(*Outputs*, *Inputs*, $n_1, \ldots, n_K$)

where

- *Outputs* are indices that select output channels.
- *Inputs* are indices that select input channels.
- $n_1, \ldots, n_K$ are indices into the array dimensions that select one model or a subset of models in the LTI array.

Note on Indexing into LTI Arrays of FRD models: For FRD models, the array indices can be followed by the keyword `'frequency'` and some expression selecting a subset of the frequency points as in

```
sys (outputs, inputs, n1,...,nk, 'frequency', SelectedFreqs)
```

See “Referencing FRD Models Through Frequencies” on page 3-7 for details on frequency point selection in FRD models.

Accessing Particular Models in an LTI Array

To access any given model in an LTI array:

- Use colon arguments (`:`, `:`) for the first two indices to select all I/O channels.

• The remaining indices specify the model coordinates within the array.

For example, if sys is a 5-by-2 array of state-space models defined by

```
sys = rss(4,3,2,5,2);
```

you can access (and display) the model located in the (3,2) position of the array sys by typing

```
sys(:,:,3,2)
```

If sys is a 5-by-2 array of 3-output, 2-input FRD models, with frequency vector [1,2,3,4,5], then you can access the response data corresponding to the middle frequency (3 rad/s), of the model in the (3,1) position by typing

```
sys(:,:,3,1,'frequency',3.0)
```

To access all frequencies of this model in the array, you can simply type

```
sys(:,:,3,1)
```

Single Index Referencing of Array Dimensions

You can also access models using single index referencing of the array dimensions.

For example, in the 5-by-2 LTI array sys above, you can also access the model located in the (3,2) position by typing

```
sys(:,:,8)
```

since this model is in the eighth position if you were to list the 10 models in the array by successively scanning through its entries along each of its columns.

For more information on single index referencing, see “Advanced Indexing” under “M-File Programming” in the MATLAB online documentation.

Extracting LTI Arrays of Subsystems

To select a particular subset of I/O channels from all the models in an LTI array, use the syntax described in “Extracting and Modifying Subsystems” on page 3-5. For example,

```
sys = rss(4,3,2,5,2);
A = sys(1, [1 2])
```

or equivalently,

```
A = sys(1,[1 2],:,:)
```

selects the first two input channels, and the first output channel in each model of the LTI array `A`, and returns the resulting 5-by-2 array of one-output, two-input subsystems.

You can also combine model selection with I/O selection within an LTI array. For example, to access both:

- The state-space model in the `(3,2)` array position
- Only the portion of that model relating the second input to the first output

type

```
sys(1,2,3,2)
```

To access the subsystem from all inputs to the first two output channels of this same array entry, type

```
sys(1:2,:,3,2)
```

Reassigning Parts of an LTI Array

You can reassign entire models or portions of models in an LTI array. For example,

```
sys = rss(4,3,2,5,2); % 5X2 array of state-space models
H = rss(4,1,1,5,2);   % 5X2 array of SISO models
sys(1,2) = H
```

reassigns the subsystem from input two to output one, for all models in the LTI array `sys`. This SISO subsystem of each model in the LTI array is replaced with the LTI array `H` of SISO models. This one-line assignment command is equivalent to the following 10-step nested `for` loop.

```
for k = 1:5
  for j = 1:2
    sys(1,2,k,j) = H(:,:,k,j);
  end
end
```

Notice that you don't have to use the array dimensions with this assignment. This is because I/O selection applies to all models in the array when the array indices are omitted.

Similarly, the commands

```
sys(:,:,3,2) = sys(:,:,4,1);
sys(1,2,3,2) = 0;
```

reassign the entire model in the (3,2) position of the LTI array sys and the (1,2) subsystem of this model, respectively.

Deleting Parts of an LTI Array

You can use indexing to delete any part of an LTI array by reassigning it to be empty ([]). For instance,

```
sys = rss(4,3,2,5,2);
sys(1,:) = [];
size(sys)

5x2 array of continuous-time state-space models
Each model has 2 outputs, 2 inputs, and 4 states.
```

deletes the first output channel from every model of this LTI array.

Similarly,

```
sys(:,:,[3 4],:) = []
```

deletes the third and fourth rows of this two-dimensional array of models.

Operations on LTI Arrays

Using LTI arrays, you can apply almost all of the basic model operations that work on single LTI models to entire sets of models at once. These basic operations include:

- The arithmetic operations: +, –, *, /,\, ', .'
- The functions: concatenation along I/O dimensions ([,], [;]), `feedback`, `append`, `series`, `parallel`, and `lft`

When you apply any of these operations to two (or more) LTI arrays (for example, `sys1` and `sys2`), the operation is implemented on a model-by-model basis. Therefore, the *k*th model of the resulting LTI array is derived from the application of the given operation to the *k*th model of `sys1` and the *k*th model of `sys2`.

For example, if `sys1` and `sys2` are two LTI arrays and

```
sysa = op(sys1,sys2)
```

then the *k*th model in the resulting LTI array `sys` is obtained by adding the *k*th models in `sys1` to the *k*th model in `sys2`

```
sysa(:,:,k) = sys1(:,:,k) + sys2(:,:,k)
```

You can also apply any of the response plotting functions such as `step`, `bode`, and `nyquist` to LTI arrays. These plotting functions are also applied on a model by model basis.

Example: Addition of Two LTI Arrays

The following diagram illustrates the addition of two 3-by-1 LTI arrays `sys1+sys2`.

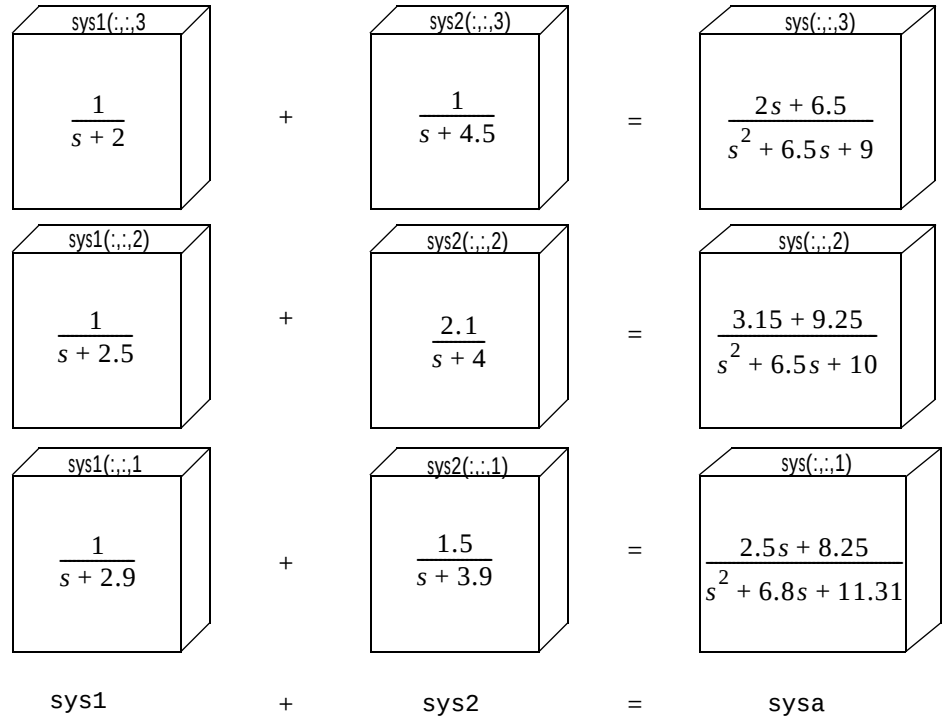


Figure 5-8: The Addition of Two LTI Arrays

The summation of these LTI arrays

```
sysa = sys1+sys2
```

is equivalent to the following model-by-model summation.

```
for k = 1:3
  sysa(:,:,k)=sys1(:,:,k) + sys2(:,:,k)
end
```

Note that:

- Each model in sys1 and sys2 must have the same number of inputs and outputs. This is required for the addition of two LTI arrays.
- The lengths of the array dimensions of sys1 and sys2 must match.

Dimension Requirements

In general, when you apply any of these basic operations to two or more LTI arrays:

- The I/O dimensions of each of the LTI arrays must be compatible with the requirements of the operation.
- The lengths of array dimensions must match.

The I/O dimensions of each model in the resulting LTI array are determined by the operation being performed. See Chapter 3, "Operations on LTI Models," for requirements on the I/O dimensions for the various operations.

For example, if sys1 and sys2 are both 1-by-3 arrays of LTI models with two inputs and two outputs, and sys3 is a 1-by-3 array of LTI models with two outputs and 1 input, then

```
sys1 + sys2
```

is an LTI array with the same dimensions as sys1 and sys2.

```
sys1 * sys3
```

is a 1-by-3 array of LTI models with two outputs and one input, and

```
[sys1,sys3]
```

is a 1-by-3 array of LTI models with two outputs and three inputs.

Special Cases for Operations on LTI Arrays

There are some special cases in coding operations on LTI arrays.

Consider

```
sysa = op(sys1,sys2)
```

where op is a symbol for the operation being applied. sys1 is an LTI array, and sysa (the result of the operation) is an LTI array with the same array

dimensions as sys1. You can use shortcuts for coding sysa = op(sys1,sys2) in the following cases:

- For operations that apply to LTI arrays, sys2 does not have to be an array. It can be a single LTI model (or a gain matrix) whose I/O dimensions satisfy the compatibility requirements for op (with those of each of the models in sys1). In this case, op applies sys2 to each model in sys1, and the *k*th model in sys satisfies

  ```
  sysa(:,:,k) = op(sys1(:,:,k),sys2)
  ```

- For arithmetic operations, such as +, *, /, and \, sys2 can be either a single SISO model, or an LTI array of SISO models, even when sys1 is an LTI array of MIMO models. This special case relies on MATLAB's scalar expansion capabilities for arithmetic operations.
  - When sys2 is a single SISO LTI model (or a scalar gain), op applies sys2 to sys1 on an entry-by-entry basis. The *ij*th entry in the *k*th model in sysa satisfies

    ```
    sysa(i,j,k) = op(sys1(i,j,k),sys2)
    ```

  - When sys2 is an LTI array of SISO models (or a multidimensional array of scalar gains), op applies sys2 to sys1 on an entry-by-entry basis for each model in sysa.

    ```
    sysa(i,j,k) = op(sys1(i,j,k),sys2(:,:,k))
    ```

Examples of Operations on LTI Arrays with Single LTI Models

Suppose you want to create an LTI array containing three models, where, for τ in the set $\{1.1, 1.2, 1.3\}$, each model $H_\tau(s)$ has the form

$$H_\tau(s) = \begin{bmatrix} \frac{1}{s+\tau} & 0 \\ -1 & \frac{1}{s} \end{bmatrix}$$

You can do this efficiently by first setting up an LTI array h containing the SISO models $1/(s+\tau)$ and then using concatenation to form the LTI array H of MIMO LTI models $H_\tau(s)$, $\tau \in \{1.1, 1.2, 1.3\}$. To do this, type

```
tau = [1.1 1.2 1.3];
for i=1:3                    % Form LTI array h of SISO models.
```

```
  h(:,:,i)=tf(1,[1 tau]);
end
H = [h 0; -1 tf(1,[1 0])]; %Concatenation: array h & single models
size(H)

3x1 array of continuous-time transfer functions
Each transfer function has 2 output(s) and 2 input(s).
```

Similarly, you can use append to perform the diagonal appending of each model in the SISO LTI array h with a fixed single (SISO or MIMO) LTI model.

```
S = append(h,tf(1,[1 3])); % Append a single model to h.
```

specifies an LTI array S in which each model has the form

$$S_\tau(s) = \begin{bmatrix} \frac{1}{s+\tau} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

You can also combine an LTI array of MIMO models and a single MIMO LTI model using arithmetic operations. For example, if h is the LTI array of three SISO models defined above,

```
[h,h] + [tf(1,[1 0]);tf(1,[1 5])]
```

adds the single one-output, two-input LTI model [1/s 1/(s + 5)] to every model in the 3-by-1 LTI array of one-output, two-input models [h,h]. The result is a new 3-by-2 array of models.

Examples: Arithmetic Operations on LTI Arrays and SISO Models

Using the LTI array of one-output, two-input state-space models [h,h], defined in the previous example,

```
tf(1,[1 3]) + [h,h]
```

adds a single SISO transfer function model to each entry in each model of the LTI array of MIMO models [h,h].

Finally,

```
G = rand(1,1,3,1);
sysa = G + [h,h]
```

adds the array of scalars to each entry of each MIMO model in the LTI array `[h,h]` on a model-by-model basis. This last command is equivalent to the following `for` loop.

```
hh = [h,h];
for k = 1:3
  sysa(:,:,k) = G(1,1,k) + hh(:,:,k);
end
```

Other Operations on LTI Arrays

You can also apply the analysis functions, such as `bode`, `nyquist`, and `step`, to LTI arrays.

6

Customization

The Control System Toolbox provides editors that allow you to set properties and preferences in the SISO Design Tool, the LTI Viewer, and in any response plots that you create from the MATLAB prompt.

Properties refer to settings that are specific to an individual response plot. This includes the following:

- Axes labels, and limits
- Data units and scales
- Plot styles, such as grids, fonts, and axes foreground colors
- Plot characteristics, such as rise time, peak response, and gain and phase margins.

Preferences refers to properties that persist either:

- Within a single session for a specific instance of an LTI Viewer or a SISO Design Tool
- Across Control System Toolbox sessions

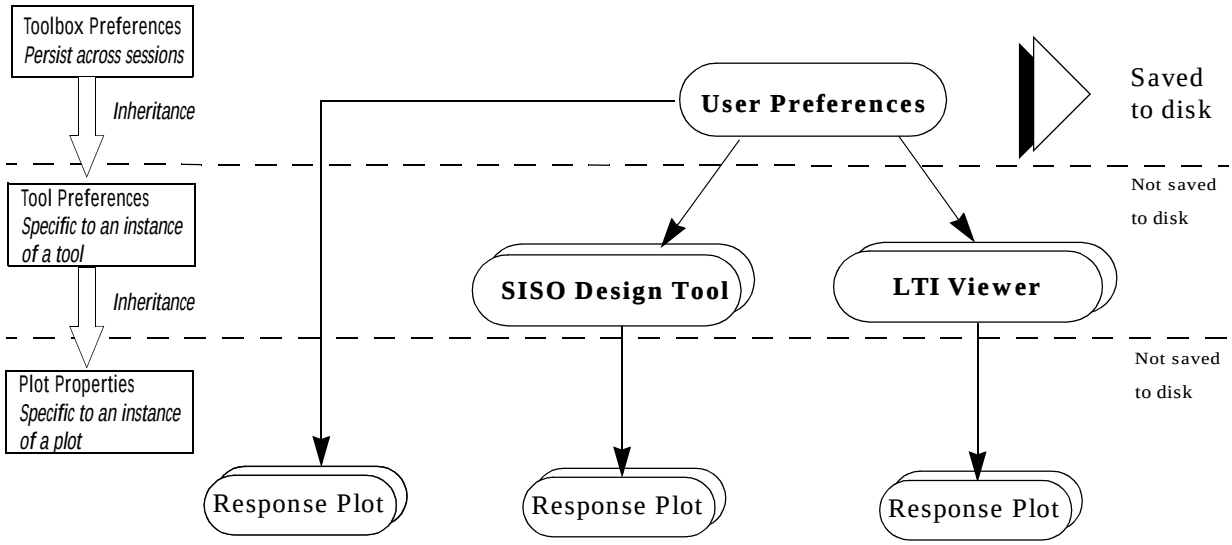
The former are called *tool preferences*, the latter *toolbox preferences*.

This document contains five sections:

- "Setting Toolbox Preferences" — Using the Toolbox Preferences Editor, you can set features that apply to all LTI Viewers, SISO Design Tools, and response plots you create. Settings here persist from session to session.
- "Setting Tool Preferences" — Using either the SISO Tool Editor or LT Viewer Editor, you can set features that apply to individual instances of LTI Viewers and SISO Design Tools.
- "Customizing Response Plot Properties" — Using the Property Editor, you can set features that apply to individual instances of response plots
  - "Property Editing for Subplots" — How to edit subplots individually using the Property Editor.
  - "Customizing Plots Inside the SISO Design Tool" — How to use the Property Editor specific to the SISO Design Tool.

The Property and Preferences Hierarchy

This diagram explains the hierarchy from properties, which are local, to toolbox preferences, which are global and persist from session to session.



7

Setting Toolbox Preferences

The Toolbox Preferences Editor allows you to set plot preferences that will persist from session to session. This is the highest level shown in “The Property and Preferences Hierarchy”.

Opening the Toolbox Preferences Editor

To open the Toolbox Preferences Editor, select **Toolbox Preferences** under the **File** menu of the LTI Viewer or the SISO Design Tool. Alternatively, you can type

```
ctrlpref
```

at the MATLAB prompt.

Note To get help on pages in the Control System Toolbox Preferences editor, click on the page tabs below.

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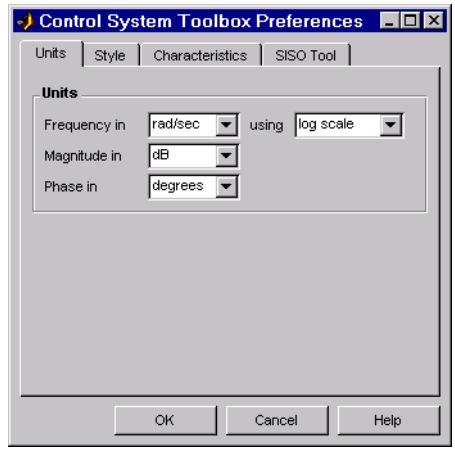
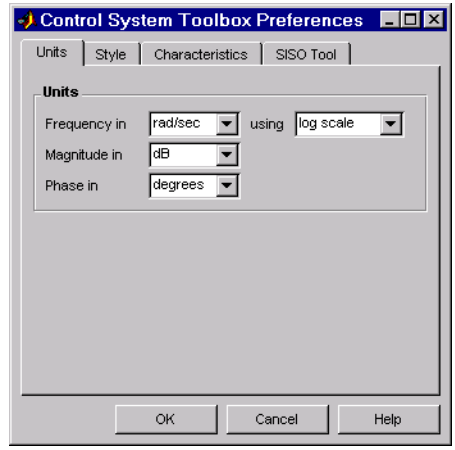


Figure 7-1: The Control System Toolbox Preferences Editor

Units Page

Note To get help on pages in the Control System Toolbox Preferences editor, click on the page tabs below.



Use the **Units** page to set preferences for the following:

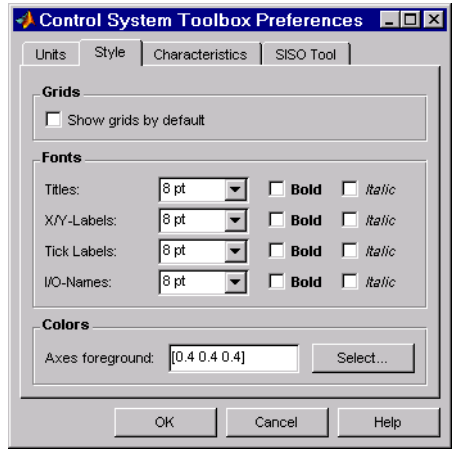
- Frequency — Radians per second (rad/sec) or Hertz (Hz)
- Magnitude — Decibels (dB) or absolute value (abs)
- Phase — Degrees or radians

For frequency and magnitude axes, you can select logarithmic or linear scales.

Style Page

Note Click on the page tabs below to get help on pages in the Control System Toolbox Preferences editor.

Use the **Style** page to toggle grid visibility and set font preferences and axes foreground colors for all plots you create using the Control System Toolbox. This figure shows the **Style** page.



You have the following choices:

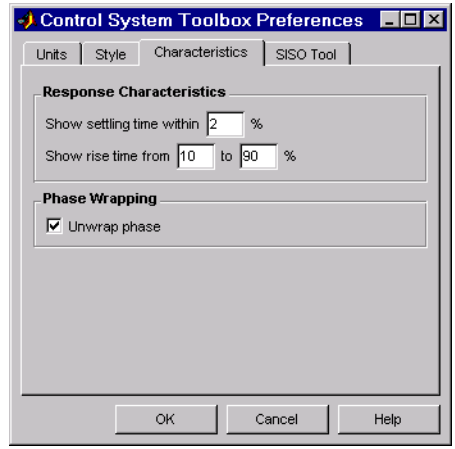
- Grid — Activate grids by default in new plots
- Font preferences — Set the font size, weight (bold), and angle (italic)
- Colors — Specify the color vector to use for the axes foreground, which includes the X-Y axes, grid lines, and tick labels. Use a three-element vector to represent red, green, and blue (RGB) values. Vector element values can range from 0 to 1.

  If you do not want to specify RGB values numerically, press the **Select** button to open the **Select Colors** window. See “Select colors” for more information.

Characteristics Page

Note Click on the page tabs below to get help on pages in the Control System Toolbox Preferences editor.

The Characteristics page has selections for response characteristics and phase wrapping. This figure shows the Characteristics page with default settings.



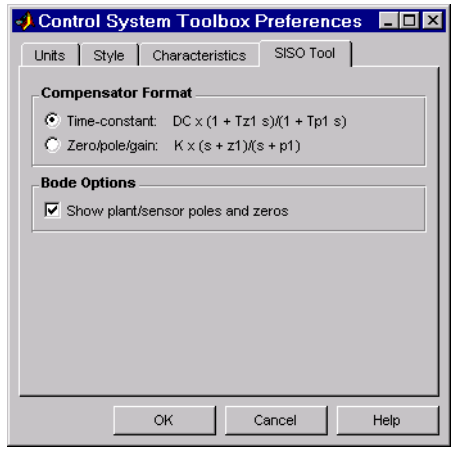
The following are the available options for the Characteristics page:

- **Response Characteristics**:
  - Specify settling time tolerance — You can set the threshold of the settling time calculation to any percentage from 0 to 100%. The default is 2%.
  - Specify rise time boundaries — The standard definition of rise time is the time it takes the signal to go from 10% to 90% of the final value. You can choose any percentages you like (from 0% to 100%), provided that the first value is smaller than the second.
- **Phase Wrapping** — By default, the phase is not wrapped. Wrap the phrase by unchecking this box. If the phase is wrapped, all phase values are shifted such that their equivalent value displays in the range [-180°, 180°).

SISO Tool Page

Note Click on the page tabs below to get help on pages in the Control System Toolbox Preferences editor.

The SISO Tool page has settings for the SISO Design Tool. This figure shows the SISO Tool page with default settings.



You can make the following selections:

- **Compensator Format** — You can select either the time-constant format or the zero/pole/gain format. The time-constant format is

$$dcgain \times \frac{(1 + Tz_1 s)}{(1 + Tp_1 s)} \ldots$$

  where Tz_1, Tz2, ..., are the zero time constants, and Tp_1, Tp_2, ..., are the pole time constants.

  The zero/pole/gain format is a variation on the time-constant format.

$$K \times \frac{(s + z_1)}{(s + p_1)}$$

  In this case, the gain is compensator gain; z_1, z_2, ... and p_1, p_2, ..., are the zero and pole locations, respectively.

- **Bode Options** — By default, the SISO Design Tool shows the plant and sensor poles and zeros as blue x's and o's, respectively. Uncheck this box to eliminate the plant's poles and zeros from the Bode plot. Note that the compensator poles and zeros (in red) will still appear.

8

Setting Tool Preferences

Both the LTI Viewer and the SISO Design Tool have Tool Preferences Editors. These editors comprise the middle layer of “The Property and Preferences Hierarchy”.

Both editors allow you to set default characteristics for specific instances of LTI Viewers and SISO Design Tools. If you open a new instance of either, each defaults to the characteristics specified in the Toolbox Preferences editor.

Opening the LTI Viewer Preferences Editor

Select **LTI Viewer Preferences** under the **Edit** menu of the LTI Viewer to open the **LTI Viewer Preferences** editor. This figure shows the editor open to its first page.

Note Click on the page tabs below to get help on LTI Viewer Preference editor pages.

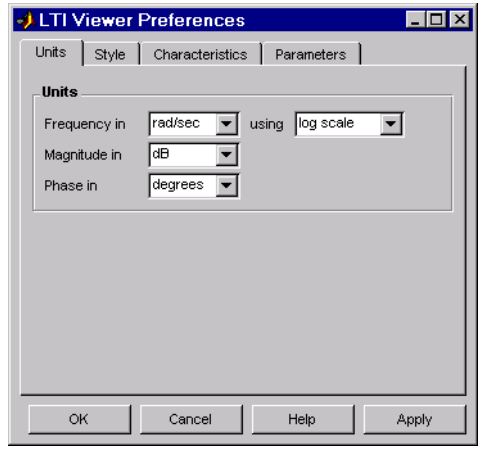
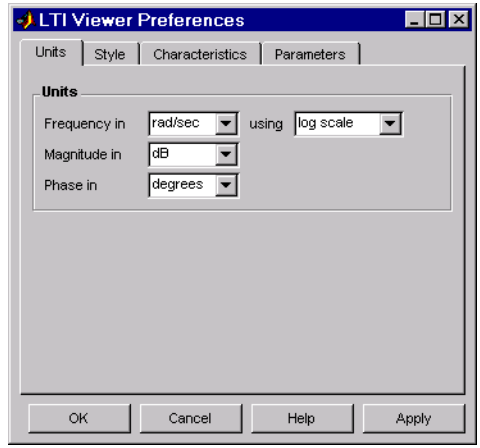


Figure 8-1: The LTI Viewer Preferences Editor

Units Page

Note Click on the page tabs below to get help on LTI Viewer Preference editor pages.



You can select the following on the **Units** page (shown in Figure 8-1):

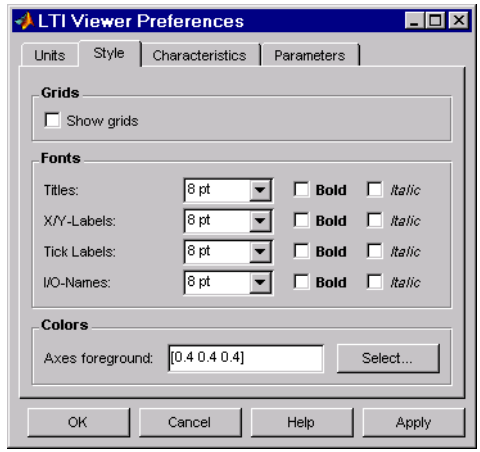
- Frequency — Radians per second (rad/sec) or Hertz (Hz)
- Magnitude — Decibels (dB) or absolute value (abs)
- Phase — Degrees or radians

For frequency and magnitude axes, you can select logarithmic or linear scales.

Style Page

Note Click on the page tabs below to get help on LTI Viewer Preference editor pages.

Use the **Style** page to toggle grid visibility and set font preferences and axes foreground colors for all plots in the LTI Viewer. This figure shows the **Style** page.



You have the following choices:

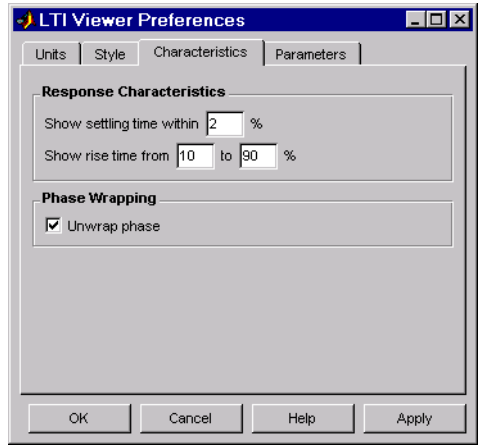
- **Grid** — Activate grids for all plots in the LTI Viewer
- **Fonts** — Set the font size, weight (bold), and angle (italic)
- **Colors** — Specify the color vector to use for the axes foreground, which includes the X-Y axes, grid lines, and tick labels. Use a three-element vector to represent red, green, and blue (RGB) values. Vector element values can range from 0 to 1.

  If you do not want to specify the RGB values numerically, press the **Select** button to open the **Select Colors** window. See “Select colors” for more information.

Characteristics Page

Note Click on the page tabs below to get help on LTI Viewer Preference editor pages.

The **Characteristics** page, shown below, has selections for response characteristics and phase wrapping.



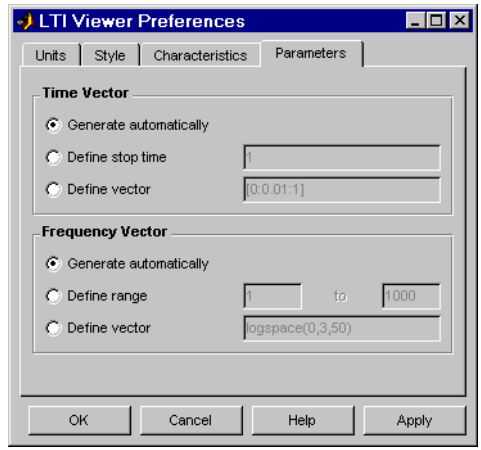
The following choices are available:

- **Response Characteristics**:
  - Specify settling time tolerance — You can set the threshold of the settling time calculation to any percentage from 0 to 100%. The default is 2%.
  - Specify rise time boundaries — The standard definition of rise time is the time it takes the signal to go from 10% to 90% of the final value. You can choose any percentages you like (from 0% to 100%), provided that the first value is smaller than the second.
- **Phase Wrapping** — By default, the phase is not wrapped. Wrap the phrase by unchecking this box. If the phase is wrapped, all phase values are shifted such that their equivalent value displays in the range [-180°, 180°).

Parameters Page

Note Click on the page tabs below to get help on LTI Viewer Preference editor pages.

Use the **Parameters** page, shown below, to specify input vectors for time and frequency simulation.



The defaults are to generate time and frequency vectors for your plots automatically. You can, however, override the defaults as follows:

- **Time Vector**:
  - Define stop time — Specify the final time value for your simulation
  - Define vector — Specify the time vector manually using equal-sized time steps
- **Frequency Vector**:
  - Define range — Specify the bandwidth of your response. Whether it's in rad/sec or Hz depends on the selection you made in the Units page.
  - Define vector — Specify the vector for your frequency values. Any real, positive, strictly monotonically increasing vector is valid.

Opening the SISO Tool Preferences Editor

Note Click on the page tabs below to get help on SISO Tool Preference editor pages.

To open the **SISO Tool Preferences** editor, select **SISO Tool Preferences** from the **Edit** menu of the SISO Design Tool. This window opens.

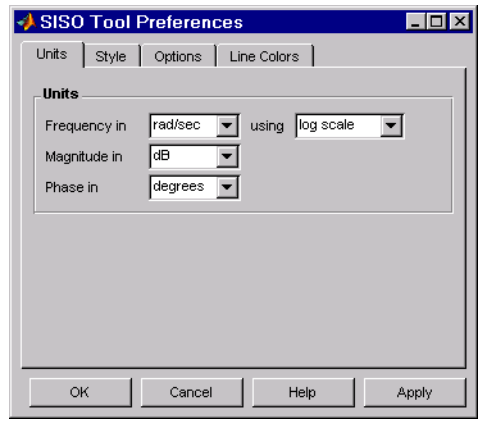
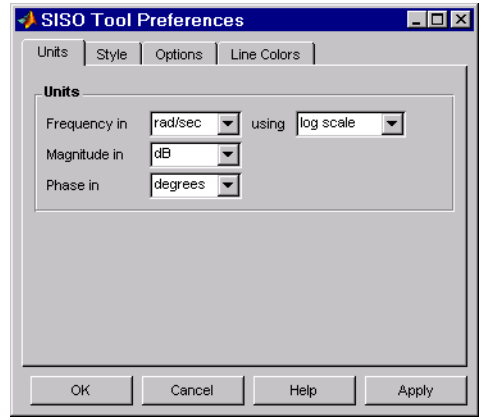


Figure 8-2: The SISO Tool Preferences Editor

Units Page

Note Click on the page tabs below to get help on SISO Tool Preference editor pages.



The **Units** page has settings for the following units:

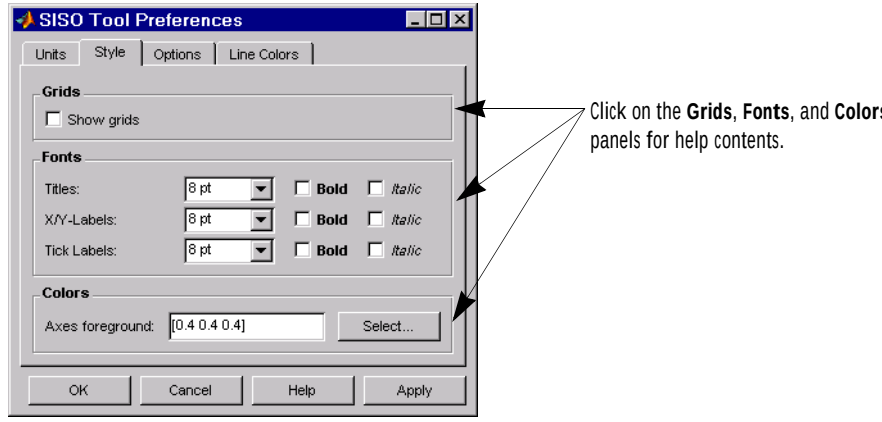
- Frequency — Radians per second (rad/sec) or Hertz (Hz)
- Magnitude — Decibels (dB) or absolute value (abs)
- Phase — Degrees or radians

For frequency and magnitude axes, you can select logarithmic or linear scales.

Style Page

Note Click on the page tabs below to get help on SISO Tool Preference editor pages.

Use the **Style** page to toggle grid visibility and set font preferences and axes foreground colors for all plots in the SISO Design Tool. This figure shows the **Style** page.



Grids Panel

Check the box to activate grids for all plots in the SISO Design Tool

Fonts Panel

Set the font size, weight (bold), and angle (italic) by using the menus and checkboxes.

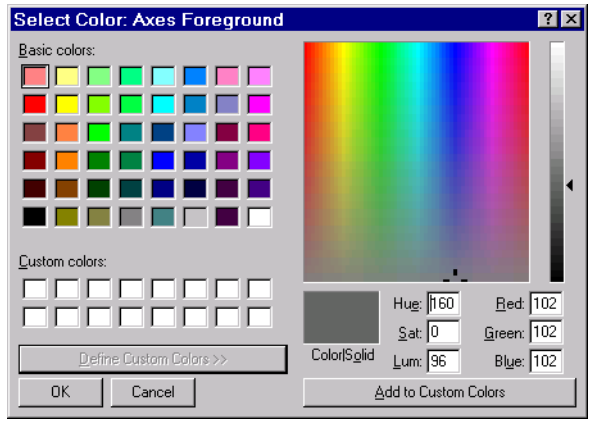
Colors Panel

Specify the color vector to use for the axes foreground, which includes the X-Y axes, grid lines, and tick labels. Use a three-element vector to represent red, green, and blue (RGB) values. Vector element values can range from 0 to 1.

Select colors. Press the **Select** button to open the Select Color window for the axes foreground.

You can use this window to choose axes foreground colors without having to set RGB (red-green-blue) values numerically. To make your selections, click on the colored rectangles and press OK. If you want a broader range of colors, press

the **Define Custom Colors** button. This extends the Select Color window, as shown in this figure.



You can pick colors from the color spectrum located in the upper right corner of the window. To select a custom color, follow these steps:

1 Place your cursor at a point in the color spectrum that has a color you want to define.

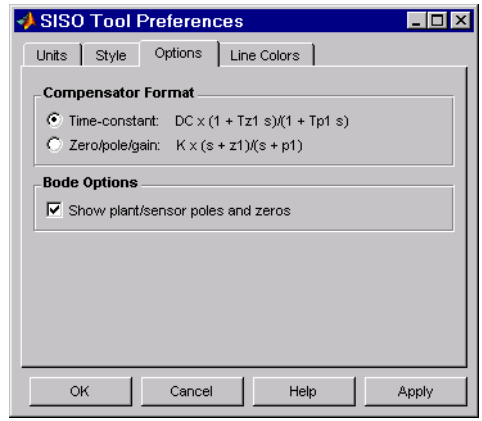
2 Left-click. Notice that the hue, saturation, luminescence (lum.), red, green, and blue fields specify the numerical values for the selected color.

3 Press **Add to Custom Colors**. This adds the selected color to the row of boxes labeled **Custom Color.** You can now use this color just like the basic colors.

Options Page

Note Click on the page tabs below to get help on SISO Tool Preference editor pages.

The **Options** page, shown below, has selections for compensator format and Bode diagrams.



You can make the following selections:

- **Compensator Format** — Select either the time-constant format or the zero/pole/gain format. The time-constant format is

  $$dcgain \times \frac{(1 + Tz_1 s)}{(1 + Tp_1 s)} \ldots$$

  where Tz_1, Tz2, ..., are the zero time constants, and Tp_1, Tp_2, ..., are the pole time constants.

  The zero/pole/gain format is a variation on the time-constant format.

  $$K \times \frac{(s + z_1)}{(s + p_1)}$$

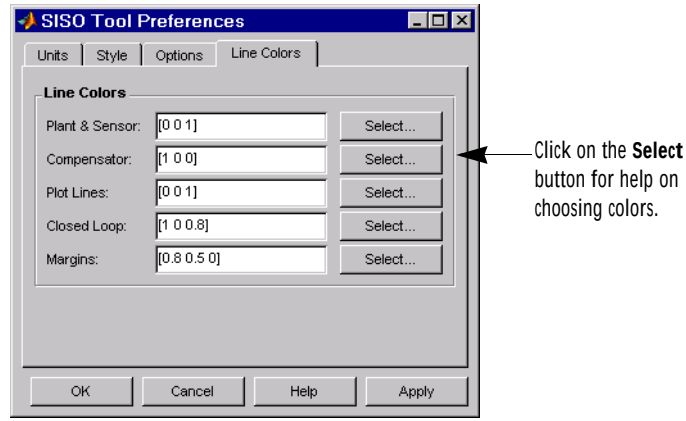
  In this case, the gain is compensator gain; z_1, z_2, ... and p_1, p_2, ..., are the zero and pole locations, respectively.

- **Bode Options** — By default, the SISO Design Tool shows the plant and sensor poles and zeros as blue x's and o's, respectively. Uncheck this box to eliminate the plant's poles and zeros from the Bode plot. Note that the compensator poles and zeros (in red) will still appear.

Line Colors Page

Note Click on the page tabs below to get help on SISO Tool Preference editor pages.

The **Line Colors** page, shown below, has selections for specify the colors of the lines in the response plots of the SISO Design Tool.



To change the colors of plot lines associated with parts of your model, specify a three-element vector to represent red, green, and blue (RGB) values. Vector element values can range from 0 to 1.

If you do not want to specify the RGB values numerically, press the **Select** button to open the **Select Colors** window. See “Select colors” for more information.

9

Customizing Response Plot Properties

The lowest level of "The Property and Preferences Hierarchy" is setting response plot properties. If you have created a response plot, there are two ways to open the Property Editor:

- Double-click in the plot region
- Select **Properties** from the right-click menu

Before looking at the Property Editor, open a step response plot using this commands.

```
load ltiexamples
step(sys_dc)
```

This creates a step plot. Select **Properties** from the right-click window. Note that when you open the **Property Editor**, a black dashed box appears around the step response, as this figure shows.

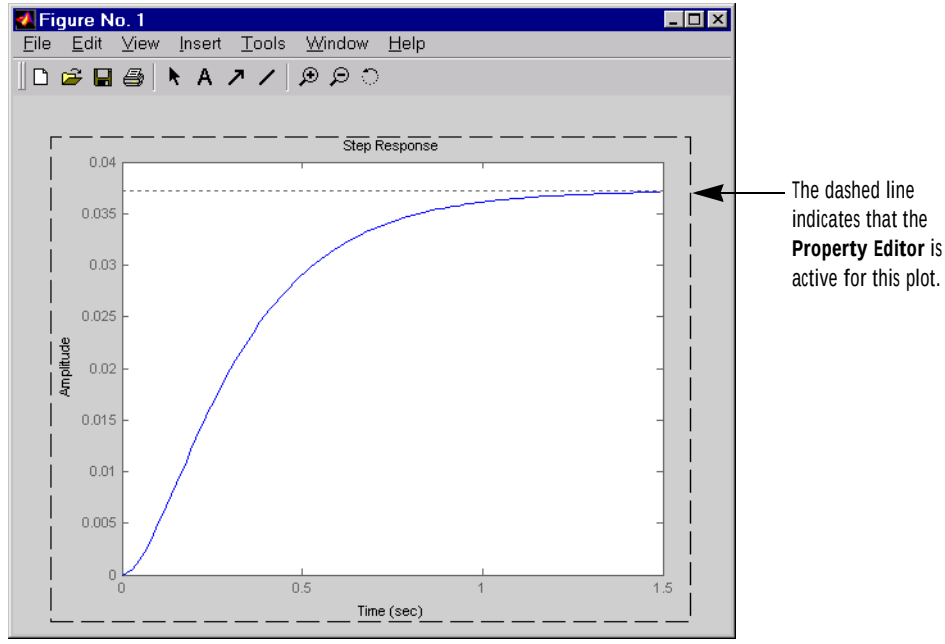


Figure 9-1: A SISO System Step Response

Property Editor

Note Click on the page tabs below to get help on pages in the Property Editor.

This figure shows the **Property Editor** for this step response.

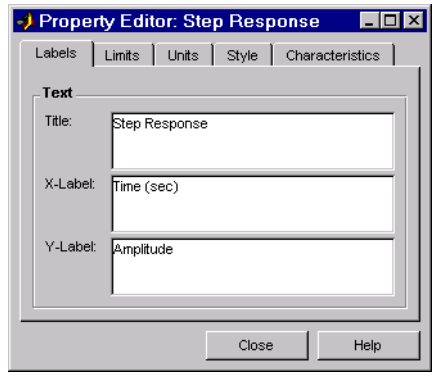


Figure 9-2: The Property Editor for the Step Response

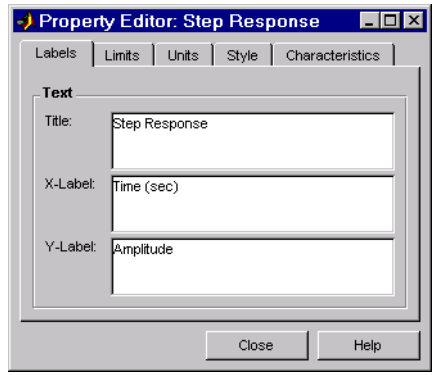
In general, you can change the following properties of response plots:

- **Labels** — Titles and X- and Y-labels
- **Limits** — Numerical ranges of the X and Y axes
- **Units** — Where applicable (e.g., rad/sec to Hertz). If you cannot customize units, as is the case with step responses, the Property Editor will display that no units ar available for the selected plot.
- **Style** — Show a grid and adjust font properties, such as font size, bold and italics
- **Characteristics** — Where applicable, these include peak response, settling time, phase and gain margins, etc. Plot characteristics change with each plot response type. The Property Editor displays only the characteristics that make sense for the selected response plot. For example, phase and gain margins are not available for step responses.

As you make changes in the Property Editor, they display immediately in the response plot. Conversely, if you make changes in a plot using right-click menus, the Property Editor for that plot automatically updates. The Property Editor and its associated plot are dynamically linked.

Labels Page

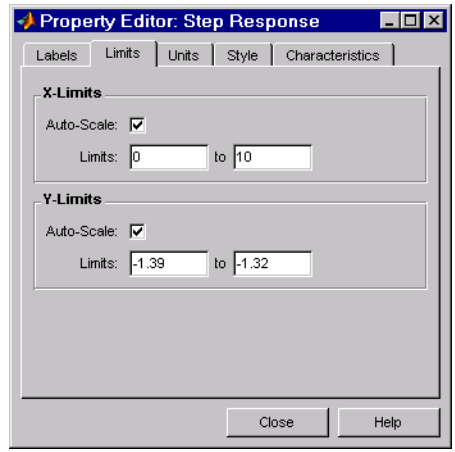
Note Click on the page tabs below to get help on pages in the Property Editor.



To specify new text for plot titles and axis labels, type the new string in the field next to the label you want to change. Note that the label changes immediately as you type, so you can see how the new text looks as you are typing.

Limits Page

Note Click on the page tabs below to get help on pages in the Property Editor.

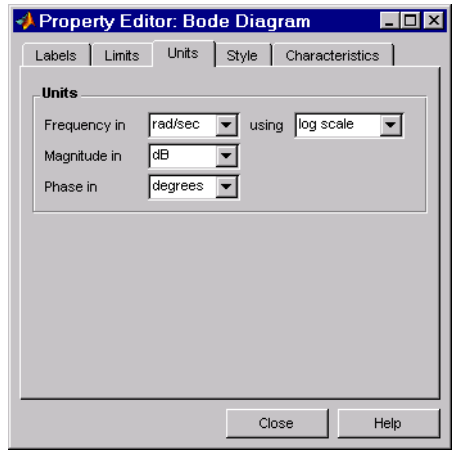


The Control System Toolbox selects default values for the axes limits to make sure that the maximum and minimum *x* and *y* values are displayed. If you want to override the default settings, change the values in the Limits fields. The **Auto-Scale** box automatically unchecks if you click on a different field. The new limits appear immediately in the response plot.

To reestablish the default values, recheck the **Auto-Scale** box.

Units Page

Note Click on the page tabs below to get help on pages in the Property Editor.



You can use the units page to change units in your response plot. The contents of this page depend on the response plot associated with the editor.

Note that for step and impulse responses, there are no alternate units available (only time and amplitude are possible in the toolbox). This table lists the options available for the other response objects. Use the menus to toggle between units.

Table 9-1: Optional Unit Conversions for Response Plots

Response Plot	Unit Conversions
Bode and Bode Magnitude	Frequency in rad/sec or Hertz (Hz) using logarithmic or linear scale Magnitude in decibels (dB) or the absolute value Phase in degrees or radians
Impulse	None
Nichols Chart and Nyquist Diagram	Frequency in rad/sec or Hertz Magnitude in decibels or the absolute value Phase in degrees or radians
Pole/Zero Map	Frequency in rad/sec or Hertz

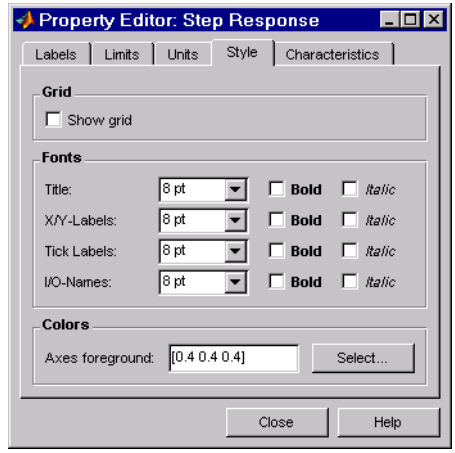
Table 9-1: Optional Unit Conversions for Response Plots

Response Plot	Unit Conversions
Singular Values	Frequency in rad/sec or Hertz using logarithmic or linear scale Magnitude in decibels or the absolute value
Step	None

Style Page

Note Click on the page tabs below to get help on pages in the Property Editor.

Use the **Style** page to toggle grid visibility and set font preferences and axes foreground colors for response plots.



You have the following choices:

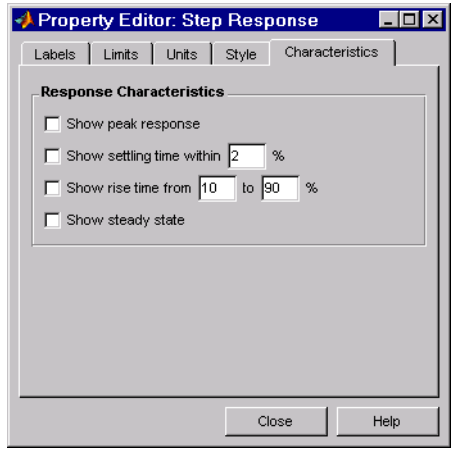
- **Grid** — Activate grids by default in new plots
- **Fonts** — Set the font size, weight (bold), and angle (italic)

- **Colors** — Specify the color vector to use for the axes foreground, which includes the X-Y axes, grid lines, and tick labels. Use a three-element vector to represent red, green, and blue (RGB) values. Vector element values can range from 0 to 1.

  If you do not want to specify RGB values numerically, press the **Select** button to open the **Select Colors** window. See "Select colors" for more information.

Characteristics Page

Note Click on the page tabs below to get help on pages in the Property Editor.



The **Characteristics** page allows you to customize response characteristics for plots. Each response plot has its own set of characteristics; the table below lists

them. Use the checkboxes to activate the feature and the fields to specify rise or settling time percentages.

Table 9-2: Response Characteristic Options for Response Plots

Plot	Customizable Feature
Bode Diagram	Show peak response Show minimum stability margins Show all stability margins Unwrap phase (default is wrapped)
Bode Magnitude	Show peak response
Impulse	Show peak response Show settling time within *xx*% (specify the percentage)
Nichols Chart	Show peak response Show minimum stability margins Show all stability margins Unwrap phase (default is wrapped)
Nyquist Diagram	Show peak response Show minimum stability margins Show all stability margins
Pole/Zero Map	None
Sigma	Show peak response
Step	Show peak response Show settling time within *xx*% (specify the percentage) Show rise time from *xx* to *yy*% (specify the percentages) Show steady state

Property Editing for Subplots

If you create more than one plot in a single figure window, you can edit each plot individually. For example, the following code creates a figure with two plots, a step and an impulse response with two randomly selected systems.

```
subplot(2,1,1)
step(rss(2,1))
subplot(2,1,2)
impulse(rss(1,1))
```

After the figure window appears, double-click in the upper (step response) plot to activate the **Property Editor**. You will see a dashed line appear around the step response, indicating that it is the active plot for the editor. To switch to the lower (impulse response) plot, just click once in the impulse response plot region. The dashed box switches to the impulse response, and the **Property Editor** updates as well.

Customizing Plots Inside the SISO Design Tool

Customizing plots inside the SISO Design Tool is similar to how you customize any response plot. The Control System Toolbox provides a property editor specific to the SISO Design Tool that you can use to create custom root locus and Bode diagrams within the SISO Design Tool.

Opening the Root Locus Plot Editor

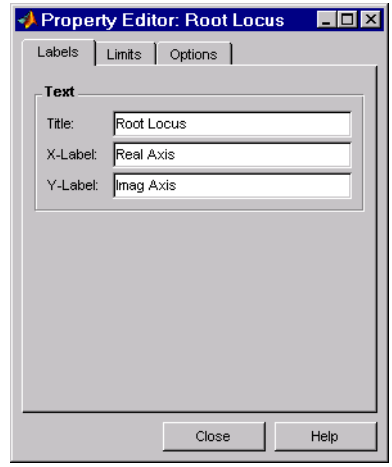
There are three ways to open the Property Editor for root locus plots:

- Double-click in the root locus away from the curve
- Select **Properties** from the right-click menu
- Select **Root Locus** and then **Properties** from Edit in the menu bar

---

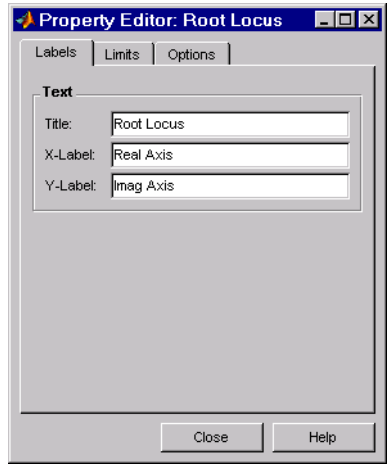
Note Click on the page tabs below to get help on pages in the Root Locus Property Editor.

This figure shows the **Property Editor: Root Locus** window.



Labels Page

Note Click on the page tabs below to get help on pages in the Root Locus Property Editor.



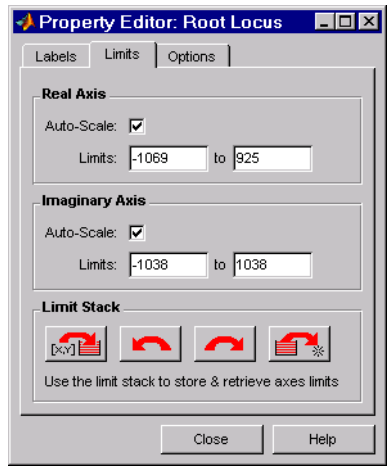
You can use the **Label** page to specify plot titles and axis labels. To specify a new label, type the string in the appropriate field. The root locus plot automatically updates.

Limits Page

Note Click on the page tabs below to get help on pages in the Root Locus Property Editor.

The SISO Design Tool specifies default values for the real and imaginary axes ranges to make sure that all the poles and zeros in your model appear in the

root locus plot. Use the **Limits** page, shown below, to override the default settings.



To change the limits, specify the new limits in the real and imaginary axes **Limits** fields. The **Auto-Scale** checkbox automatically deactivates once you click in a different field. Your root locus diagram updates immediately. If you want to reapply the default limits, recheck the **Auto-Scale** checkboxes.

The Limit Stack panel provides support for storing and retrieving custom limit specifications. There are four buttons available:

 — Add the current limits to the stack

 — Retrieve the previous stack entry

 — Retrieve the next stack entry

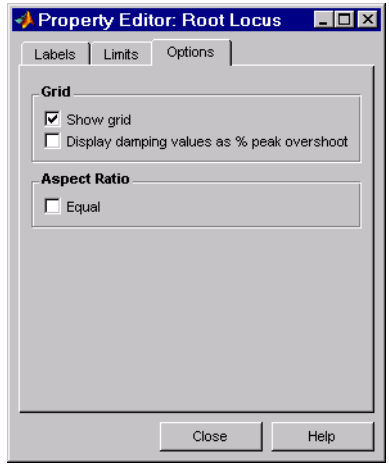
 — Remove the current limits from the stack

Using these buttons, you can store and retrieve any number of saved custom axes limits.

Options Page

Note Click on the page tabs below to get help on pages in the Root Locus Property Editor.

The **Options** page contains settings for adding a grid and changing the plot's aspect ratio. This figure shows the **Options** page.



Check **Show grid** to display a grid on the root locus. If you select **Display damping ratios as % peak overshoot**, the SISO Design Tool displays the damping ratio values along the grid lines. This figure shows both options activated for an imported model, `Gservo`. If you want to verify these settings, type

```
load ltiexamples
```

at the MATLAB prompt and import `Gservo` from the workspace into your SISO Design Tool.

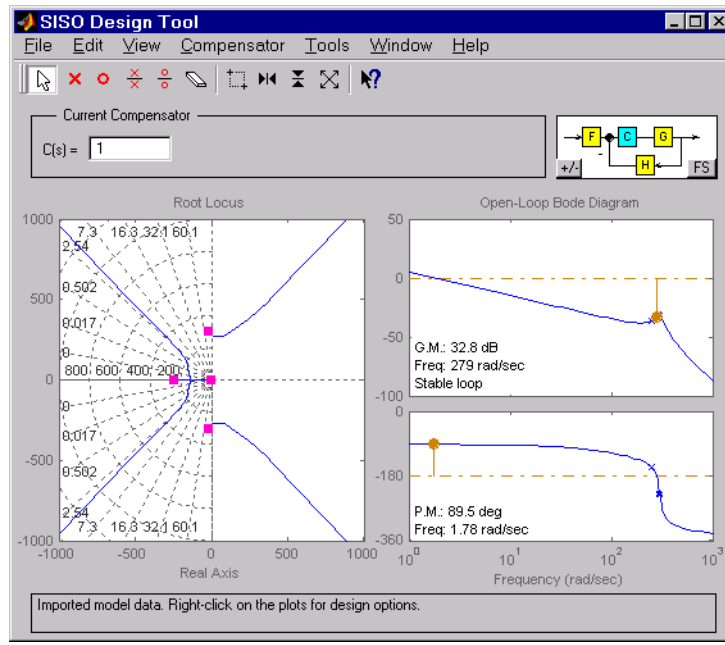


Figure 9-3: Displaying Damping Ratio Values

The numbers displayed on the root locus gridlines are the damping ratios as a percentage of the overshoot values.

If you check the **Equal** box in the **Aspect Ratio** panel, the *x* and *y*-axes are set to equal limit values.

Opening the Bode Diagram Property Editor

The Property Editor for Bode diagrams is identical to the one for root locus, with one exception, the **Options** page. As is the case with the root locus Property Editor, there are three ways to open the Bode diagram property editor:

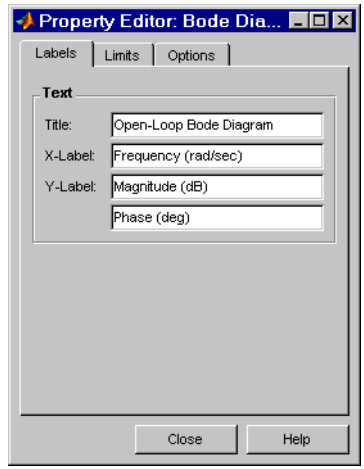
- Double-click in the root locus away from the curve
- Select **Properties** from the right-click menu

- Select **Bode** and then **Properties** from **Edit** in the menu bar

---

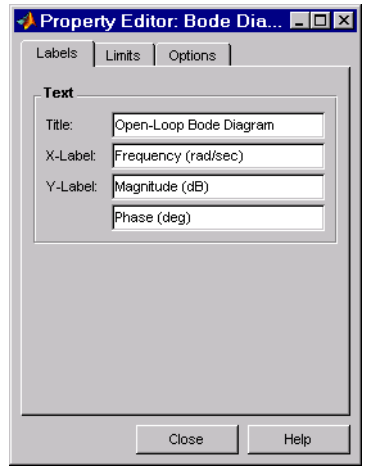
Note Click on the page tabs below to get help on pages in the Bode Diagram Property Editor.

This figure shows the **Property Editor: Bode Diagram** editor.



Labels Page

Note Click on the page tabs below to get help on pages in the Bode Diagram Property Editor.

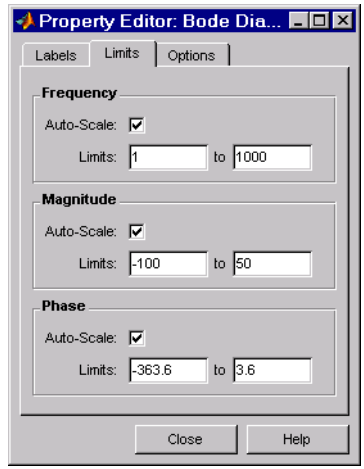


You can use the **Label** page to specify plot titles and axis labels. To specify a new label, type the string in the appropriate field. The Bode diagram automatically updates.

Limits Page

Note Click on the page tabs below to get help on pages in the Bode Diagram Property Editor.

The Control System Toolbox sets default limits for the frequency, magnitude, and phase scales for your plots. Use the **Limits** page to override the default values.



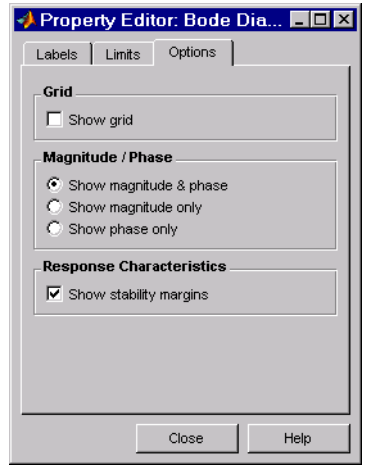
To change the limits, specify the new values in the **Limits** fields for frequency, magnitude, and phase. The **Auto-Scale** checkbox automatically deactivates once you click in a different field. The Bode diagram updates immediately.

To restore the default settings, recheck the Auto-Scale boxes.

Options Page

Note Click on the page tabs below to get help on pages in the Bode Diagram Property Editor.

This figure shows the **Options** page for Bode diagrams.



The following options are available from this page:

- **Grid** — Check **Show grid** to display grid lines.
- **Magnitude/Phase** — There are three radio buttons; you can toggle between the following displays:
  - **Show magnitude & phase**
  - **Show magnitude only**
  - **Show phase only**
- **Response Characteristics** — Check **Show stability margins** to display the phase and gain margins on your Bode diagram. The margins appear as brown stems, and the Bode diagram displays the numerical values of the margins in one of the bottom corners of the gain and phase plots.

  The Bode diagram in Figure 9-3, Displaying Damping Ratio Values, has the stability margins displayed.

10

Design Case Studies

This chapter contains four detailed case studies of control system design and analysis using the Control System Toolbox.

- "Yaw Damper for a 747 Jet Transport" — Illustrating the classical design process
- "Hard-Disk Read/Write Head Controller" — Illustrating classical digital controller design
- "LQG Regulation: Rolling Mill Example" — Using linear quadratic Gaussian techniques to regulate the beam thickness in a steel rolling mill
- "Kalman Filtering"— Kalman filtering that illustrates both steady-state and time-varying Kalman filter design and simulation

Demonstration files for these case studies are available as `jetdemo.m`, `diskdemo.m`, `milldemo.m`, and `kalmdemo.m`. To run any of these demonstrations, type the corresponding name at the command line, for example,

```
jetdemo
```

Yaw Damper for a 747 Jet Transport

This case study demonstrates the tools for classical control design by stepping through the design of a yaw damper for a 747 jet transport aircraft.

The jet model during cruise flight at MACH = 0.8 and H = 40,000 ft. is

```
A = [-0.0558   -0.9968    0.0802    0.0415
      0.5980   -0.1150   -0.0318         0
     -3.0500    0.3880   -0.4650         0
           0    0.0805    1.0000         0];

B = [ 0.0729    0.0000
     -4.7500    0.00775
      .15300    0.1430
           0         0];

C = [0     1     0     0
     0     0     0     1];

D = [0     0
     0     0];
```

The following commands specify this state-space model as an LTI object and attach names to the states, inputs, and outputs.

```
states = {'beta' 'yaw' 'roll' 'phi'};
inputs = {'rudder' 'aileron'};
outputs = {'yaw' 'bank angle'};

sys = ss(A,B,C,D,'statename',states,...
                 'inputname',inputs,...
                 'outputname',outputs);
```

You can display the LTI model sys by typing sys. MATLAB responds with

```
a =
                    beta          yaw         roll          phi
         beta      -0.0558      -0.9968       0.0802       0.0415
          yaw        0.598       -0.115      -0.0318            0
         roll        -3.05        0.388       -0.465            0
          phi            0       0.0805            1            0
```

```
b =
                  rudder      aileron
         beta     0.0729            0
          yaw      -4.75      0.00775
         roll      0.153        0.143
          phi          0            0


c =
                    beta          yaw         roll          phi
          yaw          0            1            0            0
   bank angle          0            0            0            1


d =
                  rudder      aileron
          yaw          0            0
   bank angle          0            0

   Continuous-time model.
```

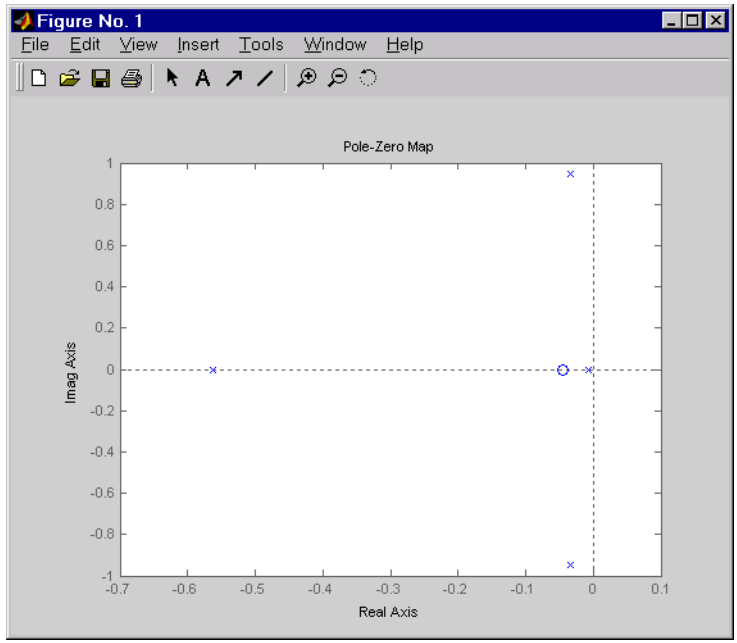
The model has two inputs and two outputs. The units are radians for `beta` (sideslip angle) and `phi` (bank angle) and radians/sec for `yaw` (yaw rate) and `roll` (roll rate). The rudder and aileron deflections are in radians as well.

Compute the open-loop eigenvalues and plot them in the *s*-plane.

```
damp(sys)
```

```
Eigenvalue                  Damping        Freq. (rad/s)

 -7.28e-003                 1.00e+000      7.28e-003
 -5.63e-001                 1.00e+000      5.63e-001
 -3.29e-002 + 9.47e-001i    3.48e-002      9.47e-001
 -3.29e-002 - 9.47e-001i    3.48e-002      9.47e-001
```

```
pzmap(sys)
```



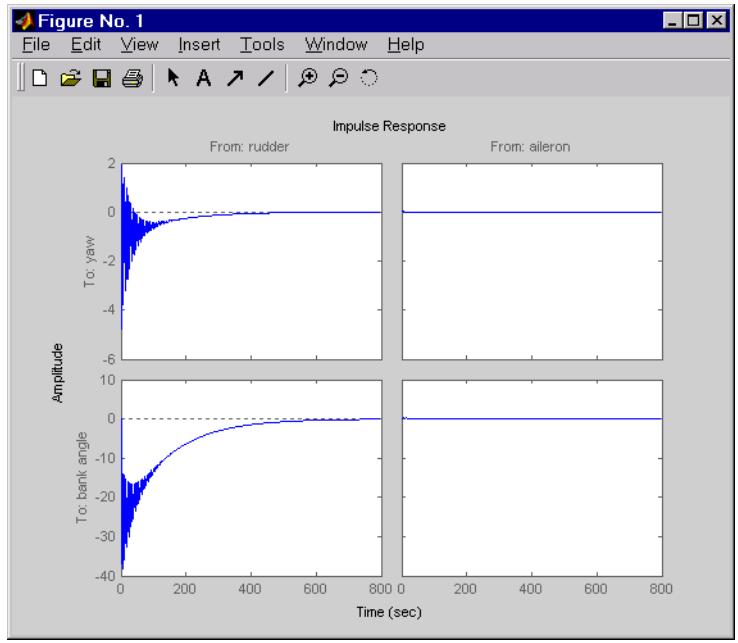
This model has one pair of lightly damped poles. They correspond to what is called the “Dutch roll mode.”

Suppose you want to design a compensator that increases the damping of these poles, so that the resulting complex poles have a damping ratio $\zeta > 0.35$ with natural frequency $\omega_n < 1$ rad/sec. You can do this using the Control System toolbox analysis tools.

Open-Loop Analysis

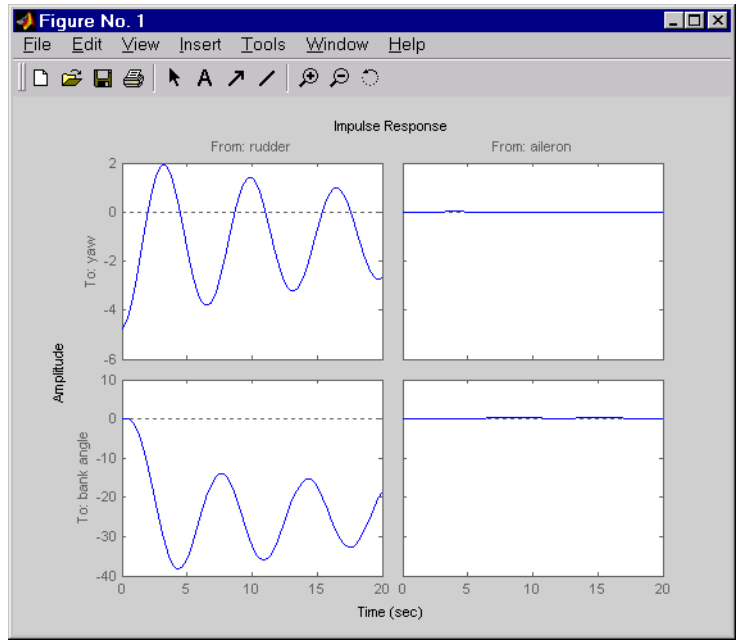
First, perform some open-loop analysis to determine possible control strategies. Start with the time response (you could use `step` or `impulse` here).

```
impulse(sys)
```



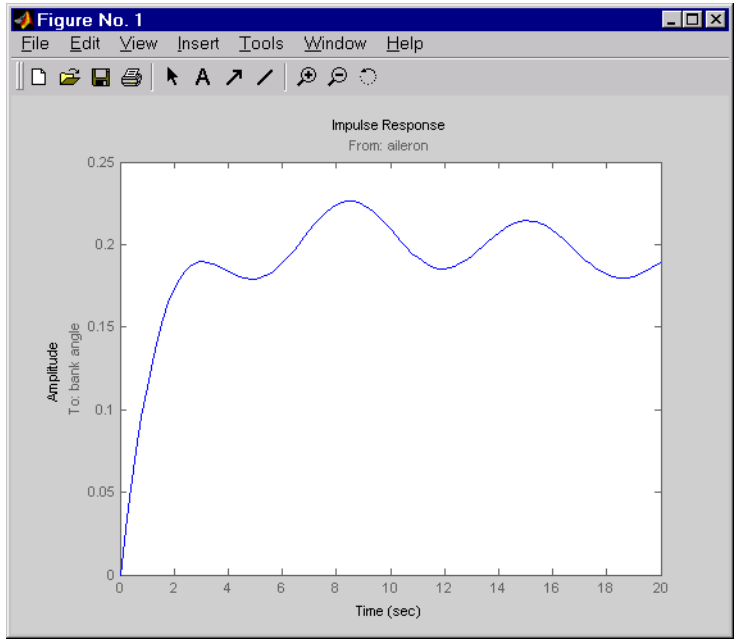
The impulse response confirms that the system is lightly damped. But the time frame is much too long because the passengers and the pilot are more concerned about the behavior during the first few seconds rather than the first few minutes. Next look at the response over a smaller time frame of 20 seconds.

```
impulse(sys,20)
```



Look at the plot from aileron (input 2) to bank angle (output 2). To show only this plot, right-click and choose **I/O Selector**, then click on the (2,2) entry. The I/O Selector should look like this.

The new figure is shown below.

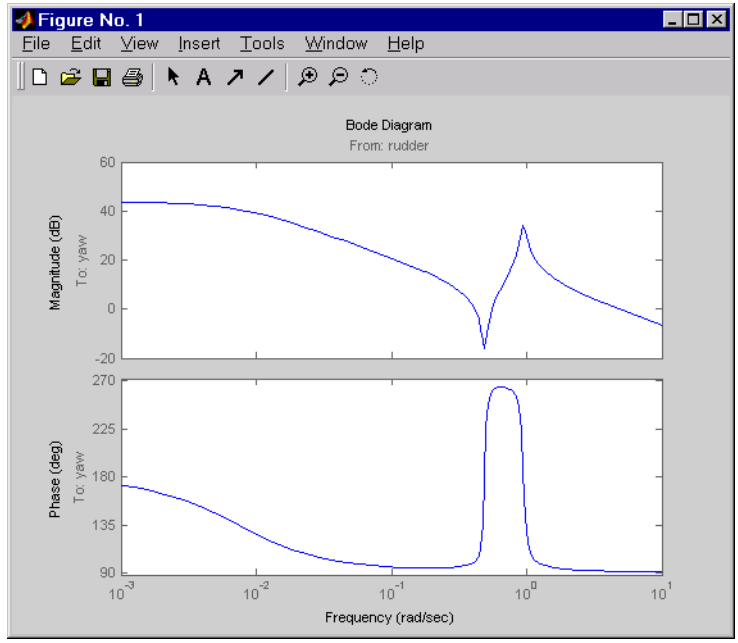


The aircraft is oscillating around a nonzero bank angle. Thus, the aircraft is turning in response to an aileron impulse. This behavior will prove important later in this case study.

Typically, yaw dampers are designed using the yaw rate as sensed output and the rudder as control input. Look at the corresponding frequency response.

```
sys11=sys('yaw','rudder') % Select I/O pair.
```

```
bode(sys11)
```



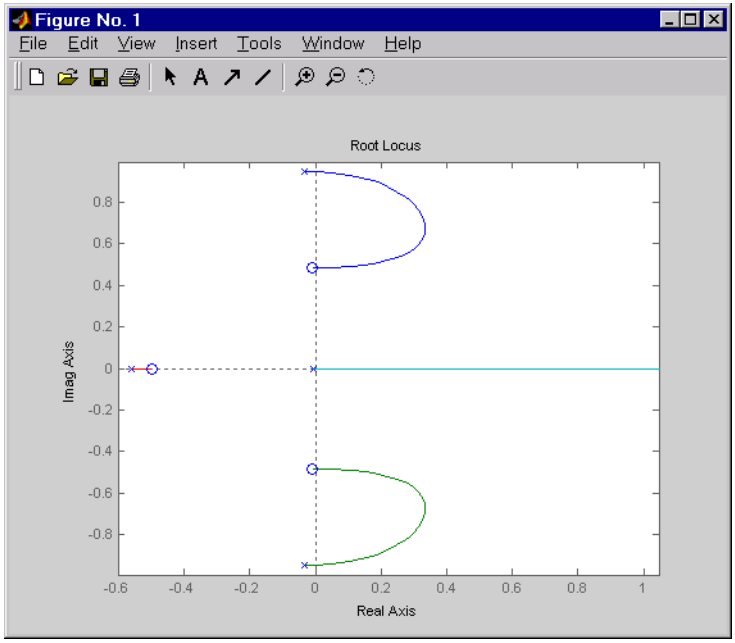
From this Bode diagram, you can see that the rudder has significant effect around the lightly damped Dutch roll mode (that is, near $\omega = 1$ rad/sec).

Root Locus Design

A reasonable design objective is to provide a damping ration $\zeta > 0.35$ with a natural frequency $\omega_n < 1.0$ rad/sec. Since the simplest compensator is a static gain, first try to determine appropriate gain values using the root locus technique.

```
% Plot the root locus for the rudder to yaw channel
```

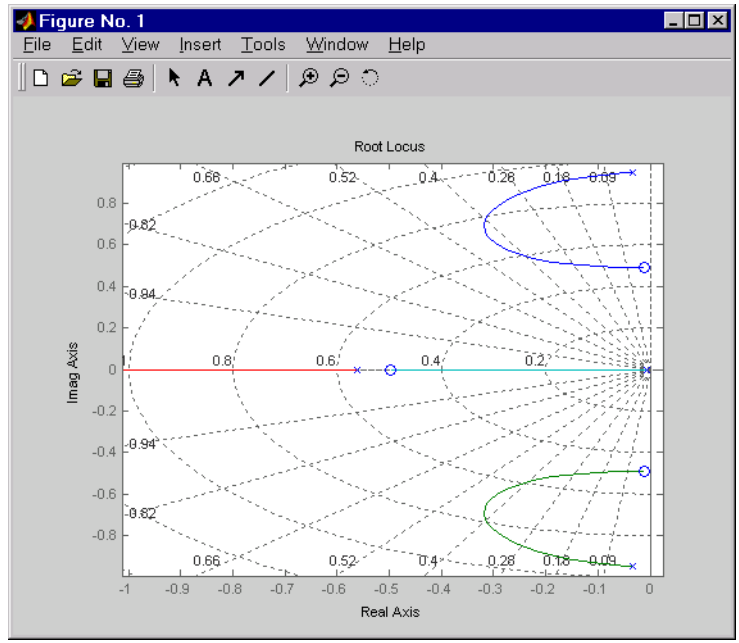
```
rlocus(sys11)
```



This is the root locus for negative feedback and shows that the system goes unstable almost immediately. If, instead, you use positive feedback, you may be able to keep the system stable.

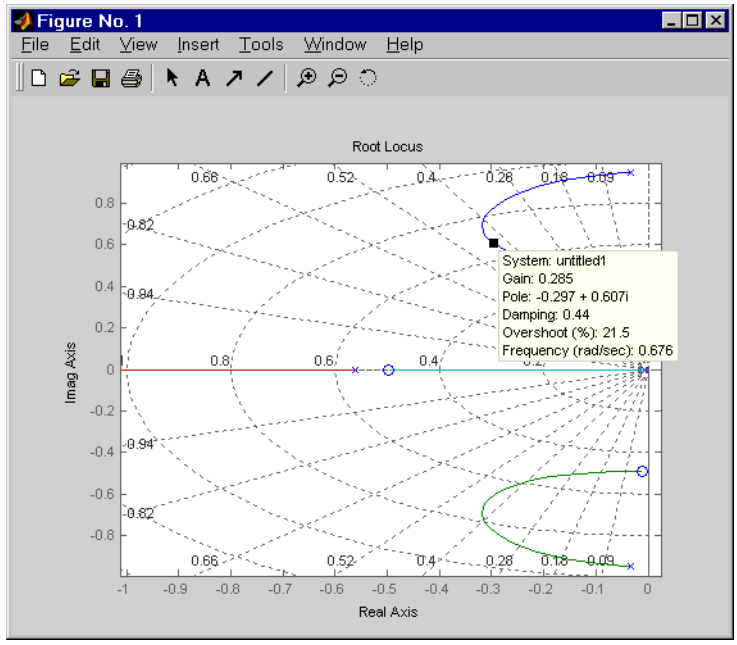
```
rlocus(-sys11)
```

```
sgrid
```



This looks better. By using simple feedback, you can achieve a damping ratio of $\zeta = 0.45$. Click on the blue curve and move the data marker to track the

gain and damping values. To achieve a 0.45 damping ratio, the gain should be about 2.85. This figure shows the data marker with similar values.

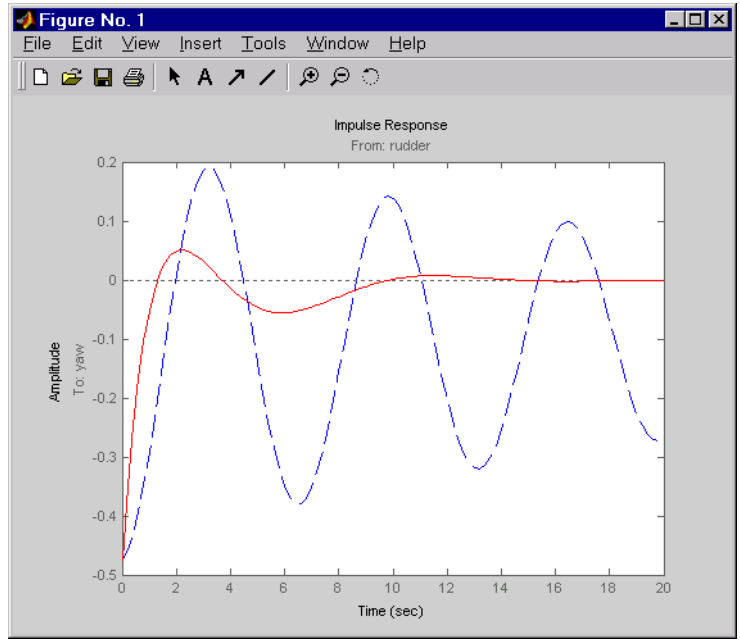


Next, close the SISO feedback loop.

```
K = 2.85;
cl11 = feedback(sys11,-K);   % Note: feedback assumes negative
                             % feedback by default
```

Plot the closed-loop impulse response for a duration of 20 seconds, and compare it to the open-loop impulse response.

```
impulse(sys11,'b--',cl11,'r',20)
```



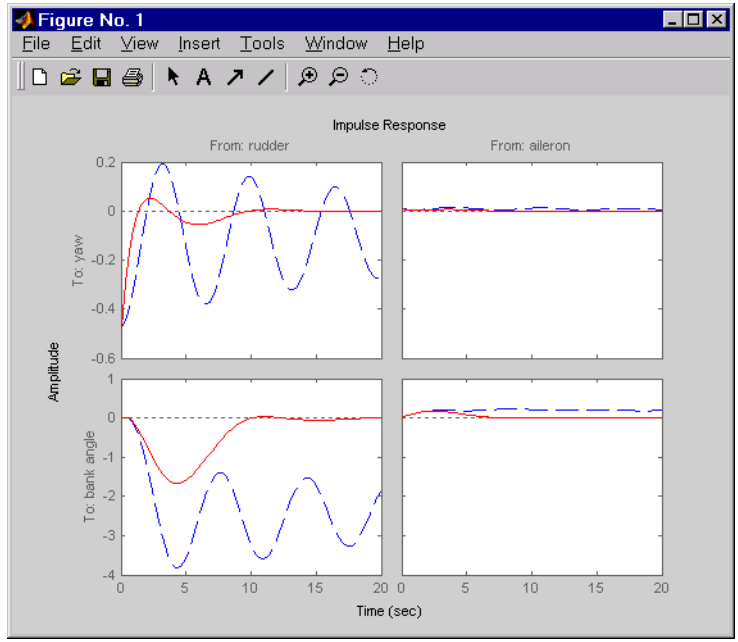
The closed-loop response settles quickly and does not oscillate much, particularly when compared to the open-loop response.

Now close the loop on the full MIMO model and see how the response from the aileron looks. The feedback loop involves input 1 and output 1 of the plant (use `feedback` with index vectors selecting this input/output pair). At the MATLAB prompt, type

```
cloop = feedback(sys,-K,1,1);
damp(cloop)    % closed-loop poles

Eigenvalue            Damping     Freq. (rad/s)

-3.42e-001                  1.00e+000      3.42e-001
-2.97e-001 + 6.06e-001i     4.40e-001      6.75e-001
-2.97e-001 - 6.06e-001i     4.40e-001      6.75e-001
-1.05e+000                  1.00e+000      1.05e+000
```

Plot the MIMO impulse response.

```
impulse(sys,'b--',cloop,'r',20)
```



The yaw rate response is now well damped, but look at the plot from aileron (input 2) to bank angle (output 2). When you move the aileron, the system no longer continues to bank like a normal aircraft. You have over-stabilized the spiral mode. The spiral mode is typically a very slow mode and allows the aircraft to bank and turn without constant aileron input. Pilots are used to this behavior and will not like your design if it does not allow them to fly normally. This design has moved the spiral mode so that it has a faster frequency.

Washout Filter Design

What you need to do is make sure the spiral mode does not move further into the left-half plane when you close the loop. One way flight control designers have addressed this problem is to use a washout filter $kH(s)$ where

$$H(s) = \frac{s}{s+a}$$

The washout filter places a zero at the origin, which constrains the spiral mode pole to remain near the origin. We choose $a = 0.2$ for a time constant of five seconds and use the root locus technique to select the filter gain `H`. First specify the fixed part $s/(s+a)$ of the washout by

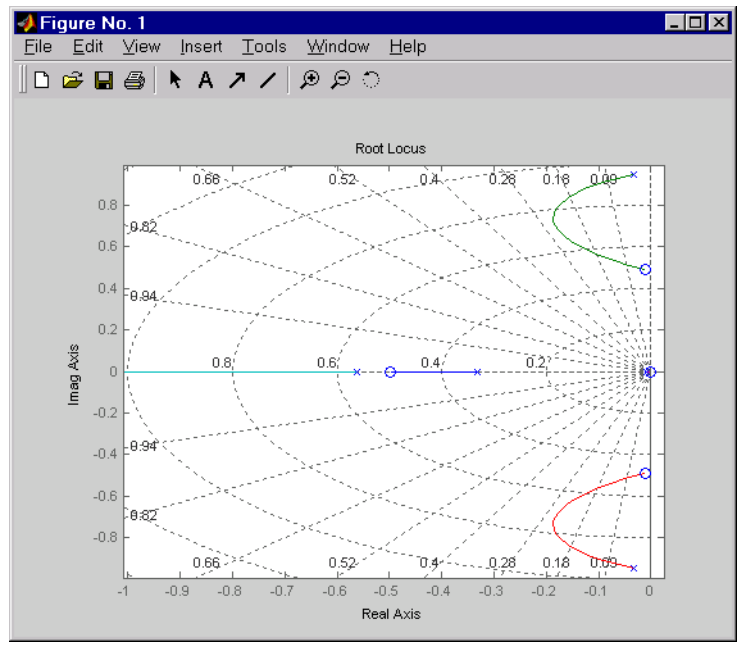
```
H = zpk(0,-0.2,1);
```

Connect the washout in series with the design model `sys11` (relation between input 1 and output 1) to obtain the open-loop model

```
oloop = H * sys11;
```

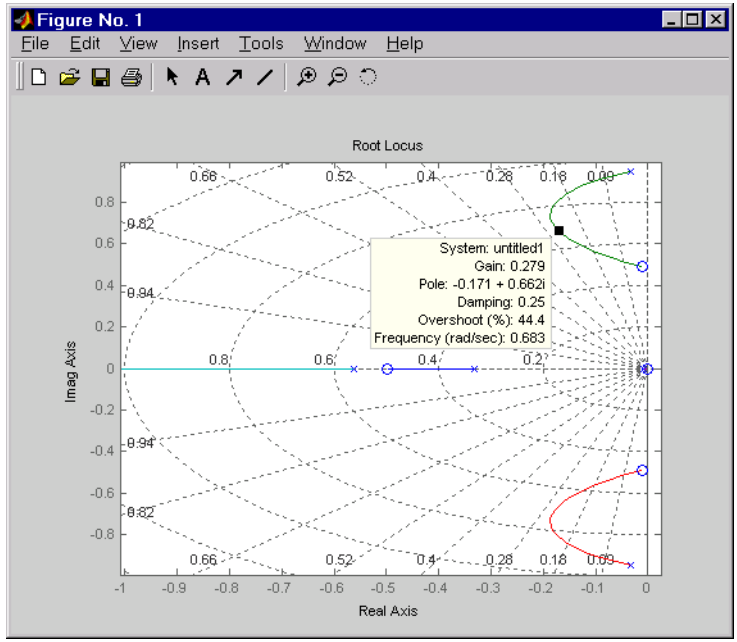
and draw another root locus for this open-loop model.

```
rlocus(-oloop)
sgrid
```



Create and drag a data marker around the upper curve to locate the maximum damping, which is about $\zeta = 0.3$.

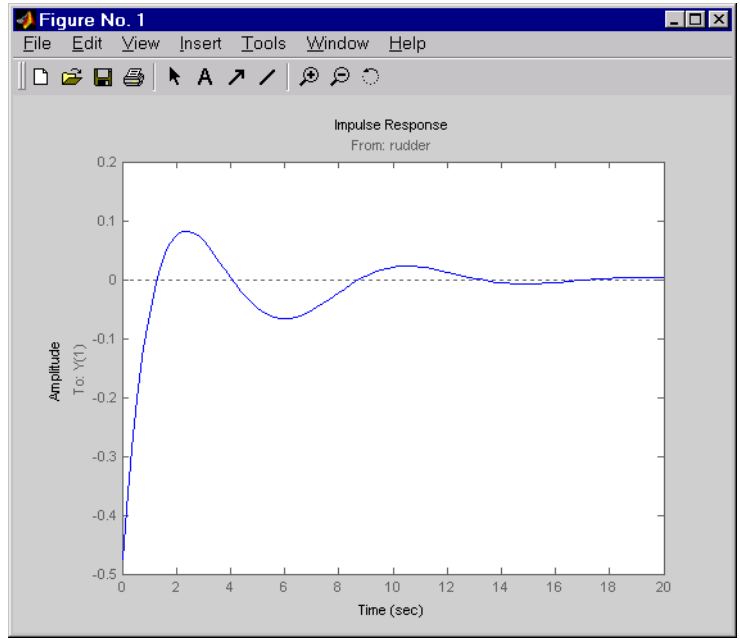
This figure shows a data marker at the maximum damping ratio; the gain is approximately 2.07.



Look at the closed-loop response from rudder to yaw rate.

```
K = 2.07;
cl11 = feedback(oloop,-K);
```

```
impulse(cl11,20)
```



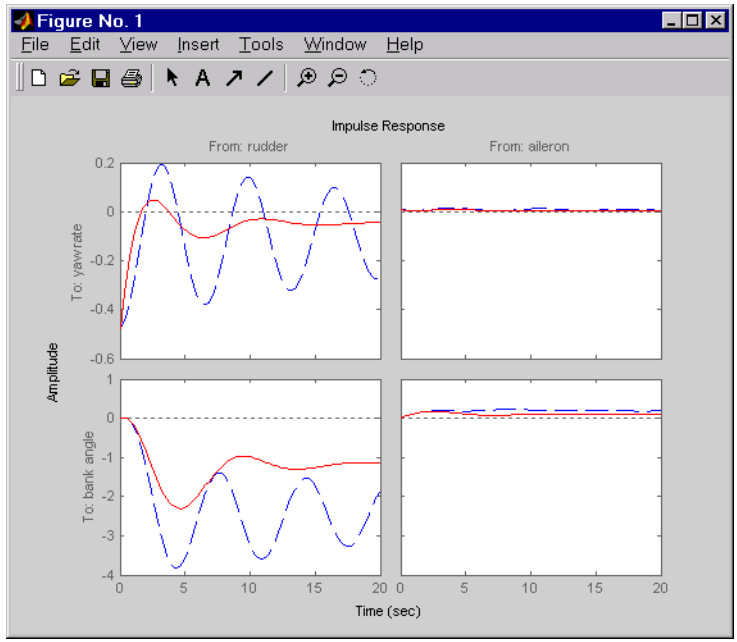
The response settles nicely but has less damping than your previous design. Finally, you can verify that the washout filter has fixed the spiral mode problem. First form the complete washout filter $kH(s)$ (washout + gain).

```
WOF = -K * H;
```

Then close the loop around the first I/O pair of the MIMO model sys and simulate the impulse response.

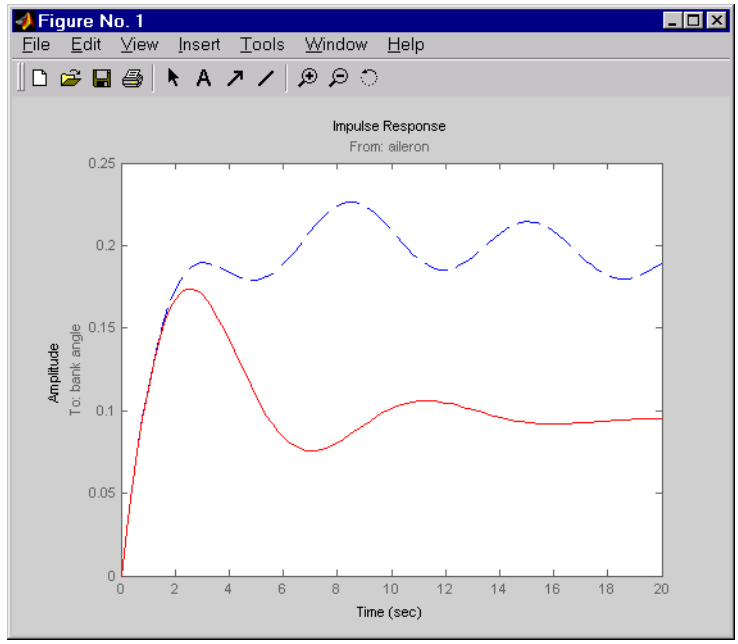
```
cloop = feedback(sys,WOF,1,1);
```

```
% Final closed-loop impulse response
impulse(sys,'b--',cloop,'r',20)
```



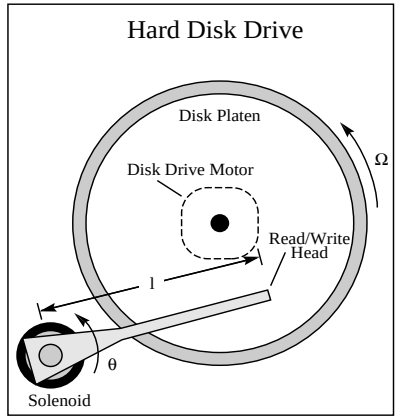
The bank angle response (output 2) due to an aileron impulse (input 2) now has the desired nearly constant behavior over this short time frame. To inspect the

response more closely, use the I/O Selector in the right-click menu to select the (2,2) I/O pair.



Although you did not quite meet the damping specification, your design has increased the damping of the system substantially and now allows the pilot to fly the aircraft normally.

Hard-Disk Read/Write Head Controller



This case study demonstrates the ability to perform classical digital control design by going through the design of a computer hard-disk read/write head position controller.

Using Newton's law, a simple model for the read/write head is the differential equation

$$J\frac{d^2\theta}{dt^2} + C\frac{d\theta}{dt} + K\theta = K_i i$$

where J is the inertia of the head assembly, C is the viscous damping coefficient of the bearings, K is the return spring constant, K_i is the motor torque constant, θ is the angular position of the head, and i is the input current.

Taking the Laplace transform, the transfer function from i to θ is

$$H(s) = \frac{K_i}{Js^2 + Cs + K}$$

Using the values $J = 0.01$ kg m^2, $C = 0.004$ Nm/(rad/sec), $K = 10$ Nm/rad, and $K_i = 0.05$ Nm/rad, form the transfer function description of this system. At the MATLAB prompt, type

```
J = .01; C = 0.004; K = 10; Ki = .05;
```

```
num = Ki;
den = [J C K];
H = tf(num,den)
```

MATLAB responds with

```
Transfer function:
         0.05
-----------------------
0.01 s^2 + 0.004 s + 10
```

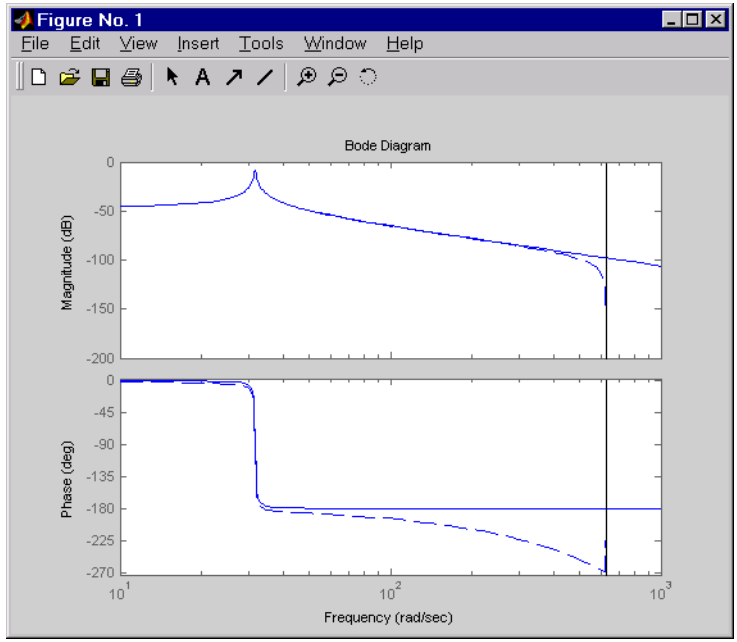
The task here is to design a digital controller that provides accurate positioning of the read/write head. The design is performed in the digital domain. First, discretize the continuous plant. Because our plant will be equipped with a digital-to-analog converter (with a zero-order hold) connected to its input, use `c2d` with the `'zoh'` discretization method. Type

```
Ts = 0.005;     % sampling period = 0.005 second
Hd = c2d(H,Ts,'zoh')
```

```
Transfer function:
6.233e-05 z + 6.229e-05
-----------------------
 z^2 - 1.973 z + 0.998

Sampling time: 0.005
```

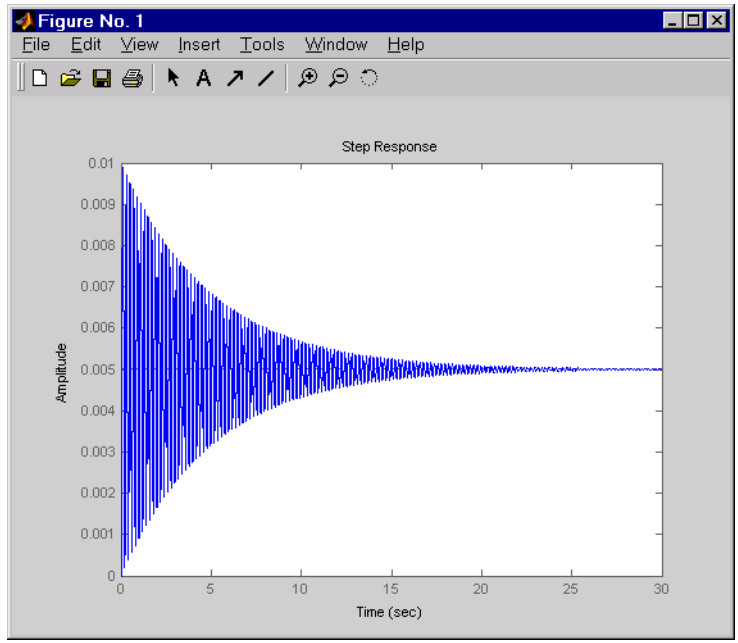
You can compare the Bode plots of the continuous and discretized models with

```
bode(H,'-',Hd,'--')
```



To analyze the discrete system, plot its step response, type

```
step(Hd)
```



The system oscillates quite a bit. This is probably due to very light damping. You can check this by computing the open-loop poles. Type

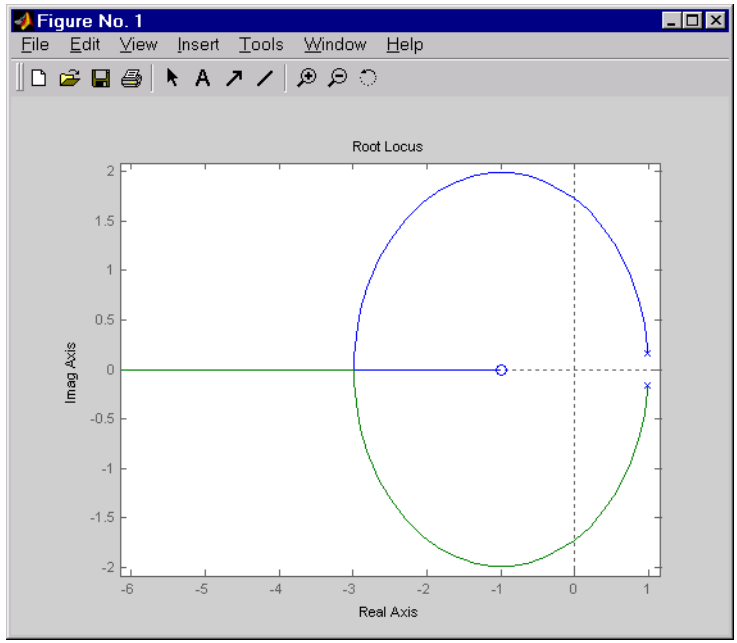
```
% Open-loop poles of discrete model
damp(Hd)
```

```
    Eigenvalue          Magnitude    Equiv. Damping    Equiv. Freq.

9.87e-01 + 1.57e-01i    9.99e-01       6.32e-03         3.16e+01
9.87e-01 - 1.57e-01i    9.99e-01       6.32e-03         3.16e+01
```

The poles have very light equivalent damping and are near the unit circle. You need to design a compensator that increases the damping of these poles.

The simplest compensator is just a gain, so try the root locus technique to select an appropriate feedback gain.

```
rlocus(Hd)
```

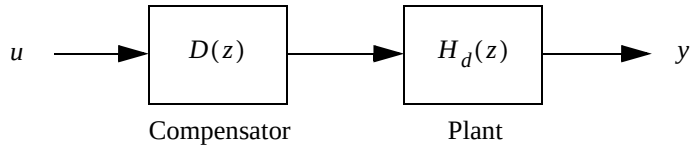


As shown in the root locus, the poles quickly leave the unit circle and go unstable. You need to introduce some lead or a compensator with some zeros. Try the compensator

$$D(z) = \frac{z + a}{z + b}$$

with $a = -0.85$ and $b = 0$.

The corresponding open-loop model



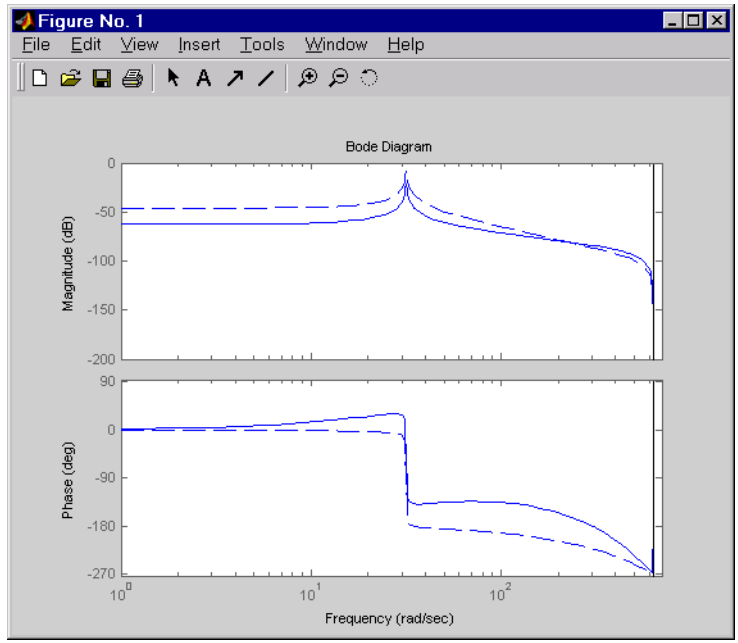
is obtained by the series connection

```
D = zpk(0.85,0,1,Ts)
oloop = Hd * D
```

Now see how this compensator modifies the open-loop frequency response.

```
bode(Hd,'--',oloop,'-')
```

The plant response is the dashed line and the open-loop response with the compensator is the solid line.

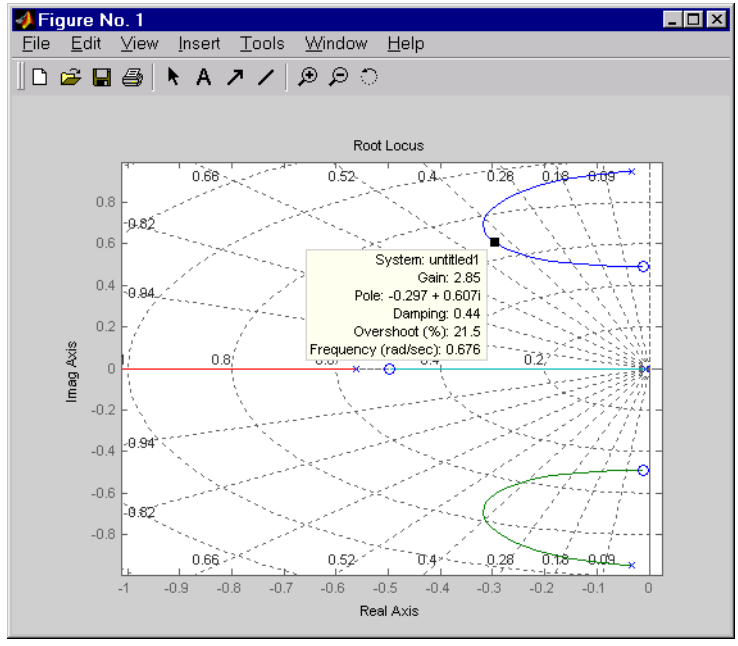


The plot above shows that the compensator has shifted up the phase plot (added lead) in the frequency range $\omega > 10$ rad/sec.

Now try the root locus again with the plant and compensator as open loop.

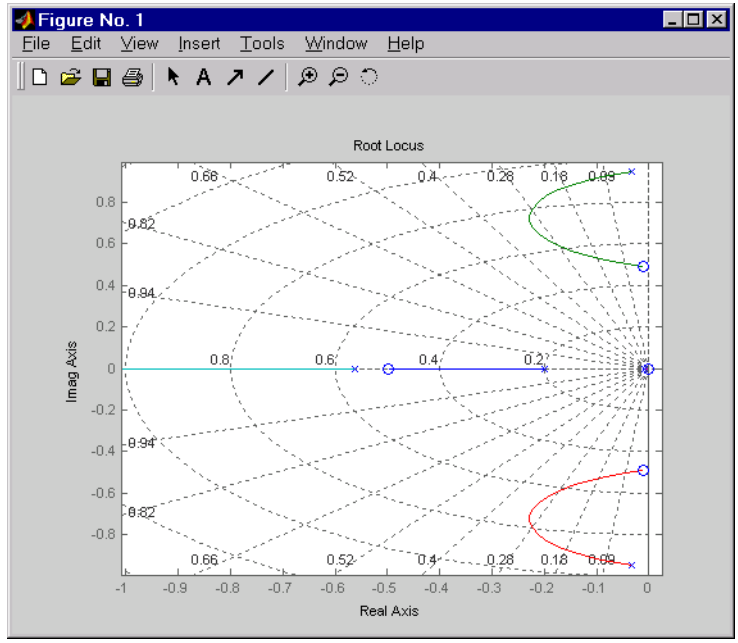
```
rlocus(oloop)
zgrid
```

Open the **Property Editor** by right-clicking in the plot away from the curve. On the **Limits** page, set the *x*-axis limits from -1 to 1.01. This figure shows the result.



This time, the poles stay within the unit circle for some time (the lines drawn by `zgrid` show the damping ratios from $\zeta = 0$ to 1 in steps of 0.1). Use a data

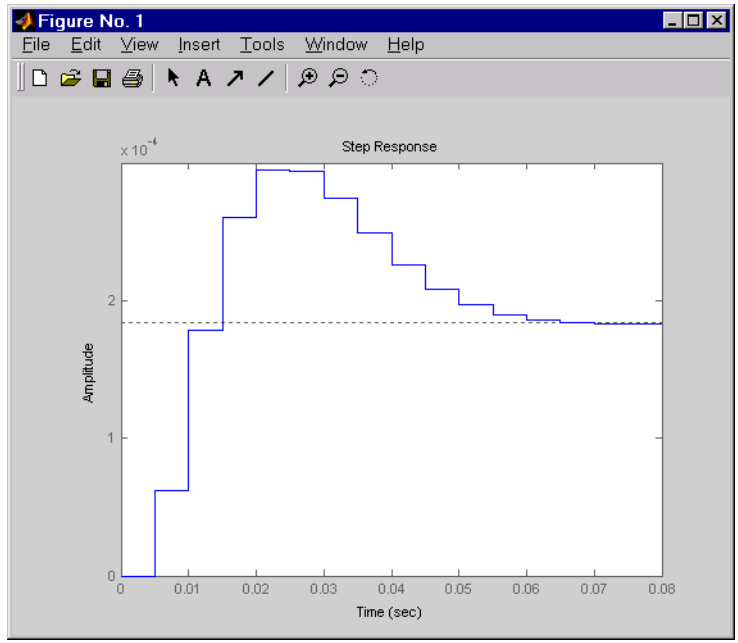
marker to find the point on the curve where the gain equals 4.111e+03. This figure shows the data marker at the correct location.



To analyze this design, form the closed-loop system and plot the closed-loop step response.

```
K = 4.11e+03;
cloop = feedback(oloop,K);
```

```
step(cloop)
```

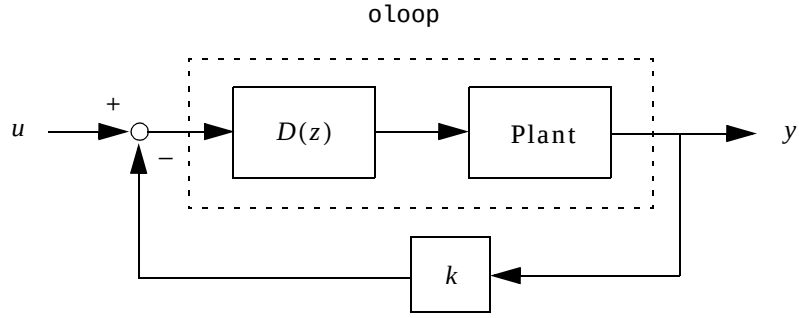


This response depends on your closed loop set point. The one shown here is relatively fast and settles in about 0.07 seconds. Therefore, this closed loop disk drive system has a seek time of about 0.07 seconds. This is slow by today's standards, but you also started with a very lightly damped system.

Now look at the robustness of your design. The most common classical robustness criteria are the gain and phase margins. Use the function `margin` to determine these margins. With output arguments, `margin` returns the gain and phase margins as well as the corresponding crossover frequencies. Without output argument, `margin` plots the Bode response and displays the margins graphically.

To compute the margins, first form the unity-feedback open loop by connecting the compensator $D(z)$, plant model, and feedback gain k in series.

```
olk = K * oloop;
```



Next apply `margin` to this open-loop model. Type

```
[Gm,Pm,Wcg,Wcp] = margin(olk);
Margins = [Gm Wcg Pm Wcp]
```

```
Margins =

    3.7987  296.7978   43.2031  106.2462
```

To obtain the gain margin in dB, type

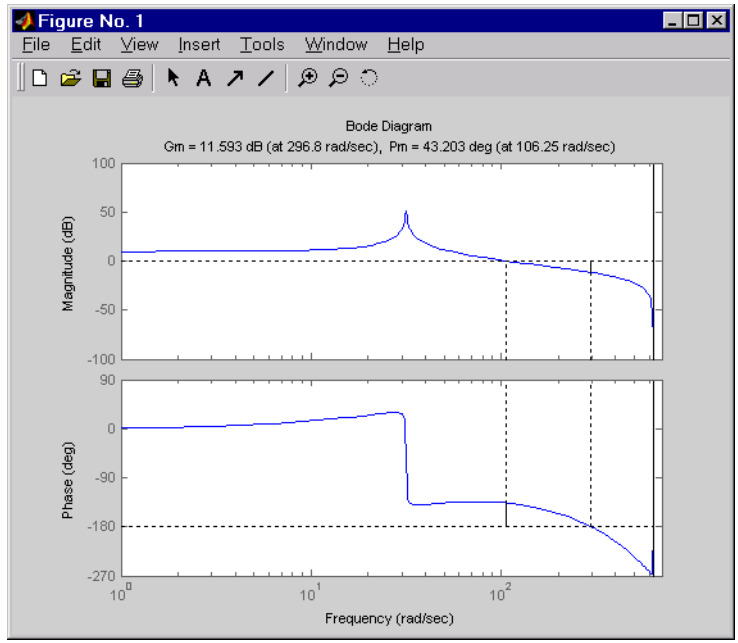
```
20*log10(Gm)
```

```
ans =
   11.5926
```

You can also display the margins graphically by typing

```
margin(olk)
```

The command produces the plot shown below.



This design is robust and can tolerate a 11 dB gain increase or a 40 degree phase lag in the open-loop system without going unstable. By continuing this design process, you may be able to find a compensator that stabilizes the open-loop system and allows you to reduce the seek time.

LQG Regulation: Rolling Mill Example

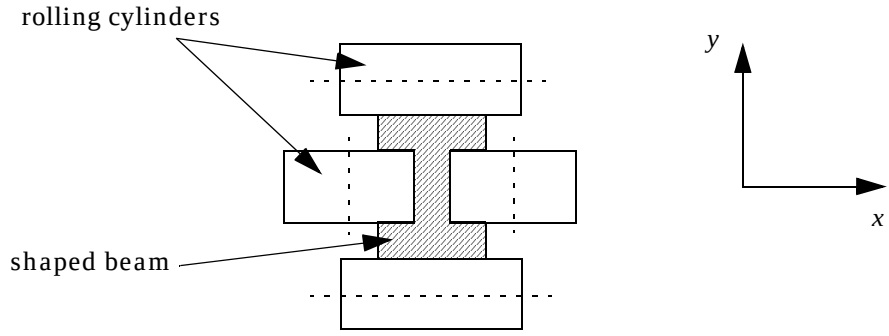
This case study demonstrates the use of the LQG design tools in a process control application. The goal is to regulate the horizontal and vertical thickness of the beam produced by a hot steel rolling mill. This example is adapted from [1]. The full plant model is MIMO and the example shows the advantage of direct MIMO LQG design over separate SISO designs for each axis. Type

```
milldemo
```

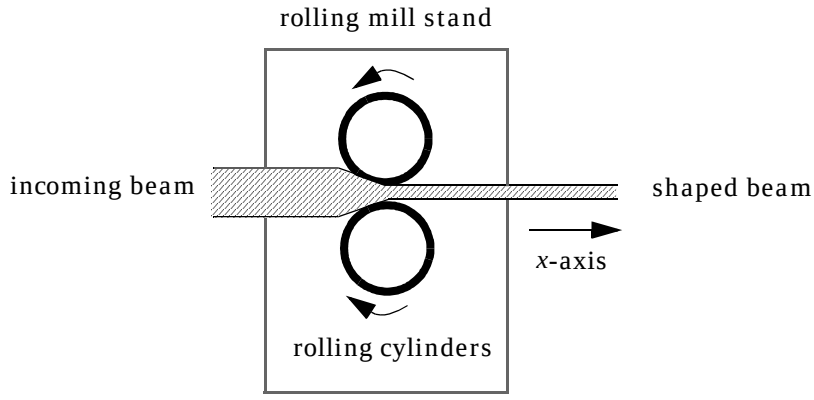
at the command line to run this demonstration interactively.

Process and Disturbance Models

The rolling mill is used to shape rectangular beams of hot metal. The desired outgoing shape is sketched below.



This shape is impressed by two pairs of rolling cylinders (one per axis) positioned by hydraulic actuators. The gap between the two cylinders is called the *roll gap*.



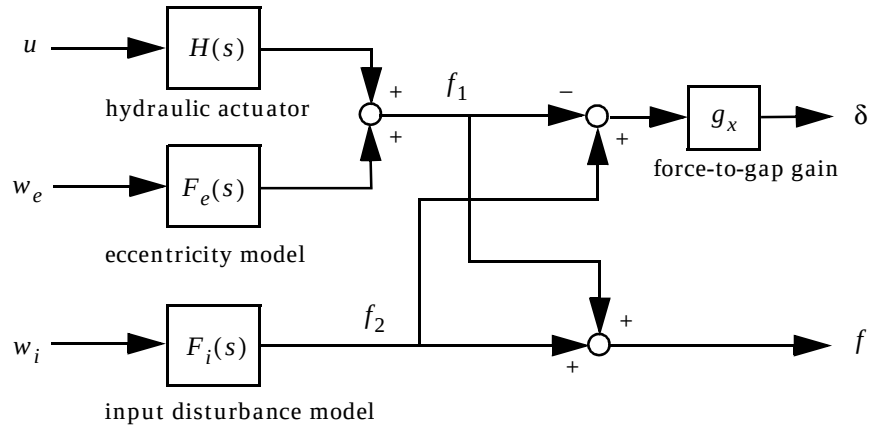
The objective is to maintain the beam thickness along the *x*- and *y*-axes within the quality assurance tolerances. Variations in output thickness can arise from the following:

- Variations in the thickness/hardness of the incoming beam
- Eccentricity in the rolling cylinders

Feedback control is necessary to reduce the effect of these disturbances. Because the roll gap cannot be measured close to the mill stand, the rolling force is used instead for feedback.

The input thickness disturbance is modeled as a low pass filter driven by white noise. The eccentricity disturbance is approximately periodic and its frequency is a function of the rolling speed. A reasonable model for this disturbance is a second-order bandpass filter driven by white noise.

This leads to the following generic model for each axis of the rolling process.



u	command
δ	thickness gap (in mm)
f	incremental rolling force
w_i, w_e	driving white noise for disturbance models

Figure 10-1: Open-loop model for x- or y-axis

The measured rolling force variation f is a combination of the incremental force delivered by the hydraulic actuator and of the disturbance forces due to eccentricity and input thickness variation. Note that:

- The outputs of $H(s)$, $F_e(s)$, and $F_i(s)$ are the incremental forces delivered by each component.
- An increase in hydraulic or eccentricity force *reduces* the output thickness gap δ.
- An increase in input thickness *increases* this gap.

The model data for each axis is summarized below.

Model Data for the x-Axis

$$H_x(s) = \frac{2.4 \times 10^8}{s^2 + 72s + 90^2}$$

$$F_{ix}(s) = \frac{10^4}{s + 0.05}$$

$$F_{ex}(s) = \frac{3 \times 10^4 \ s}{s^2 + 0.125s + 6^2}$$

$$g_x = 10^{-6}$$

Model Data for the y-Axis

$$H_y(s) = \frac{7.8 \times 10^8}{s^2 + 71s + 88^2}$$

$$F_{iy}(s) = \frac{2 \times 10^4}{s + 0.05}$$

$$F_{ey}(s) = \frac{10^5 \ s}{s^2 + 0.19s + 9.4^2}$$

$$g_y = 0.5 \times 10^{-6}$$

LQG Design for the x-Axis

As a first approximation, ignore the cross-coupling between the *x*- and *y*-axes and treat each axis independently. That is, design one SISO LQG regulator for each axis. The design objective is to reduce the thickness variations δ_x and δ_y due to eccentricity and input thickness disturbances.

Start with the *x*-axis. First specify the model components as transfer function objects.

```
% Hydraulic actuator (with input "u-x")
Hx = tf(2.4e8,[1 72 90^2],'inputname','u-x')
```

```
% Input thickness/hardness disturbance model
Fix = tf(1e4,[1 0.05],'inputn','w-ix')

% Rolling eccentricity model
Fex = tf([3e4 0],[1 0.125 6^2],'inputn','w-ex')

% Gain from force to thickness gap
gx = 1e-6;
```

Next build the open-loop model shown in Figure 10-1 above. You could use the function `connect` for this purpose, but it is easier to build this model by elementary `append` and `series` connections.

```
% I/O map from inputs to forces f1 and f2
Px = append([ss(Hx) Fex],Fix)

% Add static gain from f1,f2 to outputs "x-gap" and "x-force"
Px = [-gx gx;1 1] * Px

% Give names to the outputs:
set(Px,'outputn',{'x-gap' 'x-force'})
```

Note: To obtain minimal state-space realizations, always convert transfer function models to state space *before* connecting them. Combining transfer functions and then converting to state space may produce nonminimal state-space models.

The variable `Px` now contains an open-loop state-space model complete with input and output names.

```
Px.inputname

ans =
    'u-x'
    'w-ex'
    'w-ix'

Px.outputname
```

```
ans =
    'x-gap'
    'x-force'
```

The second output `'x-force'` is the rolling force measurement. The LQG regulator will use this measurement to drive the hydraulic actuator and reduce disturbance-induced thickness variations δ_x.

The LQG design involves two steps:

1. Design a full-state-feedback gain that minimizes an LQ performance measure of the form

$$J(u_x) = \int_0^\infty \left(q\delta_x^2 + r u_x^2 \right) dt$$

2. Design a Kalman filter that estimates the state vector given the force measurements `'x-force'`.

The performance criterion $J(u_x)$ penalizes low and high frequencies equally. Because low-frequency variations are of primary concern, eliminate the high-frequency content of δ_x with the low-pass filter $30/(s+30)$ and use the filtered value in the LQ performance criterion.

```
lpf = tf(30,[1 30])

% Connect low-pass filter to first output of Px
Pxdes = append(lpf,1) * Px
set(Pxdes,'outputn',{'x-gap*' 'x-force'})

% Design the state-feedback gain using LQRY and q=1, r=1e-4
kx = lqry(Pxdes(1,1),1,1e-4)
```

Note: `lqry` expects all inputs to be commands and all outputs to be measurements. Here the command `'u-x'` and the measurement `'x-gap*'` (filtered gap) are the first input and first output of `Pxdes`. Hence, use the syntax `Pxdes(1,1)` to specify just the I/O relation between `'u-x'` and `'x-gap*'`.

Next, design the Kalman estimator with the function `kalman`. The process noise

$$w_x = \begin{bmatrix} w_{ex} \\ w_{ix} \end{bmatrix}$$

has unit covariance by construction. Set the measurement noise covariance to 1000 to limit the high frequency gain, and keep only the measured output `'x-force'` for estimator design.

```
estx = kalman(Pxdes(2,:),eye(2),1000)
```

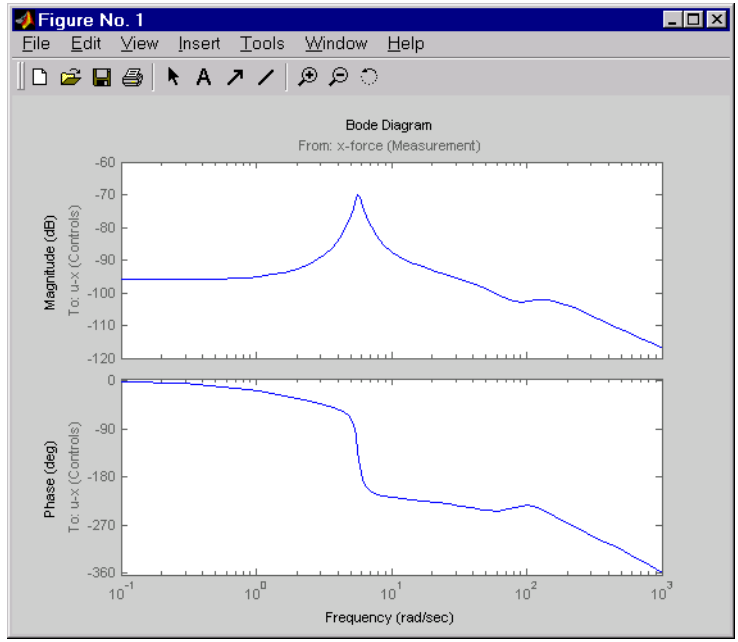
Finally, connect the state-feedback gain `kx` and state estimator `estx` to form the LQG regulator.

```
Regx = lqgreg(estx,kx)
```

This completes the LQG design for the x-axis.

Let's look at the regulator Bode response between 0.1 and 1000 rad/sec.

```
bode(Regx,{0.1 1000})
```



The phase response has an interesting physical interpretation. First, consider an increase in input thickness. This low-frequency disturbance boosts both output thickness and rolling force. Because the regulator phase is approximately 0^o at low frequencies, the feedback loop then adequately reacts by increasing the hydraulic force to offset the thickness increase. Now consider the effect of eccentricity. Eccentricity causes fluctuations in the roll gap (gap between the rolling cylinders). When the roll gap is minimal, the rolling force increases and the beam thickness diminishes. The hydraulic force must then be reduced (negative force feedback) to restore the desired thickness. This is exactly what the LQG regulator does as its phase drops to -180^o near the natural frequency of the eccentricity disturbance (6 rad/sec).

Next, compare the open- and closed-loop responses from disturbance to thickness gap. Use `feedback` to close the loop. To help specify the feedback connection, look at the I/O names of the plant `Px` and regulator `Regx`.

```
Px.inputname
ans =
```

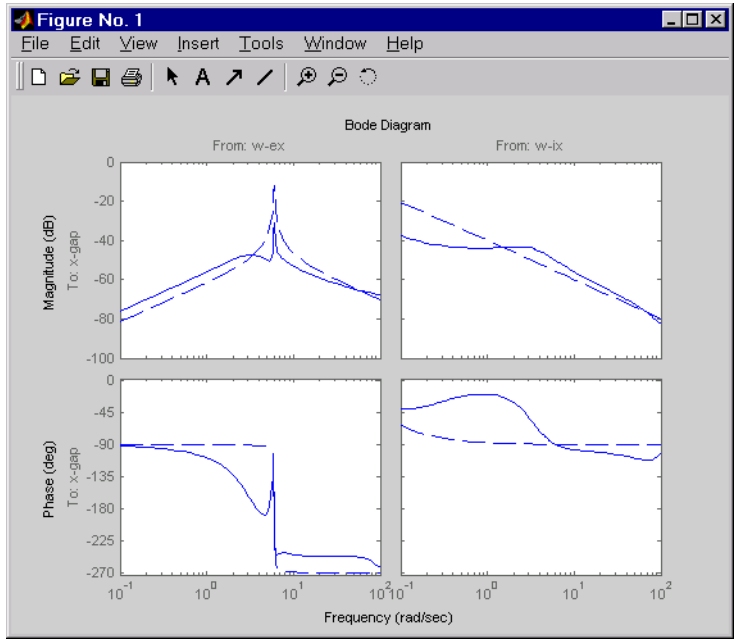
```
    'u-x'
    'w-ex'
    'w-ix'

Regx.outputname
ans =
    'u-x'

Px.outputname
ans =
    'x-gap'
    'x-force'

Regx.inputname
ans =
    'x-force'
```

This indicates that you must connect the first input and second output of `Px` to the regulator.

```
clx = feedback(Px,Regx,1,2,+1)    % Note: +1 for positive feedback
```

You are now ready to compare the open- and closed-loop Bode responses from disturbance to thickness gap.

```
bode(Px(1,2:3),'--',clx(1,2:3),'-',{0.1 100})
```

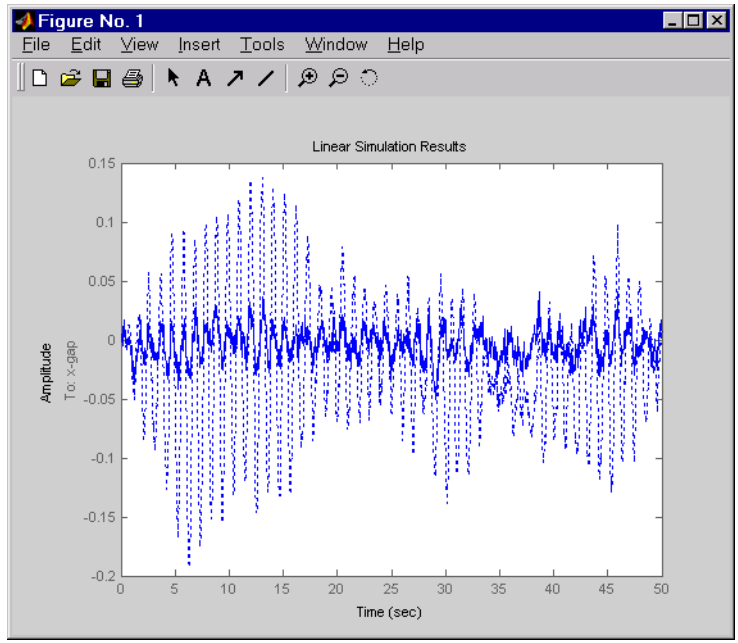


The dashed lines show the open-loop response. Note that the peak gain of the eccentricity-to-gap response and the low-frequency gain of the input-thickness-to-gap response have been reduced by about 20 dB.

Finally, use `lsim` to simulate the open- and closed-loop time responses to the white noise inputs w_{ex} and w_{ix}. Choose `dt=0.01` as sampling period for the simulation, and derive equivalent discrete white noise inputs for this sampling rate.

```
dt = 0.01
t = 0:dt:50   % time samples

% Generate unit-covariance driving noise wx = [w-ex;w-ix].
% Equivalent discrete covariance is 1/dt
wx = sqrt(1/dt) * randn(2,length(t))
```

```
lsim(Px(1,2:3),':',clx(1,2:3),'-',wx,t)
```



The dotted lines correspond to the open-loop response. In this simulation, the LQG regulation reduces the peak thickness variation by a factor 4.

LQG Design for the y-Axis

The LQG design for the *y* -axis (regulation of the *y* thickness) follows the exact same steps as for the *x* -axis.

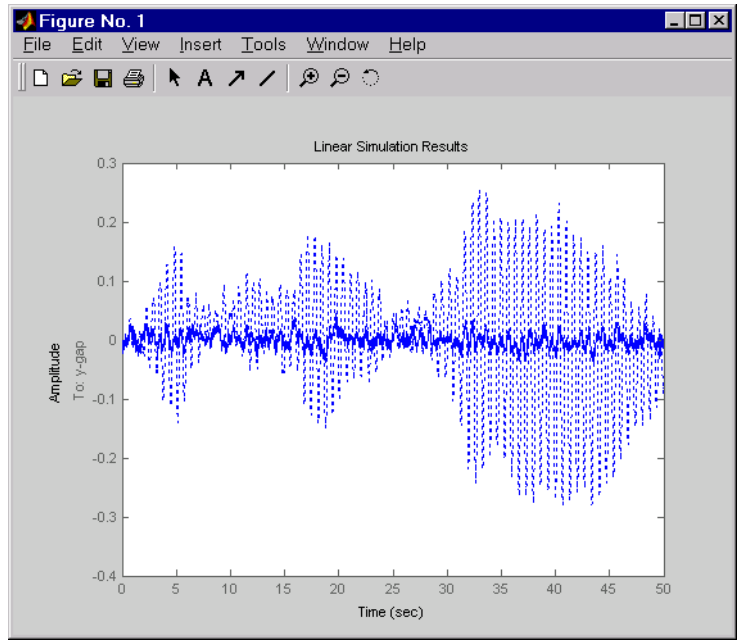
```
% Specify model components
Hy = tf(7.8e8,[1 71 88^2],'inputn','u-y')
Fiy = tf(2e4,[1 0.05],'inputn','w-iy')
Fey = tf([1e5 0],[1 0.19 9.4^2],'inputn','w-ey')
gy = 0.5e-6    % force-to-gap gain

% Build open-loop model
Py = append([ss(Hy) Fey],Fiy)
Py = [-gy gy;1 1] * Py
set(Py,'outputn',{'y-gap' 'y-force'})
```

```
% State-feedback gain design
Pydes = append(lpf,1) * Py     % Add low-freq. weigthing
set(Pydes,'outputn',{'y-gap*' 'y-force'})
ky = lqry(Pydes(1,1),1,1e-4)

% Kalman estimator design
esty = kalman(Pydes(2,:),eye(2),1e3)

% Form SISO LQG regulator for y-axis and close the loop
Regy = lqgreg(esty,ky)
cly = feedback(Py,Regy,1,2,+1)
```

Compare the open- and closed-loop response to the white noise input disturbances.

```
dt = 0.01
t = 0:dt:50
wy = sqrt(1/dt) * randn(2,length(t))
```

```
lsim(Py(1,2:3),':',cly(1,2:3),'-',wy,t)
```

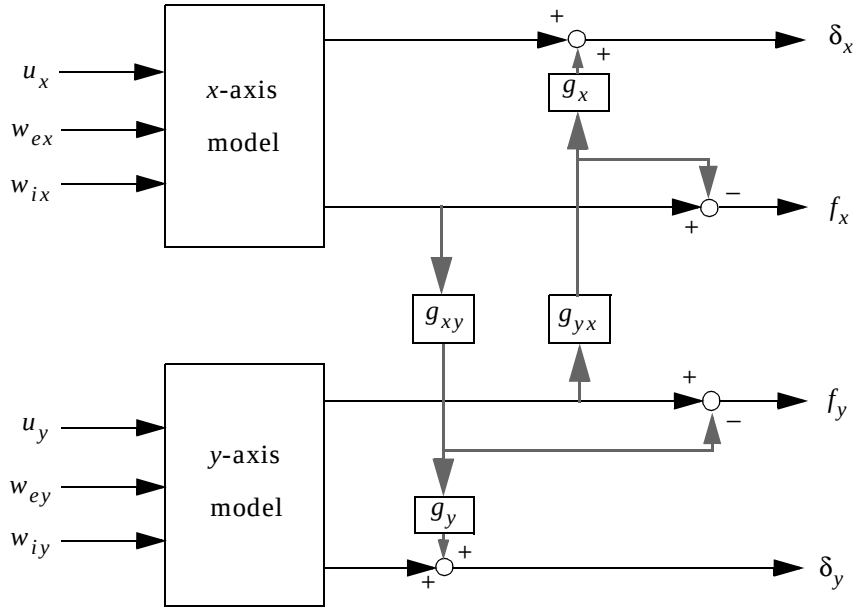


The dotted lines correspond to the open-loop response. The simulation results are comparable to those for the x-axis.

Cross-Coupling Between Axes

The x/y thickness regulation, is a MIMO problem. So far you have treated each axis separately and closed one SISO loop at a time. This design is valid as long as the two axes are fairly decoupled. Unfortunately, this rolling mill process exhibits some degree of cross-coupling between axes. Physically, an increase in hydraulic force along the x-axis compresses the material, which in turn boosts the repelling force on the y-axis cylinders. The result is an increase in y-thickness and an equivalent (relative) decrease in hydraulic force along the y-axis.

The coupling between axes is as follows.



$$g_{xy} = 0.1$$
$$g_{yx} = 0.4$$

Figure 10-2: Coupling between the x- and y-axes

Accordingly, the thickness gaps and rolling forces are related to the outputs $\delta_x, f_x, \ldots$ of the x- and y-axis models by

$$\begin{bmatrix} \delta_x \\ \delta_y \\ f_x \\ f_y \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & g_{yx}g_x \\ 0 & 1 & g_{xy}g_y & 0 \\ 0 & 0 & 1 & -g_{yx} \\ 0 & 0 & -g_{xy} & 1 \end{bmatrix}}_{\text{cross-coupling matrix}} \begin{bmatrix} \bar{\delta}_x \\ \bar{\delta}_y \\ \bar{f}_x \\ \bar{f}_y \end{bmatrix}$$

Let's see how the previous "decoupled" LQG design fares when cross-coupling is taken into account. To build the two-axes model shown in Figure 10-2, append the models Px and Py for the *x* - and *y*-axes.

```
P = append(Px,Py)
```

For convenience, reorder the inputs and outputs so that the commands and thickness gaps appear first.

```
P = P([1 3 2 4],[1 4 2 3 5 6])
P.outputname

ans =
    'x-gap'
    'y-gap'
    'x-force'
    'y-force'
```

Finally, place the cross-coupling matrix in series with the outputs.

```
gxy = 0.1; gyx = 0.4;
CCmat = [eye(2) [0 gyx*gx;gxy*gy 0] ; zeros(2) [1 -gyx;-gxy 1]]
Pc = CCmat * P
Pc.outputname = P.outputname
```

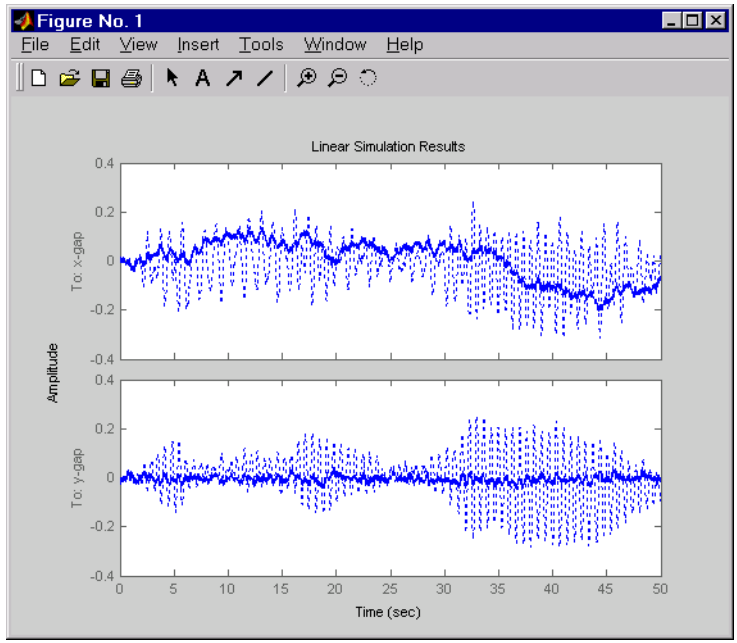
To simulate the closed-loop response, also form the closed-loop model by

```
feedin = 1:2   % first two inputs of Pc are the commands
feedout = 3:4  % last two outputs of Pc are the measurements
cl = feedback(Pc,append(Regx,Regy),feedin,feedout,+1)
```

You are now ready to simulate the open- and closed-loop responses to the driving white noises wx (for the *x*-axis) and wy (for the *y*-axis).

```
wxy = [wx ; wy]
```

```
lsim(Pc(1:2,3:6),':',cl(1:2,3:6),'-',wxy,t)
```



The response reveals a severe deterioration in regulation performance along the *x*-axis (the peak thickness variation is about four times larger than in the simulation without cross-coupling). Hence, designing for one loop at a time is inadequate for this level of cross-coupling, and you must perform a joint-axis MIMO design to correctly handle coupling effects.

MIMO LQG Design

Start with the complete two-axis state-space model `Pc` derived above. The model inputs and outputs are

```
Pc.inputname

ans =
    'u-x'
    'u-y'
    'w-ex'
    'w-ix'
```

```
    'w_ey'
    'w_iy'

P.outputname

ans =
    'x-gap'
    'y-gap'
    'x-force'
    'y-force'
```

As earlier, add low-pass filters in series with the `'x-gap'` and `'y-gap'` outputs to penalize only low-frequency thickness variations.

```
Pdes = append(lpf,lpf,eye(2)) * Pc
Pdes.outputn = Pc.outputn
```

Next, design the LQ gain and state estimator as before (there are now two commands and two measurements).

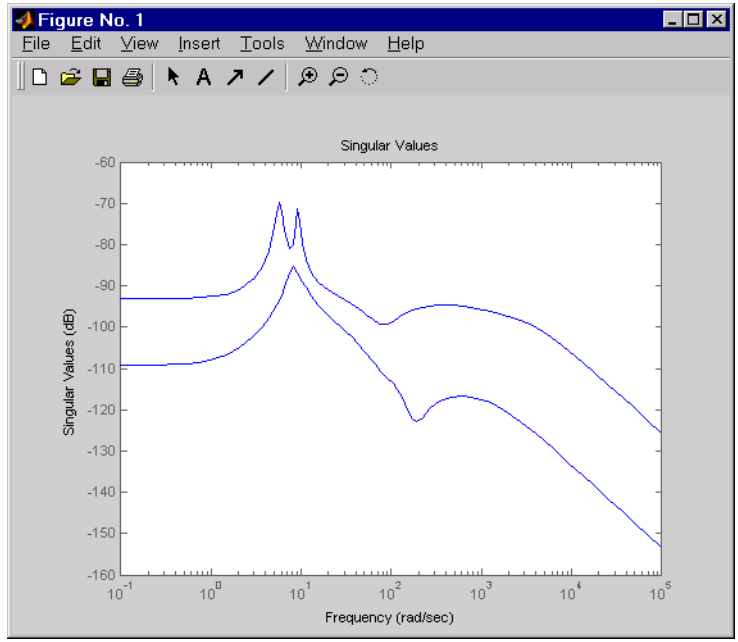
```
k = lqry(Pdes(1:2,1:2),eye(2),1e-4*eye(2))    % LQ gain
est = kalman(Pdes(3:4,:),eye(4),1e3*eye(2))   % Kalman estimator

RegMIMO = lqgreg(est,k)   % form MIMO LQG regulator
```

The resulting LQG regulator `RegMIMO` has two inputs and two outputs.

```
RegMIMO.inputname

ans =
    'x-force'
    'y-force'

RegMIMO.outputname

ans =
    'u-x'
    'u-y'
```

Plot its singular value response (principal gains).

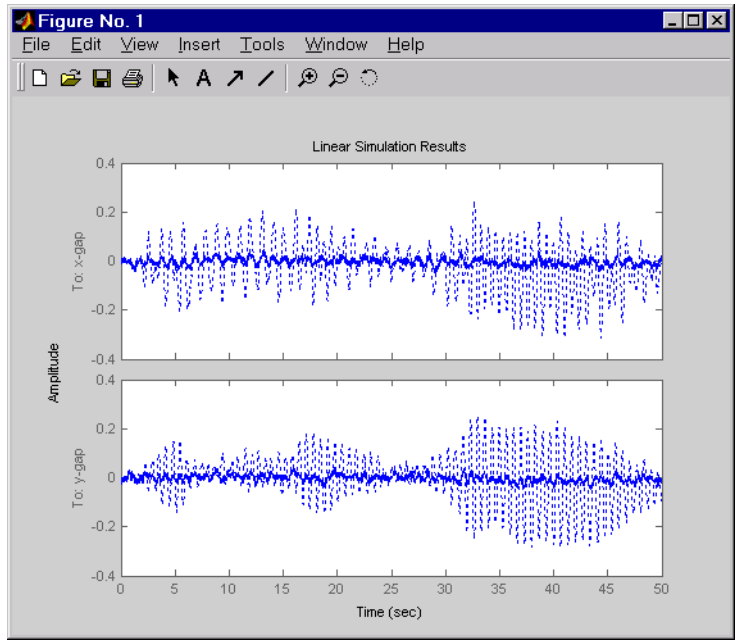
```
sigma(RegMIMO)
```



Next, plot the open- and closed-loop time responses to the white noise inputs (using the MIMO LQG regulator for feedback).

```
% Form the closed-loop model
cl = feedback(Pc,RegMIMO,1:2,3:4,+1);

% Simulate with lsim using same noise inputs
```

```
lsim(Pc(1:2,3:6),':',cl(1:2,3:6),'-',wxy,t)
```



The MIMO design is a clear improvement over the separate SISO designs for each axis. In particular, the level of x/y thickness variation is now comparable to that obtained in the decoupled case. This example illustrates the benefits of direct MIMO design for multivariable systems.

Kalman Filtering

This final case study illustrates the use of the Control System Toolbox for Kalman filter design and simulation. Both steady-state and time-varying Kalman filters are considered.

Consider the discrete plant

$$x[n+1] = Ax[n] + B(u[n] + w[n])$$
$$y[n] = Cx[n]$$

with additive Gaussian noise $w[n]$ on the input $u[n]$ and data

```
A = [1.1269   -0.4940    0.1129
     1.0000         0         0
          0    1.0000         0];

B = [-0.3832
      0.5919
      0.5191];

C = [1 0 0];
```

Our goal is to design a Kalman filter that estimates the output $y[n]$ given the inputs $u[n]$ and the noisy output measurements

$$y_v[n] = Cx[n] + v[n]$$

where $v[n]$ is some Gaussian white noise.

Discrete Kalman Filter

The equations of the steady-state Kalman filter for this problem are given as follows.

Measurement update

$$\hat{x}[n|n] = \hat{x}[n|n-1] + M(y_v[n] - C\hat{x}[n|n-1])$$

Time update

$$\hat{x}[n+1|n] = A\hat{x}[n|n] + Bu[n]$$

In these equations:

- $\hat{x}[n|n-1]$ is the estimate of $x[n]$ given past measurements up to $y_v[n-1]$
- $\hat{x}[n|n]$ is the updated estimate based on the last measurement $y_v[n]$

Given the current estimate $\hat{x}[n|n]$, the time update predicts the state value at the next sample $n+1$ (one-step-ahead predictor). The measurement update then adjusts this prediction based on the new measurement $y_v[n+1]$. The correction term is a function of the *innovation*, that is, the discrepancy.

$$y_v[n+1] - C\hat{x}[n+1|n] = C(x[n+1] - \hat{x}[n+1|n])$$

between the measured and predicted values of $y[n+1]$. The innovation gain M is chosen to minimize the steady-state covariance of the estimation error given the noise covariances

$$E(w[n]w[n]^T) = Q, \qquad E(v[n]v[n]^T) = R$$

You can combine the time and measurement update equations into one state-space model (the Kalman filter).

$$\hat{x}[n+1|n] = A(I-MC)\,\hat{x}[n|n-1] + \begin{bmatrix} B & AM \end{bmatrix} \begin{bmatrix} u[n] \\ y_v[n] \end{bmatrix}$$

$$\hat{y}[n|n] = C(I-MC)\,\hat{x}[n|n-1] + CM\,y_v[n]$$

This filter generates an optimal estimate $\hat{y}[n|n]$ of $y[n]$. Note that the filter state is $\hat{x}[n|n-1]$.

Steady-State Design

You can design the steady-state Kalman filter described above with the function `kalman`. First specify the plant model with the process noise.

$$x[n+1] = Ax[n] + Bu[n] + Bw[n] \qquad \text{(state equation)}$$
$$y[n] = Cx[n] \qquad \text{(measurement equation)}$$

This is done by

```
% Note: set sample time to -1 to mark model as discrete
Plant = ss(A,[B B],C,0,-1,'inputname',{'u' 'w'},...
```

```
                                'outputname','y');
```

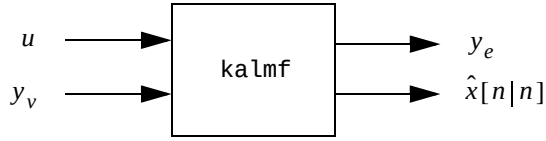
Assuming that $Q = R = 1$, you can now design the discrete Kalman filter by

```
Q = 1; R = 1;
[kalmf,L,P,M] = kalman(Plant,Q,R);
```

This returns a state-space model `kalmf` of the filter as well as the innovation gain

```
M

M =
   3.7980e-01
   8.1732e-02
  -2.5704e-01
```

The inputs of `kalmf` are u and y_v, and its outputs are the plant output and state estimates $y_e = \hat{y}[n|n]$ and $\hat{x}[n|n]$.



Kalman filter

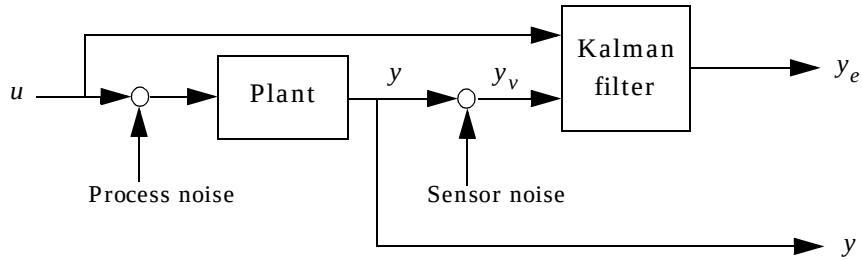
Because you are interested in the output estimate y_e, keep only the first output of `kalmf`. Type

```
kalmf = kalmf(1,:);
kalmf
a =
                    x1_e         x2_e         x3_e
        x1_e       0.7683       -0.494       0.1129
        x2_e       0.6202            0            0
        x3_e    -0.081732            1            0


b =
                       u            y
        x1_e      -0.3832       0.3586
```

```
          x2_e       0.5919       0.3798
          x3_e       0.5191     0.081732


c =
                        x1_e         x2_e         x3_e
           y_e        0.6202            0            0


d =
                           u            y
           y_e             0       0.3798

I/O groups:
       Group name      I/O    Channel(s)
       KnownInput       I         1
      Measurement       I         2
     OutputEstimate     O         1

Sampling time: unspecified
Discrete-time model.
```

To see how the filter works, generate some input data and random noise and compare the filtered response y_e with the true response y. You can either generate each response separately, or generate both together. To simulate each response separately, use `lsim` with the plant alone first, and then with the plant and filter hooked up together. The joint simulation alternative is detailed next.

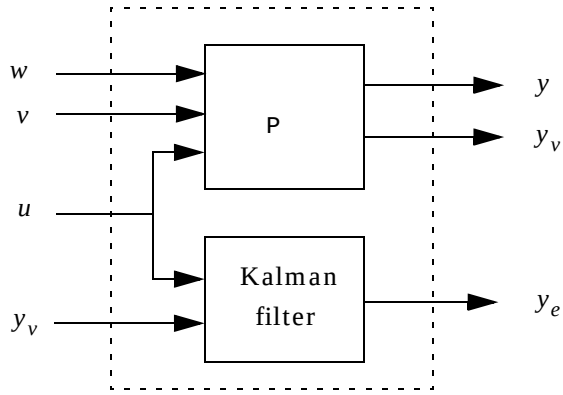
The block diagram below shows how to generate both true and filtered outputs.



You can construct a state-space model of this block diagram with the functions `parallel` and `feedback`. First build a complete plant model with u, w, v as inputs and y and y_v (measurements) as outputs.

```
a = A;
b = [B B 0*B];
c = [C;C];
d = [0 0 0;0 0 1];
P = ss(a,b,c,d,-1,'inputname',{'u' 'w' 'v'},...
                  'outputname',{'y' 'yv'});
```

Then use `parallel` to form the following parallel connection.




```
sys = parallel(P,kalmf,1,1,[],[])
```

Finally, close the sensor loop by connecting the plant output y_v to the filter input y_v with positive feedback.

```
% Close loop around input #4 and output #2
SimModel = feedback(sys,1,4,2,1)
% Delete yv from I/O list
SimModel = SimModel([1 3],[1 2 3])
```

The resulting simulation model has w, v, u as inputs and y, y_e as outputs.

```
SimModel.inputname

ans =
    'w'
```

```
    'v'
    'u'

SimModel.outputname

ans =
    'y'
    'y_e'
```

You are now ready to simulate the filter behavior. Generate a sinusoidal input u and process and measurement noise vectors w and v.

```
t = [0:100]';
u = sin(t/5);

n = length(t)
randn('seed',0)
w = sqrt(Q)*randn(n,1);
v = sqrt(R)*randn(n,1);
```

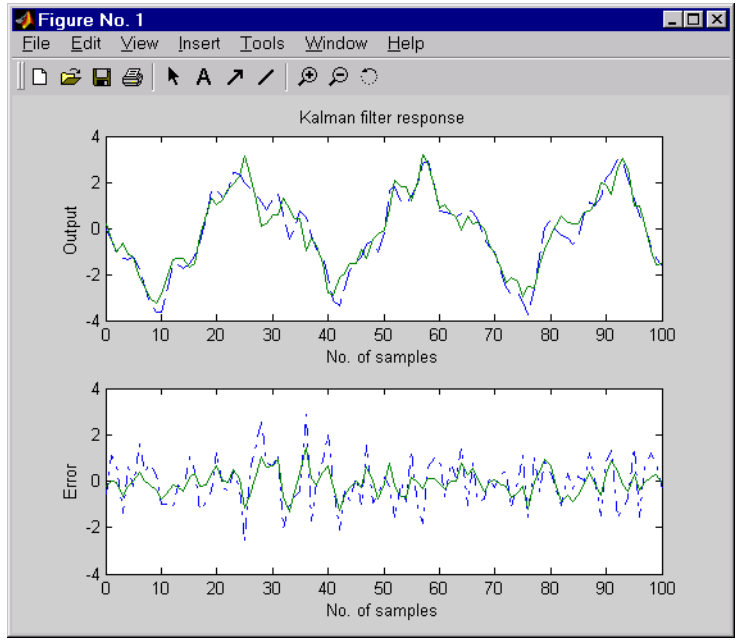
Now simulate with `lsim`.

```
[out,x] = lsim(SimModel,[w,v,u]);

y = out(:,1);   % true response
ye = out(:,2);  % filtered response
yv = y + v;     % measured response
```

and compare the true and filtered responses graphically.

```
subplot(211), plot(t,y,'--',t,ye,'-'),
xlabel('No. of samples'), ylabel('Output')
title('Kalman filter response')
subplot(212), plot(t,y-yv,'-.',t,y-ye,'-'),
```

```
xlabel('No. of samples'), ylabel('Error')
```



The first plot shows the true response y (dashed line) and the filtered output y_e (solid line). The second plot compares the measurement error (dash-dot) with the estimation error (solid). This plot shows that the noise level has been significantly reduced. This is confirmed by the following error covariance computations.

```
MeasErr = y-yv;
MeasErrCov = sum(MeasErr.*MeasErr)/length(MeasErr);
EstErr = y-ye;
EstErrCov = sum(EstErr.*EstErr)/length(EstErr);
```

The error covariance before filtering (measurement error) is

```
MeasErrCov

MeasErrCov =
    1.1138
```

while the error covariance after filtering (estimation error) is only

```
EstErrCov

EstErrCov =
    0.2722
```

Time-Varying Kalman Filter

The time-varying Kalman filter is a generalization of the steady-state filter for time-varying systems or LTI systems with nonstationary noise covariance. Given the plant state and measurement equations

$$x[n+1] = Ax[n] + Bu[n] + Gw[n]$$
$$y_v[n] = Cx[n] + v[n]$$

the time-varying Kalman filter is given by the recursions

Measurement update

$$\hat{x}[n|n] = \hat{x}[n|n-1] + M[n](y_v[n] - C\hat{x}[n|n-1])$$
$$M[n] = P[n|n-1]C^T(R[n] + CP[n|n-1]C^T)^{-1}$$
$$P[n|n] = (I - M[n]C)\ P[n|n-1]$$

Time update

$$\hat{x}[n+1|n] = A\hat{x}[n|n] + Bu[n]$$
$$P[n+1|n] = AP[n|n]A^T + GQ[n]G^T$$

with $\hat{x}[n|n-1]$ and $\hat{x}[n|n]$ as defined on page 10-50, and in the following.

$$Q[n] = E(w[n]w[n]^T)$$
$$R[n] = E(v[n]v[n]^T)$$
$$P[n|n] = E(\{x[n] - x[n|n]\}\{x[n] - x[n|n]\}^T)$$
$$P[n|n-1] = E(\{x[n] - x[n|n-1]\}\{x[n] - x[n|n-1]\}^T)$$

For simplicity, we have dropped the subscripts indicating the time dependence of the state-space matrices.

Given initial conditions $x[1|0]$ and $P[1|0]$, you can iterate these equations to perform the filtering. Note that you must update both the state estimates $x[n|.]$ and error covariance matrices $P[n|.]$ at each time sample.

Time-Varying Design

Although the Control System Toolbox does not offer specific commands to perform time-varying Kalman filtering, it is easy to implement the filter recursions in MATLAB. This section shows how to do this for the stationary plant considered above.

First generate noisy output measurements

```
% Use process noise w and measurement noise v generated above
sys = ss(A,B,C,0,-1);
y = lsim(sys,u+w);     % w = process noise
yv = y + v;            % v = measurement noise
```

Given the initial conditions

$$x[1|0] = 0\,, \qquad P[1|0] = BQB^T$$

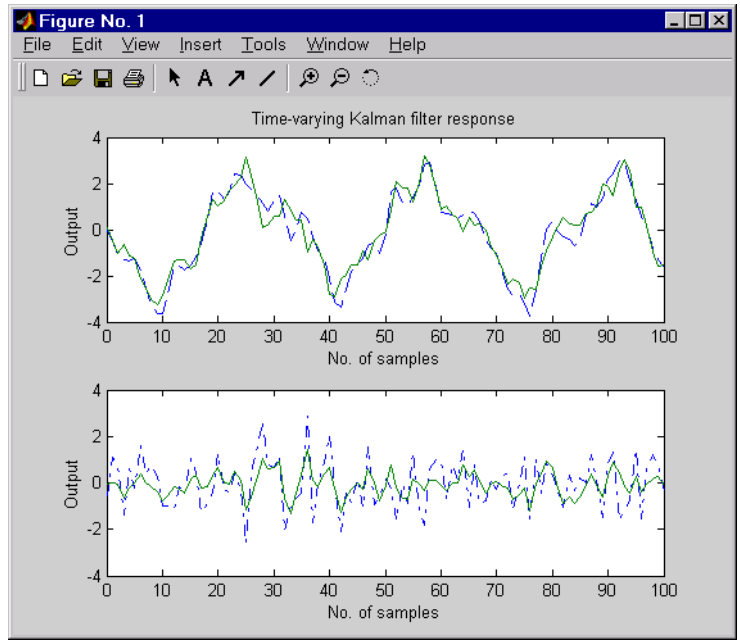
you can implement the time-varying filter with the following `for` loop.

```
P = B*Q*B';         % Initial error covariance
x = zeros(3,1);     % Initial condition on the state
ye = zeros(length(t),1);
ycov = zeros(length(t),1);

for i=1:length(t)
  % Measurement update
  Mn = P*C'/(C*P*C'+R);
  x = x + Mn*(yv(i)-C*x);   % x[n|n]
  P = (eye(3)-Mn*C)*P;      % P[n|n]

  ye(i) = C*x;
  errcov(i) = C*P*C';

  % Time update
```

```
  x = A*x + B*u(i);        % x[n+1|n]
  P = A*P*A' + B*Q*B';     % P[n+1|n]
end
```

You can now compare the true and estimated output graphically.

```
subplot(211), plot(t,y,'--',t,ye,'-')
title('Time-varying Kalman filter response')
xlabel('No. of samples'), ylabel('Output')
subplot(212), plot(t,y-yv,'-.',t,y-ye,'-')
xlabel('No. of samples'), ylabel('Output')
```

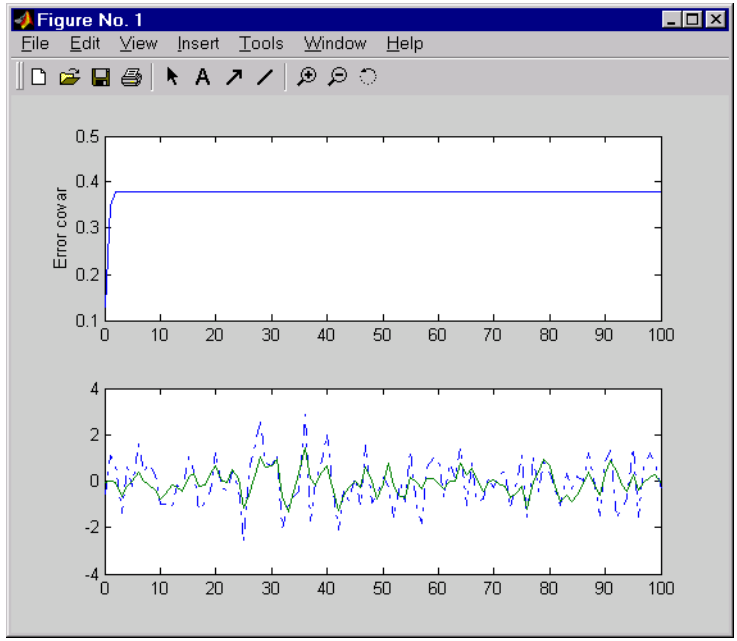


The first plot shows the true response y (dashed line) and the filtered response y_e (solid line). The second plot compares the measurement error (dash-dot) with the estimation error (solid).

The time-varying filter also estimates the covariance `errcov` of the estimation error $y - y_e$ at each sample. Plot it to see if your filter reached steady state (as you expect with stationary input noise).

```
subplot(211)
```

```
plot(t,errcov), ylabel('Error covar')
```



From this covariance plot, you can see that the output covariance did indeed reach a steady state in about five samples. From then on, your time-varying filter has the same performance as the steady-state version.

Compare with the estimation error covariance derived from the experimental data. Type

```
EstErr = y-ye;
EstErrCov = sum(EstErr.*EstErr)/length(EstErr)

EstErrCov =
    0.2718
```

This value is smaller than the theoretical value `errcov` and close to the value obtained for the steady-state design.

Finally, note that the final value $M[n]$ and the steady-state value M of the innovation gain matrix coincide.

```
Mn, M
```

```
Mn =
    0.3798
    0.0817
   -0.2570

M =
    0.3798
    0.0817
   -0.2570
```

References

[1] Grimble, M.J., *Robust Industrial Control: Optimal Design Approach for Polynomial Systems*, Prentice Hall, 1994, p. 261 and pp. 443-456.

11

Reliable Computations

When working with low-order SISO models (less than five states), computers are usually quite forgiving and insensitive to numerical problems. You generally won't encounter any numerical difficulties and MATLAB will give you accurate answers regardless of the model or conversion method you choose. For high order SISO models and MIMO models, however, the finite-precision arithmetic of a computer is not so forgiving and you must exercise caution.

In general, to get a numerically accurate answer from a computer, you need:

- A well-conditioned problem
- An algorithm that is numerically stable in finite-precision arithmetic
- A good software implementation of the algorithm

A problem is said to be well-conditioned if small changes in the data cause only small corresponding changes in the solution. If small changes in the data have the potential to induce large changes in the solution, the problem is said to be ill-conditioned. An algorithm is numerically stable if it does not introduce any more sensitivity to perturbation than is already inherent in the problem. Many numerical linear algebra algorithms can be shown to be backward stable; i.e., the computed solution can be shown to be (near) the exact solution of a slightly perturbed original problem. The solution of a slightly perturbed original problem will be close to the true solution if the problem is well-conditioned.

Thus, a stable algorithm cannot be expected to solve an ill-conditioned problem any more accurately than the data warrant, but an unstable algorithm can produce poor solutions even to well-conditioned problems. For further details and references to the literature see [5].

While most of the tools in the Control System Toolbox use reliable algorithms, some of the tools do not use stable algorithms and some solve ill-conditioned problems. These unreliable tools work quite well on some problems (low-order systems) but can encounter numerical difficulties, often severe, when pushed on higher-order problems. These tools are provided because:

- They are quite useful for low-order systems, which form the bulk of real-world engineering problems.
- Many control engineers think in terms of these tools.
- A more reliable alternative tool is usually available in this toolbox.
- They are convenient for pedagogical purposes.

At the same time, it is important to appreciate the limitations of computer analyses. By following a few guidelines, you can avoid certain tools and models when they are likely to get you into trouble. The following sections try to illustrate, through examples, some of the numerical pitfalls to be avoided. We also encourage you to get the most out of the good algorithms by ensuring, if possible, that your models give rise to problems that are well-conditioned.

Conditioning and Numerical Stability

Two of the key concepts in numerical analysis are the conditioning of problems and the stability of algorithms.

Conditioning

Consider the linear system $Ax = b$ given by

```
A =
    0.7800    0.5630
    0.9130    0.6590
b =
    0.2170
    0.2540
```

The true solution is `x = [1, -1]'` and you can calculate it approximately using MATLAB.

```
x = A\b
x =
     1.0000
    -1.0000

format long, x
x =
     0.99999999991008
    -0.99999999987542
```

Of course, in real problems you almost never have the luxury of knowing the true solution. This problem is very ill-conditioned. To see this, add a small perturbation to `A`

```
E =
     0.0010     0.0010
    -0.0020    -0.0010
```

and solve the perturbed system $(A + E)x = b$

```
xe = (A+E)\b
xe =
    -5.0000
     7.3085
```

Notice how much the small change in the data is magnified in the solution.

One way to measure the magnification factor is by means of the quantity

$$\|A\| \, \|A^{-1}\|$$

called the condition number of A with respect to inversion. The condition number determines the loss in precision due to roundoff errors in Gaussian elimination and can be used to estimate the accuracy of results obtained from matrix inversion and linear equation solution. It arises naturally in perturbation theories that compare the perturbed solution $(A + E)^{-1}b$ with the true solution $A^{-1}b$.

In MATLAB, the function `cond` calculates the condition number in 2-norm. `cond(A)` is the ratio of the largest singular value of `A` to the smallest. Try it for the example above. The usual rule is that the exponent `log10(cond(A))` on the condition number indicates the number of decimal places that the computer can lose to roundoff errors.

IEEE standard double precision numbers have about 16 decimal digits of accuracy, so if a matrix has a condition number of 10^{10}, you can expect only six digits to be accurate in the answer. If the condition number is much greater than `1/sqrt(eps)`, caution is advised for subsequent computations. For IEEE arithmetic, the machine precision, `eps`, is about 2.2×10^{-16}, and `1/sqrt(eps)` = 6.7×10^{8}.

Another important aspect of conditioning is that, in general, residuals are reliable indicators of accuracy only if the problem is well-conditioned. To illustrate, try computing the residual vector $r = Ax - b$ for the two candidate solutions `x = [0.999 -1.001]'` and `x = [0.341 -0.087]'`. Notice that the second, while clearly a much less accurate solution, gives a far smaller residual. The conclusion is that residuals are unreliable indicators of relative solution accuracy for ill-conditioned problems. This is a good reason to be concerned with computing or estimating accurately the condition of your problem.

Another simple example of an ill-conditioned problem is the n-by-n matrix with ones on the first upper-diagonal.

```
A = diag(ones(1,n-1),1);
```

This matrix has n eigenvalues at 0. Now consider a small perturbation of the data consisting of adding the number 2^{-n} to the first element in the last (n th)

row of A. This perturbed matrix has n distinct eigenvalues $\lambda_1, ..., \lambda_n$ with $\lambda_k = 1/2 \exp(j2\pi k/n)$. Thus, you can see that this small perturbation in the data has been magnified by a factor on the order of 2^n to result in a rather large perturbation in the solution (the eigenvalues of A). Further details and related examples are to be found in [7].

It is important to realize that a matrix can be ill-conditioned with respect to inversion but have a well-conditioned eigenproblem, and vice versa. For example, consider an upper triangular matrix of ones (zeros below the diagonal) given by

```
A = triu(ones(n));
```

This matrix is ill-conditioned with respect to its eigenproblem (try small perturbations in A(n,1) for, say, n=20), but is well-conditioned with respect to inversion (check its condition number). On the other hand, the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1+\delta \end{bmatrix}$$

has a well-conditioned eigenproblem, but is ill-conditioned with respect to inversion for small δ.

Numerical Stability

Numerical stability is somewhat more difficult to illustrate meaningfully. Consult the references in [5], [6], and [7] for further details. Here is one small example to illustrate the difference between stability and conditioning.

Gaussian elimination with no pivoting for solving the linear system $Ax = b$ is known to be numerically unstable. Consider

$$A = \begin{bmatrix} 0.001 & 1.000 \\ 1.000 & -1.000 \end{bmatrix} \qquad b = \begin{bmatrix} 1.000 \\ 0.000 \end{bmatrix}$$

All computations are carried out in three-significant-figure decimal arithmetic. The true answer $x = A^{-1}b$ is approximately

$$x = \begin{bmatrix} 0.999 \\ 0.999 \end{bmatrix}$$

Using row 1 as the pivot row (i.e., subtracting 1000 times row 1 from row 2) you arrive at the equivalent triangular system.

$$\begin{bmatrix} 0.001 & 1.000 \\ 0 & -1000 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.000 \\ -1000 \end{bmatrix}$$

Note that the coefficient multiplying x_2 in the second equation should be -1001, but because of roundoff, becomes -1000. As a result, the second equation yields $x_2 = 1.000$, a good approximation, but now back-substitution in the first equation

$$0.001x_1 = 1.000 - (1.000)(1.000)$$

yields $x_1 = 0.000$. This extremely bad approximation of x_1 is the result of numerical instability. The problem itself can be shown to be quite well-conditioned. Of course, MATLAB implements Gaussian elimination with pivoting.

Choice of LTI Model

Now turn to the implications of the results in the last section on the linear modeling techniques used for control engineering. The Control System Toolbox includes the following types of LTI models that are applicable to discussions of computational reliability:

- State space
- Transfer function, polynomial form
- Transfer function, factored zero-pole-gain form

The following subsections show that state space is most preferable for numerical computations.

State Space

The state-space representation is the most reliable LTI model to use for computer analysis. This is one of the reasons for the popularity of "modern" state-space control theory. Stable computer algorithms for eigenvalues, frequency response, time response, and other properties of the (A, B, C, D) quadruple are known [5] and implemented in this toolbox. The state-space model is also the most natural model in MATLAB's matrix environment.

Even with state-space models, however, accurate results are not guaranteed, because of the problems of finite-word-length computer arithmetic discussed in the last section. A well-conditioned problem is usually a prerequisite for obtaining accurate results and makes it important to have reasonable scaling of the data. Scaling is discussed further in the "Scaling" section later in this chapter.

Transfer Function

Transfer function models, when expressed in terms of expanded polynomials, tend to be inherently ill-conditioned representations of LTI systems. For systems of order greater than 10, or with very large/small polynomial coefficients, difficulties can be encountered with functions like `roots`, `conv`, `bode`, `step`, or conversion functions like `ss` or `zpk`.

A major difficulty is the extreme sensitivity of the roots of a polynomial to its coefficients. This example is adapted from Wilkinson, [6] as an illustration. Consider the transfer function

$$H(s) = \frac{1}{(s+1)(s+2)...(s+20)} = \frac{1}{s^{20} + 210s^{19} + ... + 20!}$$

The A matrix of the companion realization of $H(s)$ is

$$A = \begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ : & : & . & . & : \\ 0 & 0 & ... & . & 1 \\ -20! & . & ... & . & -210 \end{bmatrix}$$

Despite the benign looking poles of the system (at $-1,-2,...,-20$) you are faced with a rather large range in the elements of A, from 1 to $20! \approx 2.4 \times 10^{18}$. But the difficulties don't stop here. Suppose the coefficient of s^{19} in the transfer function (or $A(n, n)$) is perturbed from 210 to $210 + 2^{-23}$ ($2^{-23} \approx 1.2 \times 10^{-7}$). Then, computed on a VAX (IEEE arithmetic has enough mantissa for only $n = 17$), the poles of the perturbed transfer function (equivalently, the eigenvalues of A) are

```
eig(A)'

ans =

Columns 1 through 7
-19.9998 -19.0019 -17.9916 -17.0217 -15.9594 -15.0516 -13.9504

Columns 8 through 14
-13.0369 -11.9805 -11.0081  -9.9976  -9.0005  -7.9999  -7.0000

Columns 15 through 20
 -6.0000  -5.0000  -4.0000  -3.0000  -2.0000  -1.0000
```

The problem here is not roundoff. Rather, high-order polynomials are simply intrinsically very sensitive, even when the zeros are well separated. In this case, a relative perturbation of the order of 10^{-9} induced relative perturbations of the order of 10^{-2} in some roots. But some of the roots changed

very little. This is true in general. Different roots have different sensitivities to different perturbations. Computed roots may then be quite meaningless for a polynomial, particularly high-order, with imprecisely known coefficients.

Finding all the roots of a polynomial (equivalently, the poles of a transfer function or the eigenvalues of a matrix in controllable or observable canonical form) is often an intrinsically sensitive problem. For a clear and detailed treatment of the subject, including the tricky numerical problem of deflation, consult [6].

It is therefore preferable to work with the factored form of polynomials when available. To compute a state-space model of the transfer function $H(s)$ defined above, for example, you could expand the denominator of H, convert the transfer function model to state space, and extract the state-space data by

```
H1 = tf(1,poly(1:20))
H1ss = ss(H1)
[a1,b1,c1] = ssdata(H1)
```

However, you should rather keep the denominator in factored form and work with the zero-pole-gain representation of $H(s)$.

```
H2 = zpk([],1:20,1)
H2ss = ss(H2)
[a2,b2,c2] = ssdata(H2)
```

Indeed, the resulting state matrix `a2` is better conditioned.

```
[cond(a1)  cond(a2)]

ans =
   2.7681e+03   8.8753e+01
```

and the conversion from zero-pole-gain to state space incurs no loss of accuracy in the poles.

```
format long e
[sort(eig(a1))  sort(eig(a2))]

ans =
     9.999999999998792e-01     1.000000000000000e+00
     2.000000000001984e+00     2.000000000000000e+00
     3.000000000475623e+00     3.000000000000000e+00
     3.999999981263996e+00     4.000000000000000e+00
```

```
   5.000000270433721e+00     5.000000000000000e+00
   5.999998194359617e+00     6.000000000000000e+00
   7.000004542844700e+00     7.000000000000000e+00
   8.000013753274901e+00     8.000000000000000e+00
   8.999848908317270e+00     9.000000000000000e+00
   1.000059459550623e+01     1.000000000000000e+01
   1.099854678336595e+01     1.100000000000000e+01
   1.200255822210095e+01     1.200000000000000e+01
   1.299647702454549e+01     1.300000000000000e+01
   1.400406940833612e+01     1.400000000000000e+01
   1.499604787386921e+01     1.500000000000000e+01
   1.600304396718421e+01     1.600000000000000e+01
   1.699828695210055e+01     1.700000000000000e+01
   1.800062935148728e+01     1.800000000000000e+01
   1.899986934359322e+01     1.900000000000000e+01
   2.000001082693916e+01     2.000000000000000e+01
```

There is another difficulty with transfer function models when realized in state-space form with ss. They may give rise to badly conditioned eigenvector matrices, even if the eigenvalues are well separated. For example, consider the normal matrix

```
A = [5 4 1 1
     4 5 1 1
     1 1 4 2
     1 1 2 4]
```

Its eigenvectors and eigenvalues are given as follows.

```
[v,d] = eig(A)

v =
    0.7071   -0.0000   -0.3162    0.6325
   -0.7071    0.0000   -0.3162    0.6325
    0.0000    0.7071    0.6325    0.3162
   -0.0000   -0.7071    0.6325    0.3162

d =
    1.0000         0         0         0
         0    2.0000         0         0
         0         0    5.0000         0
```

```
         0         0         0   10.0000
```

The condition number (with respect to inversion) of the eigenvector matrix is

```
cond(v)

ans =
    1.000
```

Now convert a state-space model with the above A matrix to transfer function form, and back again to state-space form.

```
b = [1 ; 1 ; 0 ; -1];
c = [0 0 2 1];
H = tf(ss(A,b,c,0));     % transfer function
[Ac,bc,cc] = ssdata(H)   % convert back to state space
```

The new A matrix is

```
Ac =
   18.0000   -6.0625    2.8125   -1.5625
   16.0000         0         0         0
         0    4.0000         0         0
         0         0    1.0000         0
```

Note that Ac is not a standard companion matrix and has already been balanced as part of the ss conversion (see ssbal for details).

Note also that the eigenvectors have changed.

```
[vc,dc] = eig(Ac)

vc =
   -0.5017    0.2353    0.0510    0.0109
   -0.8026    0.7531    0.4077    0.1741
   -0.3211    0.6025    0.8154    0.6963
   -0.0321    0.1205    0.4077    0.6963

dc =
   10.0000         0         0         0
         0    5.0000         0         0
         0         0    2.0000         0
         0         0         0    1.0000
```

The condition number of the new eigenvector matrix

```
cond(vc)

ans =
    34.5825
```

is thirty times larger.

The phenomenon illustrated above is not unusual. Matrices in companion form or controllable/observable canonical form (like Ac) typically have worse-conditioned eigensystems than matrices in general state-space form (like A). This means that their eigenvalues and eigenvectors are more sensitive to perturbation. The problem generally gets far worse for higher-order systems. Working with high-order transfer function models and converting them back and forth to state space is numerically risky.

In summary, the main numerical problems to be aware of in dealing with transfer function models (and hence, calculations involving polynomials) are:

- The potentially large range of numbers leads to ill-conditioned problems, especially when such models are linked together giving high-order polynomials.
- The pole locations are very sensitive to the coefficients of the denominator polynomial.
- The balanced companion form produced by ss, while better than the standard companion form, often results in ill-conditioned eigenproblems, especially with higher-order systems.

The above statements hold even for systems with distinct poles, but are particularly relevant when poles are multiple.

Zero-Pole-Gain Models

The third major representation used for LTI models in MATLAB is the factored, or zero-pole-gain (ZPK) representation. It is sometimes very convenient to describe a model in this way although most major design methodologies tend to be oriented towards either transfer functions or state-space.

In contrast to polynomials, the ZPK representation of systems can be more reliable. At the very least, the ZPK representation tends to avoid the

extraordinary arithmetic range difficulties of polynomial coefficients, as illustrated in the “Transfer Function” section. The transformation from state space to zero-pole-gain is stable, although the handling of infinite zeros can sometimes be tricky, and repeated roots can cause problems.

If possible, avoid repeated switching between different model representations. As discussed in the previous sections, when transformations between models are not numerically stable, roundoff errors are amplified.

Scaling

State space is the preferred model for LTI systems, especially with higher order models. Even with state-space models, however, accurate results are not guaranteed, because of the finite-word-length arithmetic of the computer. A well-conditioned problem is usually a prerequisite for obtaining accurate results.

You should generally normalize or scale the (A, B, C, D) matrices of a system to improve their conditioning. An example of a poorly scaled problem might be a dynamic system where two states in the state vector have units of light years and millimeters. You would expect the A matrix to contain both very large and very small numbers. Matrices containing numbers widely spread in value are often poorly conditioned both with respect to inversion and with respect to their eigenproblems, and inaccurate results can ensue.

Normalization also allows meaningful statements to be made about the degree of controllability and observability of the various inputs and outputs.

A set of (A, B, C, D) matrices can be normalized using diagonal scaling matrices N_u, N_x, and N_y to scale u, x, and y.

$$u = N_u u_n \qquad x = N_x x_n \qquad y = N_y y_n$$

so the normalized system is

$$\dot{x}_n = A_n x_n + B_n u_n$$
$$y_n = C_n x_n + D_n u_n$$

where

$$A_n = N_x^{-1} A N_x \qquad B_n = N_x^{-1} B N_u$$
$$C_n = N_y^{-1} C N_x \qquad D_n = N_y^{-1} D N_u$$

Choose the diagonal scaling matrices according to some appropriate normalization procedure. One criterion is to choose the maximum range of each of the input, state, and output variables. This method originated in the days of analog simulation computers when u_n, x_n, and y_n were forced to be between ± 10 Volts. A second method is to form scaling matrices where the diagonal

entries are the smallest deviations that are significant to each variable. An excellent discussion of scaling is given in the introduction to the *LINPACK Users' Guide*, [1].

Choose scaling based upon physical insight to the problem at hand. If you choose not to scale, and for many small problems scaling is not necessary, be aware that this choice affects the accuracy of your answers.

Finally, note that the function `ssbal` performs automatic scaling of the state vector. Specifically, it seeks to minimize the norm of

$$\begin{bmatrix} N_x^{-1} A N_x & N_x^{-1} B \\ C N_x & 0 \end{bmatrix}$$

by using diagonal scaling matrices N_x. Such diagonal scaling is an economical way to compress the numerical range and improve the conditioning of subsequent state-space computations.

Summary

This chapter has described numerous things that can go wrong when performing numerical computations. You won't encounter most of these difficulties when you solve practical lower-order problems. The problems described here pertain to all computer analysis packages. MATLAB has some of the best algorithms available, and, where possible, notifies you when there are difficulties. The important points to remember are:

- State-space models are, in general, the most reliable models for subsequent computations.
- Scaling model data can improve the accuracy of your results.
- Numerical computing is a tricky business, and virtually all computer tools can fail under certain conditions.

References

[1] Dongarra, J.J., J.R. Bunch, C.B. Moler, and G.W. Stewart, *LINPACK Users Guide*, SIAM Publications, Philadelphia, PA, 1978.

[2] Franklin, G.F. and J.D. Powell, *Digital Control of Dynamic Systems*, Addison-Wesley, 1980.

[3] Kailath, T., *Linear Systems*, Prentice-Hall, 1980.

[4] Laub, A.J., "Numerical Linear Algebra Aspects of Control Design Computations," *IEEE Transactions on Automatic Control*, Vol. AC-30, No. 2, February 1985, pp. 97-108.

[5] Wilkinson, J.H., *Rounding Errors in Algebraic Processes*, Prentice-Hall, 1963.

[6] Wilkinson, J.H., *The Algebraic Eigenvalue Problem*, Oxford University Press, 1965.

12

GUI Reference

The next three chapters discuss in detail all the features of the LTI Viewer and the SISO Design Tool. The LTI Viewer is a graphical user interface (GUI) that allows you to display up to six response plots in a single window. The SISO Design Tool is a GUI that facilitates the design of compensators for single-input single-output (SISO) systems. It supports both root locus and Bode diagram design techniques.

This document is not meant as an introduction to these tools. For an introduction to the LTI Viewer, see Chapter 2, “Analyzing Models,” in Getting Started with the Control System Toolbox. For an introduction to the SISO Design Tool, see Chapter 3, “Designing Compensators,” in the same book.

13

SISO Design Tool Reference

The SISO Design Tool is a graphical-user interface (GUI) that allows you to use root-locus and Bode diagram techniques to design compensators. The SISO Design Tool by default displays the root locus and Bode diagrams for your imported systems. The two are dynamically linked; for example, if you change the gain in the root locus, it immediately affects the Bode diagrams as well.

This tool is used extensively in Getting Started with the Control System Toolbox, and in particular, you should read Chapter 4, "Designing Compensators," of that book to see how to do typical design tasks with the SISO Design Tool. This document, on the other hand, is a reference that describes all available options for the SISO Design Tool.

Type

```
sisotool
```

to open the SISO Design Tool. This picture shows the GUI and introduces some terminology.

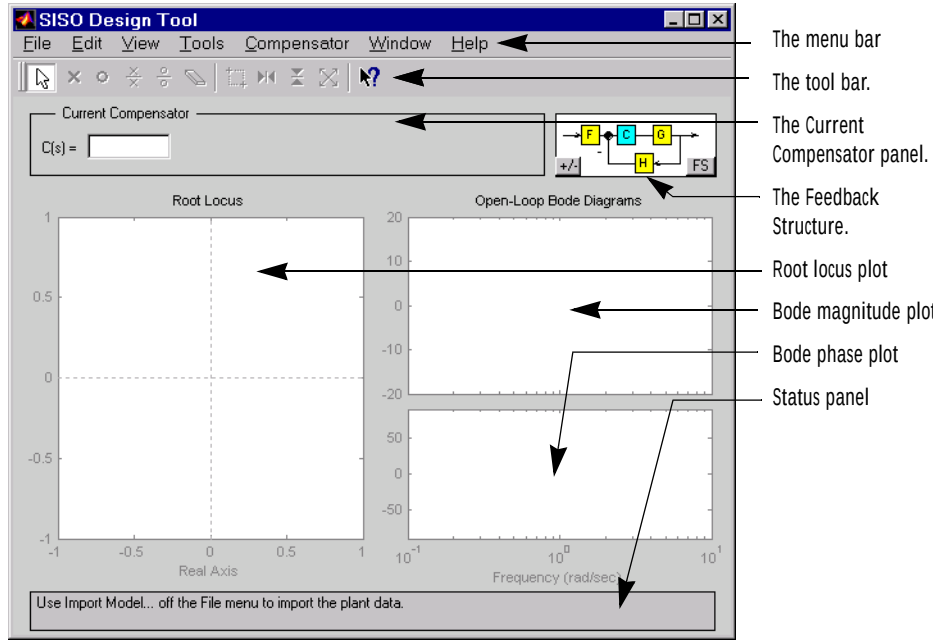


Figure 13-1: The SISO Design Tool and Some Terminology

This document describes the SISO Design Tool features left-to-right and top-to-bottom, starting with the Menu bar and ending with the Status panel at the bottom of the window.

If you want to match the SISO Design Tool pictures shown below, type

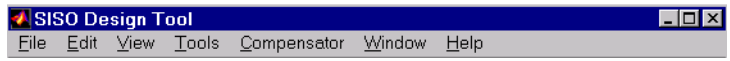
```
load ltiexamples
```

at the MATLAB prompt. This loads the same set of linear models that this document uses as examples in the GUI. The examples all use the `Gservo` system for plot displays.

Menu Bar

Note Click on items on the menu bar pictured below to get help contents.

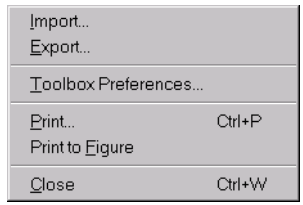
Most of the tasks you can do in the SISO Design Tool can be done from the menu bar, which this picture shows.



File

Note Click on items in the **File** menu pictured below to get help contents.

.



Using the File menu, you can:

- Import and export models
- Set toolbox preferences
- Print and print to figure
- Close the SISO Design Tool

The following sections describe the **File** menu options in turn.

Import

Select Import to import models into the SISO Design Tool. Selecting Import opens the Import System Data window, which is shown below.

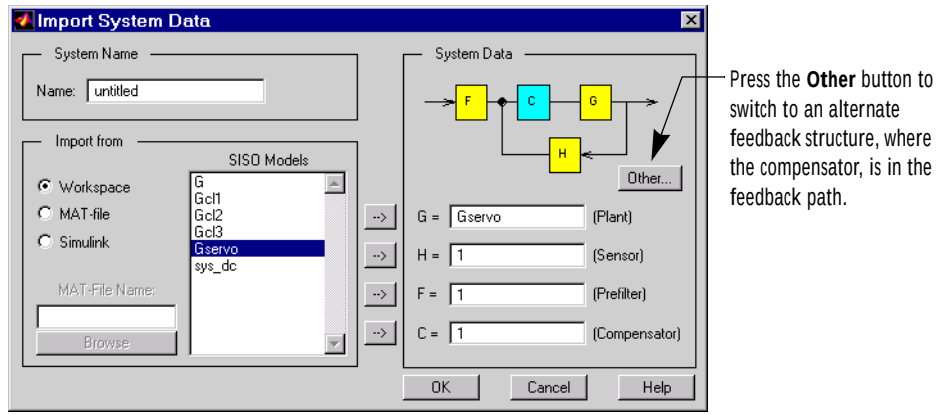


Figure 13-2: The Import System Data Window

The following sections discuss the System Name, Import from, and System Data panels of the **Import System Data** window.

System Name

Use the **Name** field to assign a name to the imported system. The default name is `untitled`.

Import From

To import models, select them from the SISO Models list and use the right arrow buttons to place the models in G (plant), H (sensor), F (prefilter), or C (compensator). You can import models from:

- The MATLAB Workspace
- A MAT-file
- Simulink (`.mdl` files)

System Data

The System Data panel performs two functions:

- Feedback structure specification — Press **Other** to toggle between placing the compensator in the forward and feedback paths
- Model import specification — You can import models for the plant (**G**), compensator (**C**), prefilter (**F**), and/or sensor (**H**). To import a model, select it from the SISO model list and press the right-arrow button next to the desired model field.

Export

Selecting Export from the File menu opens the **SISO Tool Export** Window.

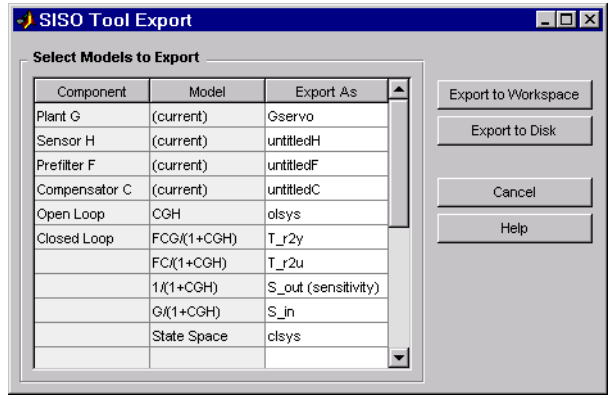


Figure 13-3: The SISO Tool Export Window

With this window, you can:

- Export models to the MATLAB Workspace or to a disk
- Rename models when exporting
- Save variations on models, including open and closed loop models, sensitivity transfer functions, and state-space representations

Exporting to the Workspace

To export models to the MATLAB workspace, follow these steps:

1 Select the model you want to export from the Component list by left-clicking the model name. To select more than one model, hold down the **Shift** key if

they are adjacent on the list. If you want to save non-adjacent models, hold down the **Ctrl** key while selecting the models.

2 For each model you want to save, specify a name in the model's cell in the **Export As** list. A default name exists if you do not want to assign a new name.

3 Press **Export to Workspace**.

Exporting to a MAT-file

If you want to save your models in a MAT-file, follow steps 1 and 2 and press **Export to Disk**, which opens this window.

Choose where you want to save the file in the **Save as** field and specify the name you want for your MAT-file in the **File name** field. Press **Save** to save the file.

Toolbox Preferences

Select Toolbox Preferences to open the Control System Toolbox Preferences menu. This picture shows the window.

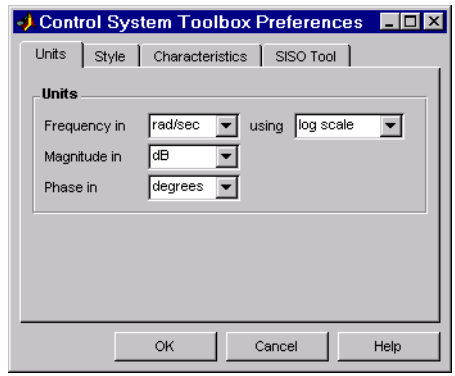


Figure 13-4: The Control System Toolbox Preferences Window

You can use this window to do the following:

- Change units
- Add grids
- Change fonts in titles, labels, and I/O names
- Change plot characteristics, where applicable
- Alter the Compensator format
- Show or hide system poles and zeros on Bode plots

Any changes you make to the toolbox preferences are saved when you close the SISO Design Tool and when you end a MATLAB session. When you restart a session, the changes are the new defaults for your session. These defaults are not only for the SISO Design Tool, but for any response object (e.g., step, impulse, nyquist) available in the Control System Toolbox.

For a discussion of properties and preferences, see Setting Plot Properties and Preferences in the online help.

Print

Use **Print** to send a picture of the SISO Design Tool to your printer.

Print to Figure

Print to Figure opens a separate figure window containing the root locus and/or Bode diagrams in your current SISO Design Tool.

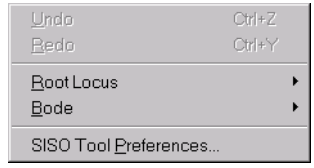
Close

Use **Close** to close the SISO Design Tool.

Edit

Note Click on items in the **Edit** menu pictured below to get help contents.

.



Undo and Redo

Use **Undo** and **Redo** to go back and forward in the design steps. Note that both **Undo** and **Redo** menus change when the task you have just performed changes. For example, if you change the compensator gain, the menu items become **Undo Gain** and **Redo Gain**.

Root Locus and Bode Diagrams

Root Locus and **Bode Diagrams** replicate the functionality of the right-click menus. See “Root Locus Right-Click Menus” and “Bode Diagram Right-Click Menus” for complete descriptions of these menus.

SISO Tool Preferences

SISO Tool Preferences opens the **SISO Tool Preferences** editor. This picture shows the open window.

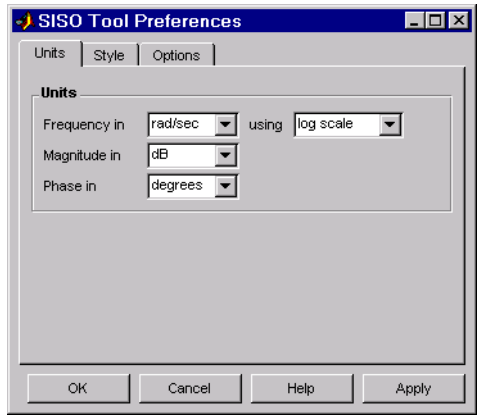


Figure 13-5: The SISO Tool Preferences Editor

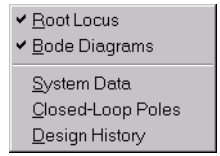
You can use this window to do the following:

- Change units
- Add plot grids, change font styles for titles, labels, etc., and change axes foreground colors
- Change the compensator format and

For a complete description of properties and preferences, see SISO Design Preferences.

View

Note Click on items in the **View** menu pictured below to get help contents.



Root Locus and Bode Diagrams

By default, the SISO Design Tool displays the root locus and Bode magnitude and phase diagrams. You can deselect either to show only the root locus or the Bode diagram.

System Data

System Data opens the window shown below.

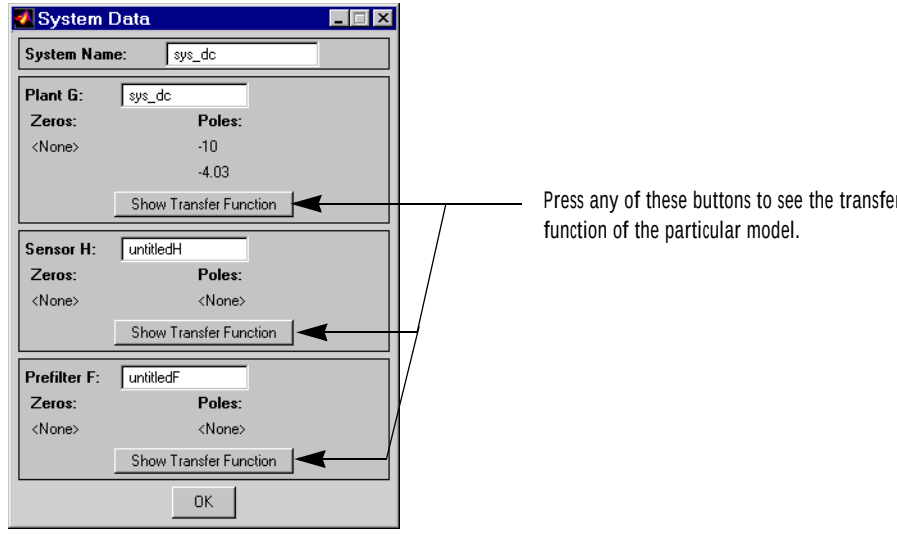


Figure 13-6: The System Data Window

The System Data window displays basic information about the models you've imported.

Closed Loop Poles

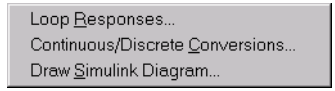
Use this menu item to display the closed-loop pole values of the current system.

Design History

Selecting Design History opens the Design History window, which displays all the actions you've performed during a design session. You can save the history to an ASCII flat text file.

Tools

Note Click on items in the **Tools** menu pictured below to get help contents.



Loop Responses

Select **Loop Responses** to open an LTI Viewer that is dynamically linked to your SISO Design Tool. When you make changes to the design in the SISO Design Tool, the response plots in the LTI Viewer automatically change to reflect the new design's responses.

For examples that use LTI Viewers linked with the SISO Design Tool, see Designing Compensators in *Getting Started with the Control System Toolbox*. See the "LTI Viewer" for a complete description of all the features of the LTI Viewer.

You have the following choices for which plot appears when opening an LTI Viewer from the SISO Design Tool:

- **Plant Output (Step)** — The closed-loop step response of your system
- **Control Signal (Step)** — The open-loop step response of your system
- **Compensator Bode** — The open-loop Bode diagram for your compensator
- **Closed-Loop Bode** — The closed-loop Bode diagram for your system

- **Open-Loop Nyquist** — The open-loop Nyquist plot for your system
- **Open-Loop Nichols** — The open-loop Nichols plot for your system

Customizing Loop Responses

If you choose **Custom** from the list of loop responses, the **Response Plot Setup** window opens.

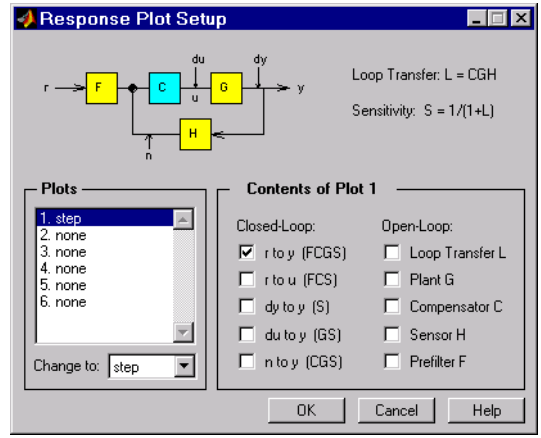


Figure 13-7: Response Plot Setup Window

This window has many options for creating more specialized response plots, but to use it properly you should be familiar with the basic features of the LTI Viewer. If you are not, see Analyzing Models in *Getting Started with the Control System Toolbox* or "LTI Viewer" in the online documentation.

Loop diagram. At the top of the Response Plod Setup window is a loop diagram. This block diagram shows the feedback structure of your system. The diagram in Figure 13-7 shows the default configuration; the compensator is in the forward path. If your system has the compensator in the feedback path, this window correctly displays the alternate feedback structure.

The loop block diagram shows the arrangement of the components of your system and the input/output structure. When selecting open- and closed-loop responses in the Contents of Plots panel, refer to the loop diagram for definitions of the responses.

Note that window lists two transfer functions next to the loop diagram:

- Loop transfer — This is defined as the compensator (**C**), the plant (**G**), and the sensor (**H**) multiplied together (**CGH**). If you haven't defined a sensor, its default value is 1.
- Sensitivity function — This is defined as $\frac{1}{1+L}$, where L is the loop transfer function.

Some of the open- and closed-loop responses use these definitions. See "Contents of plots" for more information.

Plots. You can have up to six plots in one LTI Viewer. By default, the Response Plot Setup window specifies one step response plot. To reconfigure the LTI Viewer for more plots, start by selecting "2. None" from the list of plots and then specify a new plot type in the **Change to** field. Plot types available include the following:

- Step
- Impulse
- Bode
- Bode Magnitude
- Nyquist
- Nichols
- Sigma
- Pzmap (pole/zero map)
- None (deselect a plot)

Note that you do not have to select adjacent numbers; for example, if you specify plot #1 to be a step response, plot #2 to be none, and plot #3 to be an impulse response, the LTI Viewer will open with two plots, a step and an impulse response. There will not be an empty plot region.

Contents of plots. Once you have selected a plot, you can specify various open- and closed-loop transfer functions. You can plot open-loop responses for each of the components of your system, including your compensator (**C**), plant (**G**), prefilter (**F**), or sensor (**H**). In addition, loop transfer and sensitivity transfer functions are available. Their definitions are listed in the Response Plot Setup window.

See the block diagram in Figure 13-7, Response Plot Setup Window for definitions of the input/output points for closed-loop responses.

Continuous/Discrete Conversions

Selecting **Continuous/Discrete Conversions** opens the **Continuous/Discrete Conversions** window, which you can use to convert between continuous to discrete designs.You can select the following:

- Conversion method
- Sample time
- Critical frequency (where applicable)

This picture shows the window.

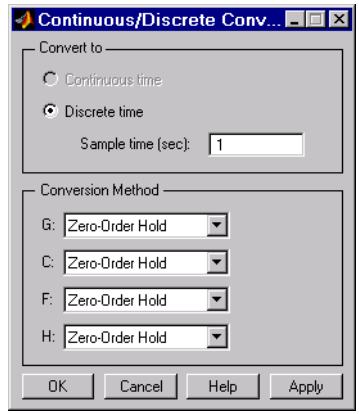


Figure 13-8: The Continuous/Discrete Conversion Window

Conversion domain. If your current model is continuous-time, the upper panel of the Continuous/Discrete Conversion window automatically selects the **Discrete time** radio button. If your model is in discrete-time, see "Discrete-time domain".

To convert to discrete time, you must specify a positive number for the sample time in the **Sample time (sec)** field.

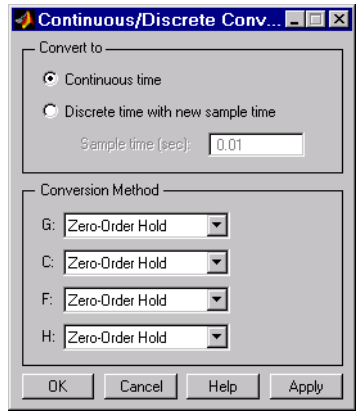
You can perform continuous to discrete conversions on any of the components of your model: the plant (**G**), the compensator (**C**), the prefilter (**F**), or the sensor (**H**). Select the method you want to use from the menus next to the model elements.

Conversion method. The following are the available continuous-to-discrete conversion methods:

- Zero-order hold
- First-order hold
- Tustin
- Tustin with prewarping
- Matched pole/zero

If you choose Tustin with prewarping, you must specify the critical frequency in rad/sec.

Discrete-time domain. If you currently have a discrete-time system, the Continuous/Discrete Conversion window looks like this figure.



You can either change the sample time of the discrete system (resampling) or do a discrete-to-continuous conversion.

To resample your system, select **Discrete time with new sample time** and specify the new sample time in the **Sample time (sec)** field. The sample time must be a positive number.

To convert from discrete-time to continuous-time, you have the following options for the conversion method:

- Zero-order hold
- Tustin

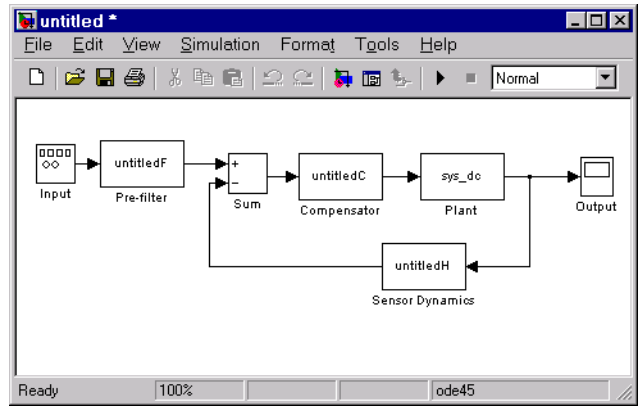
- Tustin with prewarping
- Matched pole/zero

Again, if you choose Tustin with prewarping, you must specify the critical frequency.

Draw a Simulink Diagram

Note: You must have a license for Simulink to use this feature. If you do not have Simulink, you will not see this option under the **Tools** menu.

Select **Draw a Simulink Diagram** to draw a block diagram of your system (plant, compensator, prefilter, and sensor). For the DC motor example described in Getting Started with the Control System Toolbox, this picture is the result.

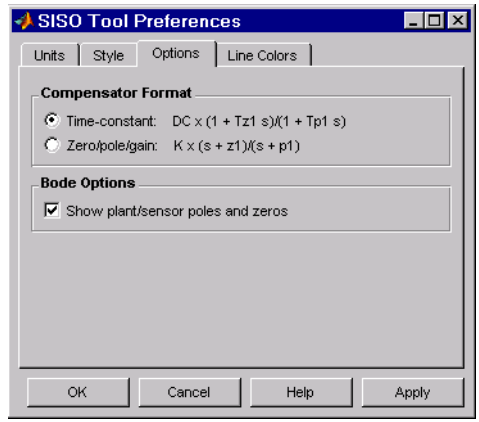


Compensator

Note Click on items in the **Compensator** menu pictured below to get help contents.

Format

Selecting **Format** under **Compensator** activates the SISO Tool Preferences editor with the **Options** page open. This figure shows the **Options** page.



Use the radio buttons to toggle between time constant and zero/pole/gain compensator formats.

By default, the SISO Design Tool shows the plant poles and zeros on the root locus and Bode diagrams as red x's and o's, respectively. Uncheck the Show plant/sensor poles and zeros box to hide the plant and sensor poles and zeros.

For a general description of the SISO Tool Preferences editor, see SISO Design Tool Preferences in the online documentation.

Edit

Selecting **Edit** under the **Compensator** menu opens the **Edit Compensator** window, which is shown below.

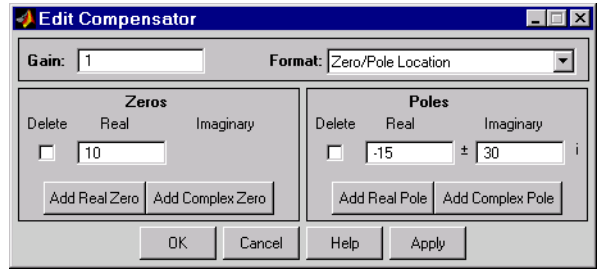


Figure 13-9: The Edit Compensator Window

You can use this window to do the following:

- Change the compensator gain
- Change the format for specifying compensator pole and zero locations
- Add compensator poles and zeros
- Delete compensator poles and zeros

Note that you can open this window by selecting Edit Compensator from either the Root locus or Bode Diagram right-click menus.

Changing the gain

To change the compensator gain, enter the new value in the **Gain** field.

Changing the format

The default is Zero/Pole Location, which means that you must specify the numerical values of the poles and zeros, but you can change the format to Damping/Natural Frequency. In the latter format, you must specify the damping and the natural frequency of poles and zeros.

Use the **Compensator Format** menu to toggle between the two formats.

Adding poles and zeros

To add real poles to your compensator, press **Add Real Pole**. This action opens an empty field in the Poles panel. Specify the pole value in the field. To add a pair of complex poles, press **Add Complex Pole**. In this case, two fields appear: one for the real and another for the imaginary part of the poles. Note that you must specify the a negative sign for the real part of the pole if you want to specify a pair left-plane poles, but that the imaginary part is defined as +/-, so you do not have to specify the sign for that part.

If you specify the damping/natural frequency format, there is no distinction between the real and complex pole specifications. Pressing either button opens two fields: one for specifying the damping and another for the natural frequency. If you pressed **Add Real Pole**, you only need to specify the natural frequency since the **Edit Compensator** window automatically places a 1 in the damping field in this case.

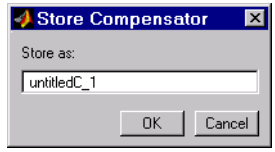
Adding zeros is exactly the same; press **Add Real Zero** or **Add Complex Zero** and proceed as above.

Deleting poles and zeros

Whenever you add poles or zeros using the **Edit Compensator** window, a delete box appears to the left of the fields used to specify the pole/zero values. Check this box anytime you want to delete the pole or zero specified next to it.

Store

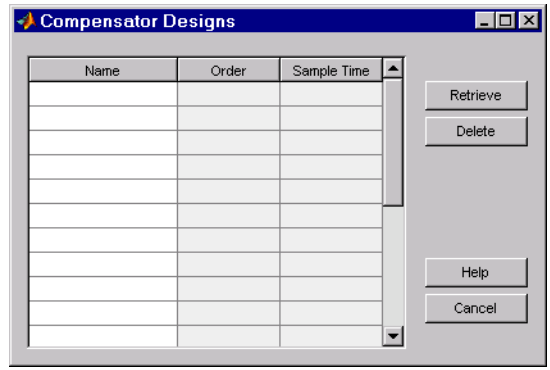
Use **Store** to open the **Store Compensator** window, shown in the figure below.



To save the compensator in the MATLAB workspace, specify the name you want to save the compensator under and press **OK**.

Retrieve

Retrieve opens the **Compensator Designs** window, shown in the figure below.



This window lists all the compensator designs you have stored during a Control System Toolbox session. To retrieve a stored design, left-click on the compensator's name to select it and press **Retrieve**. To delete a design, select it and press the **Delete** button.

Clear

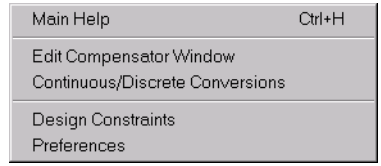
Select Clear to eliminate any compensator dynamics and set the gain to 1.

Window

The Window menu item lists all window open in MATLAB. The first item is always the MATLAB Command Window. After that, windows you have opened are listed in the order in which you invoked them. Any window you select from the list become the active window.

Help

Help brings you to various places within this document. This picture shows the Help menu.



Each topics takes you to a different place in the online documentation:

- **Main Help** — The top of this document (SISO Tool Reference documentation)
- **Edit Compensator Window** — "The Edit Compensator Window" section of this document.
- **Continuos/Discrete Conversions** — The "Continuous/Discrete Conversions" section of this document
- **Design Constraints** — The "Design Constraints" section of this document
- **Preferences** — Setting Plot Properties and Preferences, a separate online document

Tool Bar

The tool bar performs the following operations:

- Add and delete real and complex poles and zeros
- Zoom in and out
- Invoke the SISO Design Tool's context-sensitive help

This picture shows the tool bar.

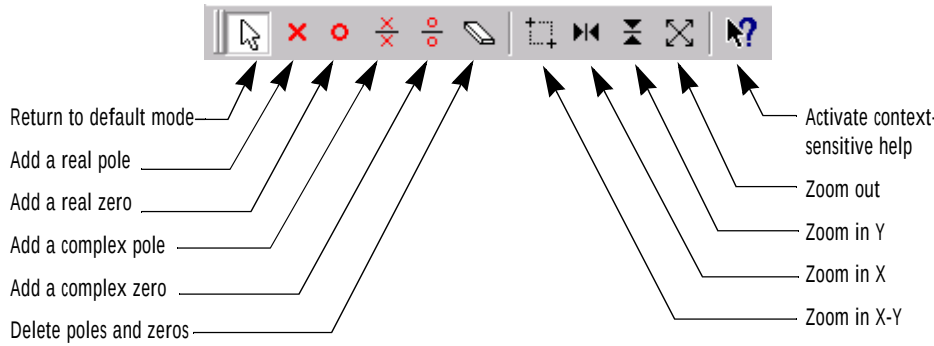


Figure 13-10: Options Available From the Tool Bar

You can use the tool tips feature to find out what a particular icon does. Just place your mouse over the icon in question, and you will see a brief description of what it does.

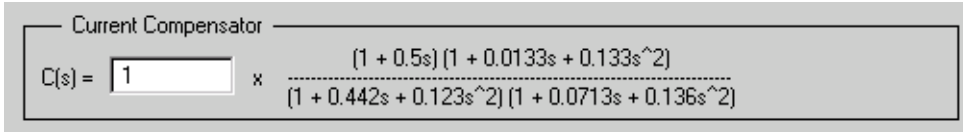
Once you've selected an icon, your mouse stays in that mode until you press the icon again.

You can reach all of these options from two other places:

- Right-click menus for root locus and Bode diagrams
- From **Root Locus** and **Bode Diagrams** under **Edit** in the menu bar

Current Compensator

The **Current Compensator** panel shows the structure of the compensator you are designing. The default compensator structure is a unity gain with no dynamics. Once you add poles and/or zeros, the Current Compensator panel displays the compensator in zero/pole/gain format. This picture shows a Current Compensator panel with Gcl1 entered as the compensator.



You can change the gain of the compensator by changing the number in the text field. If you want to change the poles and zeros of the compensator, click on the window to open the “The Edit Compensator Window”.

Feedback Structure

To the right of the **Current Compensator** panel is the **Feedback Structure** panel, which is shown in its default configuration below.

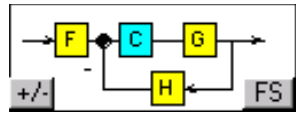


Figure 13-11: The Feedback Structure Panel

To switch to the alternate feedback structure, press the **FS** button. This figure shows the new feedback structure.

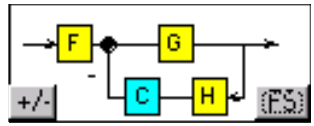


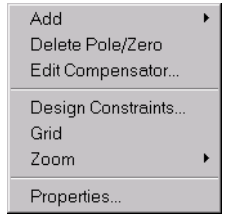
Figure 13-12: Alternate Feedback Structure with the Compensator in the Feedback Loop

Pressing the **+/-** button toggles between positive and negative feedback signs. Negative feedback is the default.

Root Locus Right-Click Menus

Note Click on items in the right-click menu pictured below to get help contents.

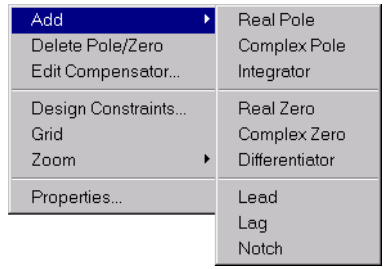
.



Note that the menus are slightly different from those of the Bode diagram right-click menus. See “Bode Diagram Right-Click Menus” for a description of those menus.

Add

The **Add** menu options give you the ability to add dynamics to your compensator design. This figure shows the **Add** submenu.



The following pole/zero configurations are available:

- **Real Pole**
- **Complex Pole**
- **Integrator** (pole at 0)
- **Real Zero**

- **Complex Zero**
- **Differentiator** (zero at 0)
- **Lead**
- **Lag**
- **Notch**

In all but the integrator and differentiator, once you select the configuration, your cursor changes to an 'x'. To add the item to your compensator design, place the x at the desired location on the plot and left-click your mouse. You will see the root locus design automatically update to include the new compensator dynamics.

The notch filter has three adjustable parameters. For a discussion about how to add and adjust notch filters, see Adding a Notch Filter in *Getting Started with the Control System Toolbox*.

Delete Pole/Zero

Select **Delete Pole/Zero** to delete poles and zeros from your compensator design. When you make this selection, your cursor changes to an eraser. Place the eraser over the pole or zero you want to delete and left-click your mouse.

Note the following:

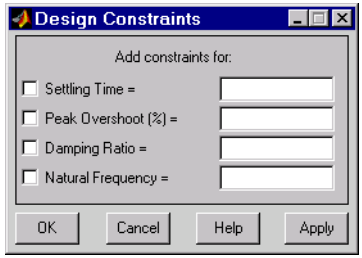
- You can only delete compensator poles and zeros. Plant (**G** in the feedback structure panel) poles and zeros cannot be altered.
- If you delete one of a pair of poles or zeros, the other member of the pair is also removed.

Edit Compensator

Edit Compensator opens the **Edit Compensator** window, which you can use to change the compensator gain and add or remove compensator poles and zeros from your design. See "Edit" for a discussion of this window.

Design Constraints

Select **Design Constraints** to open the **Design Constraints** window, which is shown below.



You have the following options:

- Settling Time
- Peak Overshoot (%)
- Damping Ratio
- Natural Frequency

Settling Time

You can

Peak Overshoot

Damping Ratio

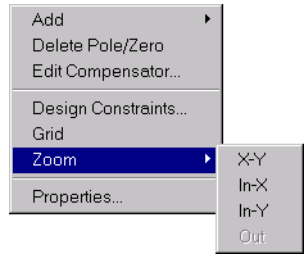
Natural Frequency

Grid

Grid adds a grid to the root locus.

Zoom

Selecting **Zoom** opens this submenu.

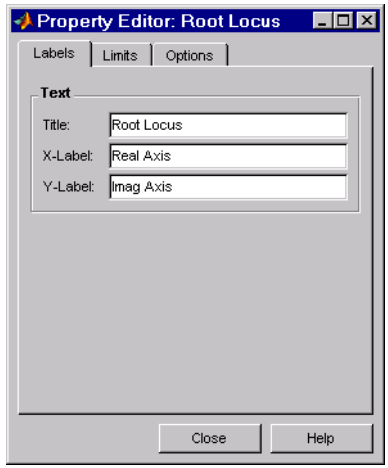


You have the following zooming options:

- **X-Y** — Enlarge a selected area in the X-Y region. To do this, select **X-Y**, hold down your mouse's left button, and drag to create a box region on the root locus. When you release the left button, the selected area becomes the entire plot region.
- **In-X** — Zoom in, X-axis only. To do this, select **In-X**, hold down your mouse's left button, and drag horizontally to create a line parallel to the X-axis. When you release the left button, the selected area becomes the new X-axis limits.
- **In-Y** — Zoom in, Y-axis only. To do this, select **In-Y**, hold down your mouse's left button, and drag vertically to create a line parallel to the Y-axis. When you release the left button, the selected area becomes the new Y-axis limits.
- **Out** — Select **Out** to undo the last zoom in that you did. If you have not done any zooming, or if you have undid all your zoom enlargements, the **Out** menu item is grayed out.

Properties

Properties opens the **Property Editor** for the root locus. This picture shows the open window.



You can use this window to change titles and axis labels, reset axes limits, add grid lines, and change the aspect ratio of the plot. For a complete discussion of the **Property Editor** for root locus, see Customizing Plots Inside the SISO Design Tool.

You can also activate this menu by double-clicking anywhere in the root locus away from the curve.

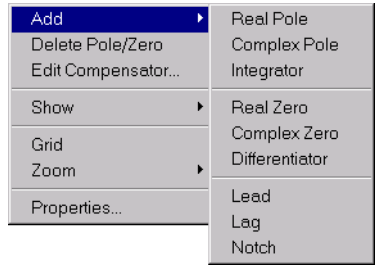
Bode Diagram Right-Click Menus

Note Click on items in the right-click menu pictured below to get help contents.

.

Add

The **Add** menu options give you the ability to add dynamics to your compensator design. This figure shows the **Add** submenu.



The following pole/zero configurations are available:

- **Real Pole**
- **Complex Pole**
- **Integrator** (pole at 0)
- **Real Zero**
- **Complex Zero**
- **Differentiator** (zero at 0)

- **Lead**
- **Lag**
- **Notch**

In all but the integrator and differentiator, once you select the configuration, your cursor changes to an 'x'. To add the item to your compensator design, place the x at the desired location on the plot and left-click your mouse. You will see the root locus design automatically update to include the new compensator dynamics.

The notch filter has three adjustable parameters. For a discussion about how to add and adjust notch filters, see Adding a Notch Filter in *Getting Started with the Control System Toolbox*.

Delete Pole/Zero

Select **Delete Pole/Zero** to delete poles and zeros from your compensator design. When you make this selection, your cursor changes to an eraser. Place the eraser over the pole or zero you want to delete and left-click your mouse.

Note the following:

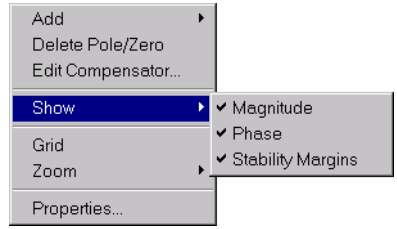
- You can only delete compensator poles and zeros. Plant (**G** in the feedback structure panel) poles and zeros cannot be altered.
- If you delete one of a pair of poles or zeros, the other member of the pair is also removed.

Edit Compensator

Edit Compensator opens the **Edit Compensator** window, which you can use to change the compensator gain and add or remove compensator poles and zeros from your design. See "Edit" for a discussion of this window.

Show

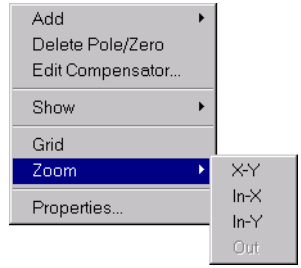
Use **Show** to select/deselect the display of magnitude, phase, and stability margins. This figure displays the Show submenu.



Selecting any of these three options toggles between showing and hiding the feature. A check next to the feature means that it is currently displayed on the Bode diagram plots.

Zoom

Selecting **Zoom** opens this submenu.



You have the following zooming options:

- **X-Y** — Enlarge a selected area in the X-Y region. To do this, select **X-Y**, hold down your mouse's left button, and drag to create a box region on the root locus. When you release the left button, the selected area becomes the entire plot region.
- **In-X** — Zoom in, X-axis only. To do this, select **In-X**, hold down your mouse's left button, and drag horizontally to create a line parallel to the X-axis. When you release the left button, the selected area becomes the new X-axis limits.

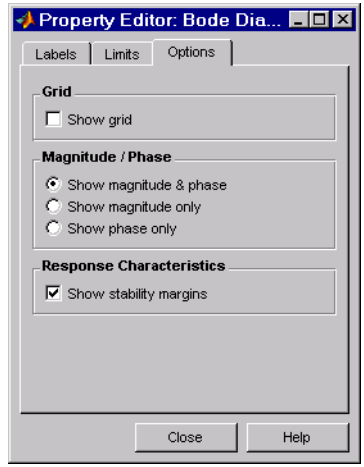
- **In-Y** — Zoom in, Y-axis only. To do this, select **In-Y**, hold down your mouse's left button, and drag vertically to create a line parallel to the Y-axis. When you release the left button, the selected area becomes the new Y-axis limits.
- **Out** — Select **Out** to undo the last zoom in that you did. If you have not done any zooming, or if you have undid all your zoom enlargements, the **Out** menu item is grayed out.

Grid

Grid adds a grid to the Bode diagram.

Properties

Properties opens the **Property Editor** for Bode diagrams. This window is exactly the same as the Property Editor for root locus, except for the **Options** page, which is shown below.



The options are customized for features that apply to Bode diagrams only.

For a complete discussion of the Property Editor for Bode diagrams, see Customizing Plots Inside the SISO Design Tool.

You can also activate this window by double-clicking anywhere in the gain or phase plots away from the curves.

Status Panel

The **Status Panel** is located at the bottom of the SISO Design Tool. It displays the most recent action you have performed and occasionally provides advice on how to use the SISO Design Tool.

14

LTI Viewer Reference

LTI Viewer

The LTI Viewer is a graphical user interface (GUI) that supports ten plot responses, including step, impulse, Bode, Nyquist, Nichols, zero/pole, sigma (singular values), `lsim`, and `initial` plots. The latter two are only available at the initialization of the LTI Viewer; see `ltiview` for more information.

The LTI Viewer is configurable and can display up to six plot type and any number of models in a single viewer. In addition, you can display information specific to the response plots, such as peak response, gain and phase margins, and so on.

You can open the LTI Viewer by typing

```
ltiview
```

at the MATLAB prompt. See the `ltiview` command for command syntax options. You can also open an LTI Viewer from the SISO Design Tool; see "SISO Design Tool Reference" for more information.

Note Click on any of the plots of the LTI Viewer, shown below, to get help on the plot. Click on the menu bar to get help on its contents. Click on the right-click menus, also shown below, to get help on right-click menu features.

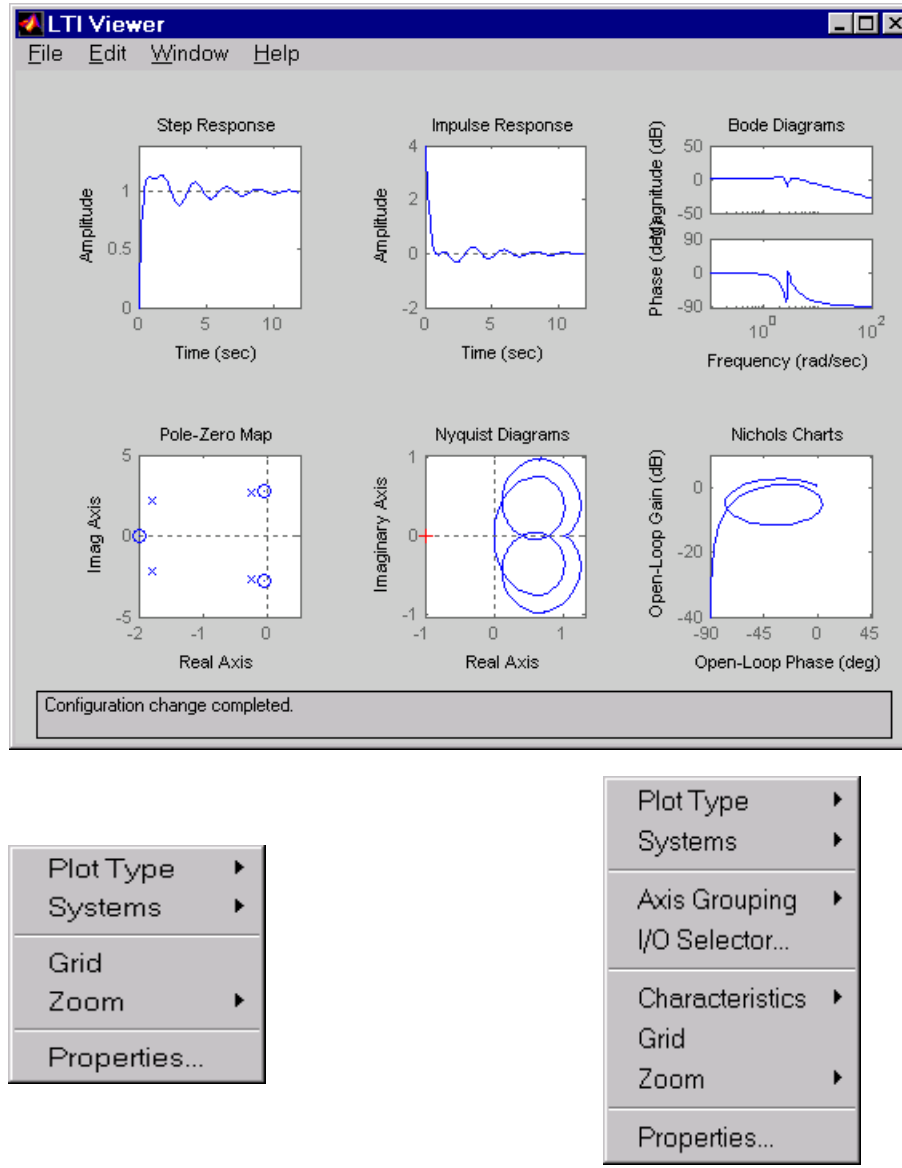


Figure 14-1: The LTI Viewer and Right-Click Menus for SISO and MIMO/LTI Array Models. Click on the Plots or the Menus for Help Contents.

LTI Viewer Menu Bar

Note Click on **File**, **Edit**, **Window**, or **Help** on the menu bar pictured below to get help on the menu items.

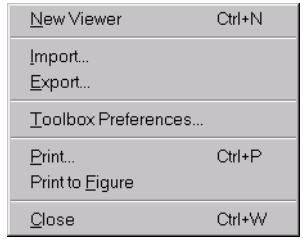
This picture shows the LTI Viewer menu bar.

Tasks that you can perform using the LTI Viewer menu bar include:

- Importing and exporting models
- Printing plot responses
- Reconfiguring the Viewer (add or remove plot responses)
- Displaying critical values (peak responses, etc.) and markers on each plot

File

Note Click on any of the items listed in the **File** menu pictured below to get help contents.



You can use the **File** menu to do the following:

- Open a new LTI Viewer

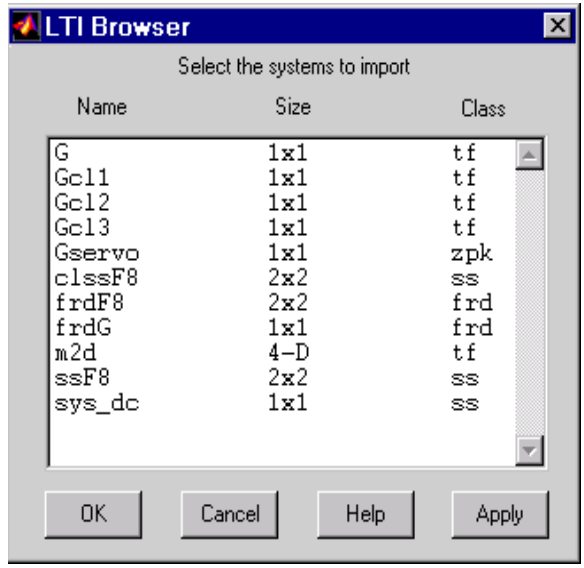
- Import and export models
- Set plot preferences for all the plots generated by the Control System Toolbox
- Print response plots
- Close the LTI Viewer

New Viewer

Select this option to open a new LTI Viewer.

Import Using the LTI Browser

Import in the **File** menu opens the LTI Browser, shown below.



The LTI Browser is used to import LTI models into or from the LTI Viewer workspace.

To import a model

- Click on the desired model in the LTI Browser List. To perform multiple selections:
  - **a** Click and drag over several variables in the list.

b Hold the Control key and click on individual variables.

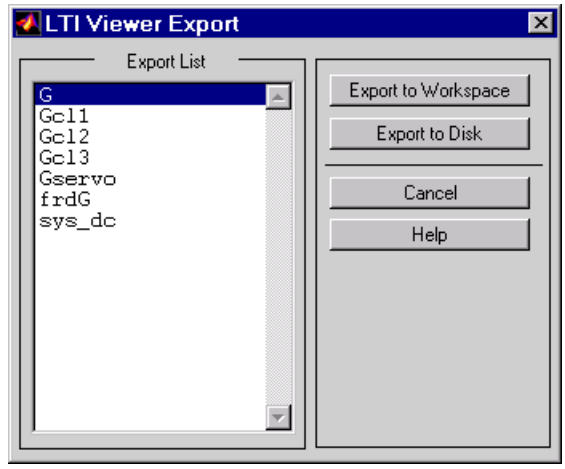
c Hold the Shift key while clicking, to select a range.

- Press the **OK** or **Apply** Button

For importing, the LTI Browser lists only the LTI models in the main MATLAB workspace.

Export Using the LTI Viewer Export Window

Export in the **File** menu opens the **LTI Viewer Export** window, shown below.



The LTI Viewer Export window lists all the models with responses currently displayed in your LTI Viewer. You can export models back to the MATLAB workspace or to disk. In the latter case, the Control System Toolbox saves the files as MAT-files.

Toolbox Preferences

Select **Toolbox Preferences** to open the Toolbox Preferences Editor, which sets preferences for all response objects in the Control System Toolbox, including the viewer.

Print

Print sends the entire LTI Viewer window to your printer.

Print to Figure

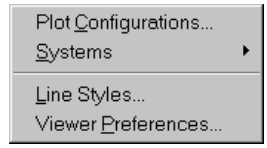
Print to Figure sends a picture of the selected system to a new figure window. Note that this new figure is a MATLAB figure window and not an LTI Viewer.

Close

Close closes the LTI Viewer.

Edit

Note Click on any of the items listed in the **Edit** menu pictured below to get help contents.

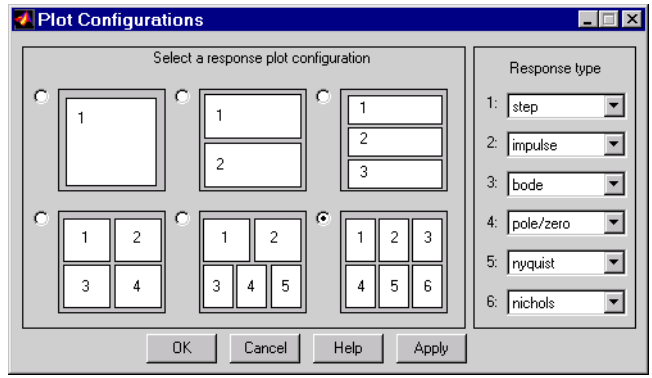


The **Edit** menu contains the following options:

- Plot Configurations — Opens the “Plot Configurations Window”
- Systems — The Systems menu item has two selections:
  - **Refresh** updates imported models to reflect any changes made in the MATLAB workspace since you imported them.
  - **Delete** opens the LTI Browser for System Deletion.
- Line Styles — Opens the “Line Styles Editor”
- Viewer Preferences — Opens the Viewer Preferences Editor

Plot Configurations Window

Plot Configuration under the **Edit** menu opens the **Plot Configurations** window.



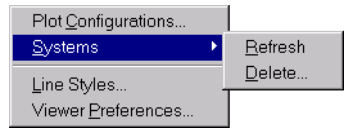
There are six possible configurations of the LTI Viewer; you can plot up to six response plots in a single viewer. Click the radio button to the upper left of the configuration you want the viewer to use.

You can select among eight response types for each plot in the viewer. These are the available response types:

- Step
- Impulse
- Bode — Plots the Bode magnitude and phase
- Bode mag. — Plots the Bode magnitude only
- Nyquist
- Nichols
- Sigma
- Pole/Zero map

Systems

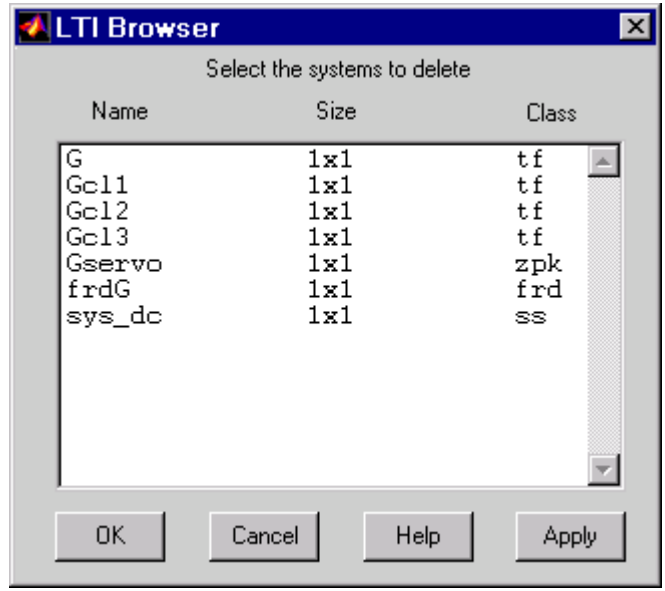
The Systems menu item has two selections, **Refresh** and **Delete**. This figure shows the two options.



Refresh updates imported models to reflect any changes made in the MATLAB workspace since you imported them. Delete opens the LTI Browser for System Deletion.

Delete Using the LTI Browser for System Deletion

Delete under **Systems** in the **Edit** menu opens the LTI Browser, shown below.



To delete a model

- Click on the desired model in the LTI Browser List. To perform multiple selections:

a Click and drag over several variables in the list.

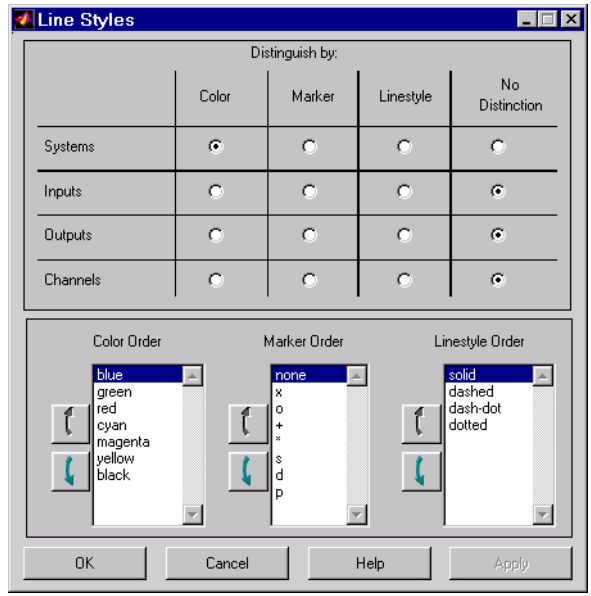
b Hold the Control key and click on individual variables.

c Hold the Shift key while clicking, to select a range.

- Press the **OK** or **Apply** Button

Line Styles Editor

Select **Line Styles** under the **Edit** menu to open the Line Styles editor, shown below.



The Line Styles editor is particularly useful when you have mutiple systems imported. You can use it change line colors, add and rearrange markers, and alter line styes (solid, dashed, and so on).

The Linestyle Preferences window allows you to customize the appearance of the response plots by specifying:

- The line property used to distinguish different systems, inputs, or outputs
- The order in which these line properties are applied

Each LTI Viewer has its own Linestyle Preferences window.

Setting Preferences

You can use the “Distinguish by” matrix to specify the line property that will vary throughout the response plots. You can group mutliple plot curves by systems, inputs, outputs, or channels (individual input/output relationships). Note that the Line Styles editor uses radio buttons, which means that you can only assign one property setting for each grouping (system, input, etc.).

Ordering Properties

The Order field allows you to change the default property order used when applying the different line properties. You can reorder the colors, markers, and linestyles (e.g., solid or dashed).

To change any of the property orders, press the up or down arrow button to the left of the associated property list to move the selected property up or down in the list

Viewer Preferences

Viewer Preferences opens the LTI Viewer Preferences editor, which you can use to set response plot defaults for the LTI Viewer that is currently open.

For a complete description of the LTI Viewer Preference editor, as well as all the property and preference editors available in the Control System Toolbox, see Setting Plot Properties and Preferences in the online documentation. To go directly to the LTI Viewer Preferences editor documentation, see LTI Viewer Preferences in the same document.

Window

Use the **Window** menu to select which of your MATLAB windows is active. This menu lists any window associated with MATLAB and the Control System Toolbox. The MATLAB Command Window is always listed first.

Help

The **Help** menu links to this help file.

Right-Click Menus for SISO Systems

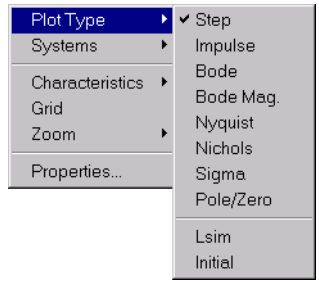
Note Click on items in the right-click menu pictured below for help contents.

This right-click menu appears when you have a SISO system imported into your LTI Viewer. If you have a MIMO system, or an LTI array containing multiple models, there are additional menu options. See “Right-Click Menus for MIMO and LTI Arrays” for more information.

You can use the right-click menus to perform the following tasks:

- Change the plot type in the viewer
- Select and deselect imported models for display
- Add or remove grid lines
- Zoom in on areas of plots
- Open the Property Editor

Plot Type

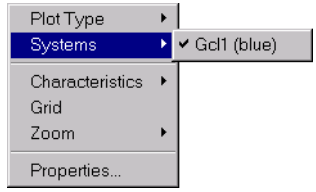


Select which plot type you want to display. The LTI Viewer shows a check to mark which plot is currently displayed. These are the available options:

- **Step** — Step response
- **Impulse** — Impulse response
- **Bode** — Magnitude and phase plots
- **Bode Mag.** — Magnitude only
- **Nyquist** — Nyquist diagram
- **Nichols** — Nichols chart
- **Sigma** — Singular values plot
- **Pole/Zero** — Pole/Zero map

You cannot switch to Lsim or Initial. To access these options, use `'lsim'` and `'initial'` flags when invoking the LTI Viewer. See `ltiview` for more information.

Systems



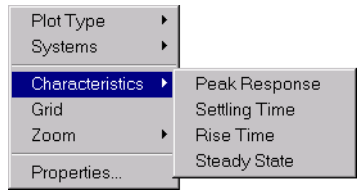
Use **Systems** to select which of the imported systems to display. Selecting a system causes a check mark to appear beside the system. To deselect a system, select it again; the menu toggles between selected and deselected.

Characteristics

The **Characteristics** menu changes for each plot response type. The next sections describe the menu for each of the eight plot types.

Step Response

Step plots the model's response to a step input.

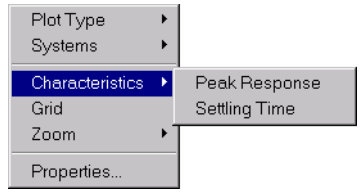


You can display the following types of information in the step response:

- **Peak Response** — The largest deviation from the steady-state value of the step response
- **Settling Time** — The time required for the step response to decline and stay at 5% of its final value
- **Rise Time** — The time require for the step response to rise from 10% to 90% of its final value
- **Steady-State** — The final value for the step response

Impulse Response

Impulse Response plots the model's response to an impulse.

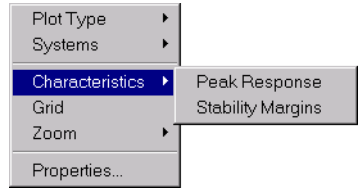


The LTI Viewer can display the following types of information in the impulse response:

- **Peak Response** — The maximum positive deviation from the steady-state value of the impulse response
- **Settling Time** — The time required for the step response to decline and stay at 5% of its final value

Bode Diagram

Bode plots the open-loop Bode phase and magnitude diagrams for the model.

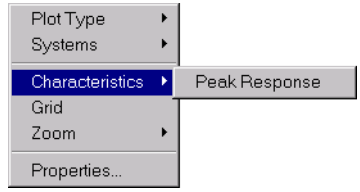


The LTI Viewer can display the following types of information in the Bode diagram:

- **Peak Response** — The maximum value of the Bode magnitude plot over the specified region
- **Stability Margins** — The phase and gain margins. The gain margin is defined to the gain (in dB) when the phase first crosses -180°. The phase margin is the distance, in degrees, of the phase from -180° when the gain magnitude is 0 dB.

Bode Magnitude

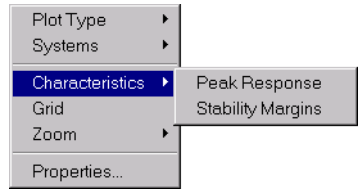
Bode Magnitude plots the Bode magnitude diagram for the model.



The LTI Viewer can display the **Peak Response**, which is the maximum value of the Bode magnitude in decibels (dB), over the specified range of the diagram.

Nyquist Diagrams

Nyquist plots the Nyquist diagram for the model.

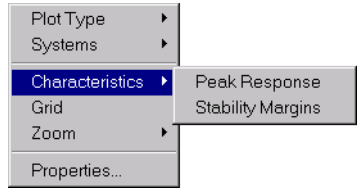


The LTI Viewer can display the following types of information in the Nyquist diagram:

- **Peak Response** — The maximum value of the Nyquist diagram over the specified region
- **Stability Margins** — The gain and phase margins for the Nyquist diagram. The gain margin is the distance from the origin to the phase crossover of the Nyquist curve. The phase crossover is where the curve meets the real axis. The phase margin is the angle subtended by the real axis and the gain crossover on the circle of radius 1.

Nichols Charts

Nichols plots the Nichols Chart for the model.

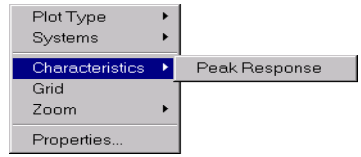


The LTI Viewer can display the following types of information in the Nichols chart:

- **Peak Response** — The maximum value of the Nichols chart in the plotted region.
- **Stability Margins** — The gain and phase margins for the Nichols chart.

Sigma

Sigma plots the singular values for the model.



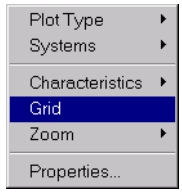
The LTI Viewer can display the **Peak Response**, which is the largest magnitude of the Sigma plot over the plotted region.

Pole/Zero

Pole/Zero plots the poles and zeros of the model with 'x' for poles and 'o' for zeros. There are no **Characteristics** available for pole-zero plots.

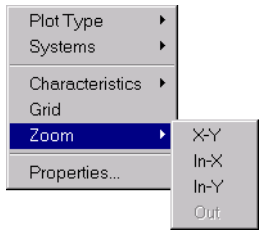
Grid

The **Grid** command activates a grid appropriate to the plot in the region you select.



Zoom

The **Zoom** command zooms in and out of the plot region selected.



There are four options:

- **In-X** — Zoom in on the specified strip of the x axis.
- **In-Y** — Zoom in on the specified strip of the y axis.
- X-Y — Zoom in on the specified box region of the x and y axes.
- **Out** — Zoom out.

When you select **In-X** or **In-Y**, left-click the mouse to specify the region of the *x* or *y* axis that you want to zoom in on. Similarly, for the **X-Y** option, left-click and drag your mouse to create a rectangular region that you want to zoom in on.

Out restores the previous appearance of the plot. Note that **Out** is grey when you have reached the limit of zooming out.

Properties

Use **Properties** to open the Property Editor. This GUI allows you to customize labels, axes limits and units, grids and font styles, and response characteristics (e.g., rise time) for your plot.

For a full description of the Property Editor, see Setting Response Plot Properties online.

Right-Click Menus for MIMO and LTI Arrays

All of the menu options described in “Right-Click Menus for SISO Systems” hold when you have imported a MIMO model or LTI Array containing multiple models.

Note, however, that when you have a MIMO model or LTI array displayed, the right-click menus contain additional options: **Axis Grouping** and **I/O selector**. These features allow you to quickly reshuffle multiple plots in a single LTI Viewer

---

Note Click on items in the right-click menu pictured below to get help contents.

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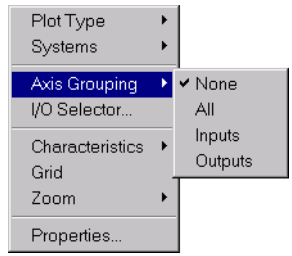
Axis Grouping

You can usse Axis Grouping to change the grouping of plots in your LTI Viewer. This picture shows the menu options.

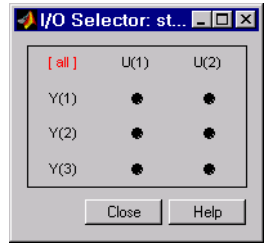


There are four options:

- **None** — By default, there is no axis grouping. For example, if you display the step responses for a 3-input, 2- output system, there will be six plots in your LTI Viewer.
- **All** — Groups all the responses into a single plot
- **Inputs** — Groups all the responses by inputs. For example, for a 3-input, 2-output system, selecting Inputs reconfigures the viewer so that there are 3 plots. Each plot contains two curves.
- **Outputs** — Groups all the responses by outputs. For example, for a 3-input, 2-output system, selecting Inputs reconfigures the viewer so that there are 2 plots. Each plot contains three curves.

I/O Selector

I/O Selector opens the **I/O Selector** window, shown below.



The **I/O Selector** window contains buttons corresponding to each I/O pair. In this example, there are 2 inputs and 3 outputs, so there are six buttons. By

default, all the I/O pairs are selected. If you click on a button, that I/O pair alone is displayed in the LTI Viewer. The other buttons automatically deselect.

To select a column of inputs, click on the input name above the column. The names are **U(1)**, **U(2)**, and so on. The LTI Viewer displays the responses from the specified input to all the outputs.

To select a row of output, click on the output name to the left of the row. The names are **Y(1)**, **Y(2)**, and so on. The LTI Viewer displays the responses from all the inputs to the specified output.

To reestablish the default setting, click **[all]**. The LTI Viewer displays all the I/O pairs.

Status Panel

The Status Panel is located at the bottom of the LTI Viewer. It contains useful information about changes you have made to the LTI Viewer.

15

Right-Click Menus for Response Plots

All the response plots that the Control System Toolbox creates have right-click menus available. The plots include the following:

- `bode`
- `bodemag`
- `impulse`
- `initial`
- `nichols`
- `nyquist`
- `pzmap`
- `sigma`
- `step`

Note Click on any of the items in the right-click menus, shown below, to get help on the feature.

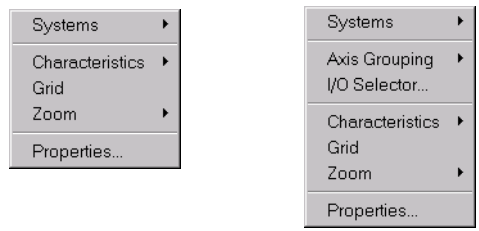


Figure 15-1: Right-Click Menus for SISO and MIMO/LTI Array Models.

You can do the following using the right-click menus for response plots:

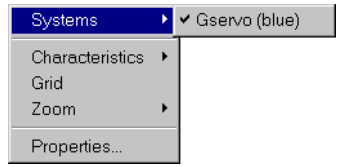
- Select and deselect imported systems
- Change plot characteristics
- Add and remove grid lines
- Zoom in and out of selected plot regions
- Open the Property Editor the the selected plot
- In the MIMO/LTI array case:

- regroup the plots
- Select subsets of I/O pairs

Right-Click Menus for SIXXSO Systems

When you create a response plot for a SISO system, you have available a set of right-click menu options, which are desribed in the following sections.

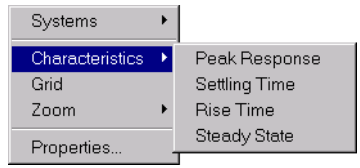
Systems



Use **Systems** to select which of the imported systems to display. Selecting a system causes a check mark to appear beside the system. To deselect a system, select it again; the menu toggles between selected and deselected.

Characteristics

The **Characteristics** menu changes for each plot response type. This picture shows the options for a step response.



The following table lists the characteristics available for each response plot type.

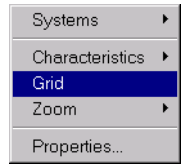
Table 15-1: Options Available from the Characteristics Menu

Function	Characteristics
`bode`	Peak Response
`bodemag`	Peak Response
`impulse`	Peak Response Settling Time

Table 15-1: Options Available from the Characteristics Menu

Function	Characteristics
`initial`	Peak Response
`nichols`	Peak Response
`nyquist`	Peak Response
`pzmap`	None
`sigma`	Peak Response
`step`	Peak Response Settling Time Rise Time Steady State

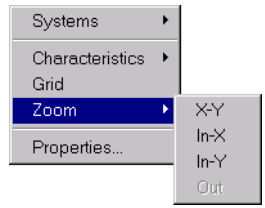
Grid



The **Grid** command activates a grid appropriate to the plot in the region you select.

Zoom

The **Zoom** command zooms in and out of the plot region selected.



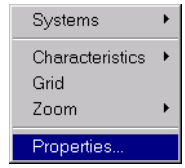
There are four options:

- **In-X** — Zoom in on the specified strip of the x axis.
- **In-Y** — Zoom in on the specified strip of the y axis.
- X-Y — Zoom in on the specified box region of the x and y axes.
- **Out** — Zoom out.

When you select **In-X** or **In-Y**, left-click the mouse to specify the region of the *x* or *y* axis that you want to zoom in on. Similarly, for the **X-Y** option, left-click and drag your mouse to create a rectangular region that you want to zoom in on.

Out restores the previous appearance of the plot. Note that **Out** is grey when you have reached the limit of zooming out.

Properties



Use **Properties** to open the Property Editor. This GUI allows you to customize labels, axes limits and units, grids and font styles, and response characteristics (e.g., rise time) for your plot.

For a full description of the Property Editor, see Setting Response Plot Properties online.

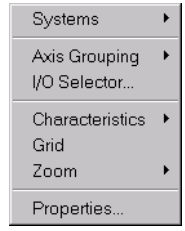
Right-Click Menus for MIMO and LTI Arrays

All of the menu options described in “Right-Click Menus for SIXXSO Systems” hold when you have generated a response plot for a MIMO model or an LTI Array.

Note, however, that when you have a MIMO model or LTI array displayed, the right-click menus contain additional options: **Axis Grouping** and **I/O selector**. These features allow you to quickly reshuffle multiple plots in a single window.

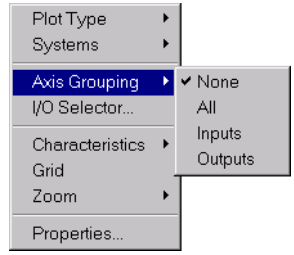
Note Click on items in the right-click menu pictured below to get help contents.

.



Axis Grouping

You can usse Axis Grouping to change the grouping of plots in a single plot window. This picture shows the menu options.

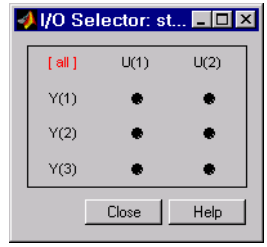


There are four options:

- **None** — By default, there is no axis grouping. For example, if you display the step responses for a 3-input, 2- output system, there will be six plots in your window.
- **All** — Groups all the responses into a single plot
- **Inputs** — Groups all the responses by inputs. For example, for a 3-input, 2-output system, selecting Inputs reconfigures the viewer so that there are 3 plots. Each plot contains two curves.
- **Outputs** — Groups all the responses by outputs. For example, for a 3-input, 2-output system, selecting Inputs reconfigures the viewer so that there are 2 plots. Each plot contains three curves.

I/O Selector

I/O Selector opens the **I/O Selector** window, shown below.



The **I/O Selector** window contains buttons corresponding to each I/O pair. In this example, there are 2 inputs and 3 outputs, so there are six buttons. By default, all the I/O pairs are selected. If you click on a button, that I/O pair alone is displayed in the plot window. The other buttons automatically deselect.

To select a column of inputs, click on the input name above the column. The names are **U(1)**, **U(2)**, and so on. The plot window displays the responses from the specified input to all the outputs.

To select a row of output, click on the output name to the left of the row. The names are **Y(1)**, **Y(2)**, and so on. The plot window displays the responses from all the inputs to the specified output.

To reestablish the default setting, click **[all]**. The plot window displays all the I/O pairs.

16

Function Reference

This chapter contains detailed descriptions of all Control System Toolbox functions. It begins with a list of functions grouped by subject area and continues with the reference entries in alphabetical order. Information is also available through the online Help facility.

Functions by Category

Table 16-1: LTI Models

Function Name	Description
drss	Generate random discrete state-space model
dss	Create descriptor state-space model
filt	Create discrete filter with DSP convention
frd	Create a frequency response data (FRD) model
frdata	Retrieve data from an FRD model
get	Query LTI model properties
rss	Generate random continuous state-space model
set	Set LTI model properties
ss	Create state-space model
ssdata, dssdata	Retrieve state-space data
tf	Create transfer function
tfdata	Retrieve transfer function data
totaldelay	Provide the aggregate delay for an LTI model
zpk	Create zero-pole-gain model
zpkdata	Retrieve zero-pole-gain data

Table 16-2: Model Characteristics

Function Name	Description
`class`	Display model type (`'tf'`, `'zpk'`, `'ss'`, or `'frd'`)
`hasdelay`	Test true if LTI model has any type of delay
`isa`	Test true if LTI model is of specified type
`isct`	Test true for continuous-time models
`isdt`	Test true for discrete-time models
`isempty`	Test true for empty LTI models
`isproper`	Test true for proper LTI models
`issiso`	Test true for SISO models
`ndims`	Display the number of model/array dimensions
`size`	Display output/input/array dimensions

Table 16-3: Model Conversion

Function Name	Description
`c2d`	Convert from continuous- to discrete-time models
`chgunits`	Convert the `units` property for FRD models
`d2c`	Convert from discrete- to continuous-time models
`d2d`	Resample discrete-time models
`delay2z`	Convert delays in discrete-time models or FRD models
`frd`	Convert to a frequency response data model

Table 16-3: Model Conversion (Continued)

Function Name	Description
pade	Compute the Padé approximation of delays
reshape	Change the shape of an LTI array
residue	Provide partial fraction expansion
ss	Convert to a state space model
tf	Convert to a transfer function model
zpk	Convert to a zero-pole-gain model

Table 16-4: Model Order Reduction

Function Name	Description
balreal	Calculate an I/O balanced realization
minreal	Calculate minimal realization or eliminate pole/zero pairs
modred	Delete states in I/O balanced realization
sminreal	Calculate structured model reduction

Table 16-5: State-Space Realizations

Function Name	Description
canon	Canonical state-space realizations
ctrb	Controllability matrix
ctrbf	Controllability staircase form
gram	Controllability and observability grammians

Table 16-5: State-Space Realizations (Continued)

Function Name	Description
obsv	Observability matrix
obsvf	Observability staircase form
ss2ss	State coordinate transformation.
ssbal	Diagonal balancing of state-space realizations.

Table 16-6: Model Dynamics

Function Name	Description
damp	Calculate natural frequency and damping
dcgain	Calculate low-frequency (DC) gain
covar	Calculate covariance of response to white noise
dsort	Sort discrete-time poles by magnitude
esort	Sort continuous-time poles by real part
norm	Calculate norms of LTI models (H_2 and L_∞)
pole, eig	Calculate the poles of an LTI model
pzmap	Plot the pole/zero map of an LTI model
rlocus	Calculate and plot root locus
roots	Calculate roots of polynomial
sgrid,zgrid	Superimpose s- and z-plane grids for root locus or pole/zero maps
zero	Calculate zeros of an LTI model

Table 16-7: Model Interconnections

Function Name	Description
append	Append models in a block diagonal configuration
augstate	Augment output by appending states
connect	Connect the subsystems of a block-diagonal model according to an interconnection scheme of your choice
feedback	Calculate the feedback connection of models
lft	Form the LFT interconnection (star product)
ord2	Generate second-order model
parallel	Create a generalized parallel connection
series	Create a generalized series connection
stack	Stack LTI models into a model array

Table 16-8: Time Response

Function Name	Description
gensig	Generate an input signal
impulse	Calculate and plot impulse response
initial	Calculate and plot initial condition response
lsim	Simulate response of LTI model to arbitrary inputs
ltiview	Open the LTI Viewer for linear response analysis
step	Calculate step response

Table 16-9: Time Delays

Function Name	Description
`delay2z`	Convert delays in discrete-time models or FRD models
`pade`	Compute the Padé approximation of delays
`totaldelay`	Provide the aggregate delay for an LTI model

Table 16-10: Frequency Response

Function Name	Description
`allmargin`	Calculate all crossover frequencies and associated gain, phase, and delay margins
`bode`	Calculate and plot Bode response
`bodemag`	Calculate and plot Bode magnitude only
`evalfr`	Evaluate response at single complex frequency
`freqresp`	Evaluate frequency response for selected frequencies
`interp`	Interpolate FRD model between frequency points
`linspace`	Create a vector of evenly spaced frequencies
`logspace`	Create a vector of logarithmically spaced frequencies
`ltiview`	Open the LTI Viewer for linear response analysis
`margin`	Calculate gain and phase margins
`ngrid`	Superimpose grid lines on a Nichols plot
`nichols`	Calculate Nichols plot

Table 16-10: Frequency Response (Continued)

Function Name	Description
nyquist	Calculate Nyquist plot
sigma	Calculate singular value plot

Table 16-11: SISO Feedback Design

Function Name	Description
allmargin	Calculate all crossover frequencies and associated gain, phase, and delay margins
margin	Calculate gain and phase margins
rlocus	Calculate and plot root locus
sisotool	Open the SISO Design Tool

Table 16-12: Pole Placement

Function Name	Description
acker	Calculate SISO pole placement design
place	Calculate MIMO pole placement design
estim	Form state estimator given estimator gain
reg	Form output-feedback compensator given state-feedback and estimator gains

Table 16-13: LQG Design

Function Name	Description
lqr	Calculate the LQ-optimal gain for continuous models
dlqr	Calculate the LQ-optimal gain for discrete models

Table 16-13: LQG Design (Continued)

Function Name	Description
lqry	Calculate the LQ-optimal gain with output weighting
lqrd	Calculate the discrete LQ gain for continuous models
kalman	Calculate the Kalman estimator
kalmd	Calculate the discrete Kalman estimator for continuous models
lqgreg	Form LQG regulator given LQ gain and Kalman filter

Table 16-14: Equation Solvers

Function Name	Description
care	Solve continuous-time algebraic Riccati equations
dare	Solve discrete-time algebraic Riccati equations
lyap	Solve continuous-time Lyapunov equations
dlyap	Solve discrete-time Lyapunov equations

Table 16-15: Graphical User Interfaces for Control System Analysis and Design

Function Name	Description
ltiview	Open the LTI Viewer for linear response analysis
sisotool	Open the SISO Design GUI

Purpose

Pole placement design for single-input systems

Syntax

```
k = acker(A,b,p)
```

Description

Given the single-input system

$$\dot{x} = Ax + bu$$

and a vector `p` of desired closed-loop pole locations, `acker(A,b,p)` uses Ackermann's formula [1] to calculate a gain vector `k` such that the state feedback $u = -kx$ places the closed-loop poles at the locations `p`. In other words, the eigenvalues of $A - bk$ match the entries of `p` (up to ordering). Here `A` is the state transmitter matrix and `b` is the input to state transmission vector.

You can also use `acker` for estimator gain selection by transposing the matrix `A` and substituting `c'` for `b` when $y = cx$ is a single output.

```
l = acker(a',c',p).'
```

Limitations

`acker` is limited to single-input systems and the pair (A, b) must be controllable.

Note that this method is not numerically reliable and starts to break down rapidly for problems of order greater than 5 or for weakly controllable systems. See `place` for a more general and reliable alternative.

See Also

`lqr`	Optimal LQ regulator
`place`	Pole placement design
`rlocus`	Root locus design

References

[1] Kailath, T., *Linear Systems*, Prentice-Hall, 1980, p. 201.

allmargin

Purpose

Compute all crossover frequencies and corresponding stability margins

Syntax

```
S = allmargin(sys)
```

Description

`allmargin` computes the gain, phase, and delay margins and the corresponding crossover frequencies of the SISO open-loop model `sys`. `allmargin` is applicable to any SISO model, including models with delays.

The output `S` is a structure with the following fields:

- GMFrequency — All -180 degree crossover frequencies (in rad/sec)
- GainMargin — Corresponding gain margins, defined as 1/G where G is the gain at crossover
- PMFrequency — All 0 dB crossover frequencies in rad/sec
- PhaseMargin — Corresponding phase margins in degrees
- DMFrequency and DelayMargin — Critical frequencies and the corresponding delay margins. Delay margins are given in seconds for continuous-time systems and multiples of the sample time for discrete-time systems.
- Stable — 1 if the nominal closed-loop system is stable, 0 otherwise.

See Also

`ltimodels`	Help on LTI models
`ltiview`	LTI system viewer
`margin`	Gain and phase margins for SISO open-loop systems

Purpose

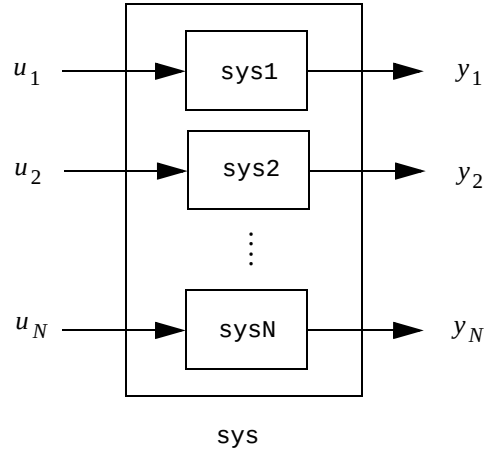
Group LTI models by appending their inputs and outputs

Syntax

```
sys = append(sys1,sys2,...,sysN)
```

Description

append appends the inputs and outputs of the LTI models sys1,...,sysN to form the augmented model sys depicted below.



For systems with transfer functions $H_1(s),...,H_N(s)$, the resulting system sys has the block-diagonal transfer function

$$\begin{bmatrix} H_1(s) & 0 & .. & 0 \\ 0 & H_2(s) & . & : \\ : & . & . & 0 \\ 0 & .. & 0 & H_N(s) \end{bmatrix}$$

For state-space models sys1 and sys2 with data (A_1, B_1, C_1, D_1) and (A_2, B_2, C_2, D_2), append(sys1,sys2) produces the following state-space model.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}A_1 & 0\\ 0 & A_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}B_1 & 0\\ 0 & B_2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}C_1 & 0\\ 0 & C_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}D_1 & 0\\ 0 & D_2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

Arguments

The input arguments `sys1`,..., `sysN` can be LTI models of any type. Regular matrices are also accepted as a representation of static gains, but there should be at least one LTI object in the input list. The LTI models should be either all continuous, or all discrete with the same sample time. When appending models of different types, the resulting type is determined by the precedence rules (see Precedence Rules for details).

There is no limitation on the number of inputs.

Example

The commands

```
sys1 = tf(1,[1 0])
sys2 = ss(1,2,3,4)
sys = append(sys1,10,sys2)
```

produce the state-space model

```
sys

a =
                     x1           x2
           x1         0            0
           x2         0      1.00000

b =
                     u1           u2           u3
           x1      1.00000         0            0
           x2         0            0      2.00000


c =
                     x1           x2
           y1      1.00000         0
```

```
            y2          0          0
            y3          0    3.00000

d =
                       u1         u2         u3
            y1          0          0          0
            y2          0   10.00000          0
            y3          0          0    4.00000

  Continuous-time system.
```

See Also

`connect`	Modeling of block diagram interconnections
`feedback`	Feedback connection
`parallel`	Parallel connection
`series`	Series connection

augstate

Purpose

Append the state vector to the output vector

Syntax

```
asys = augstate(sys)
```

Description

Given a state-space model `sys` with equations

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

(or their discrete-time counterpart), `augstate` appends the states x to the outputs y to form the model

$$\dot{x} = Ax + Bu$$
$$\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} C \\ I \end{bmatrix} x + \begin{bmatrix} D \\ 0 \end{bmatrix} u$$

This command prepares the plant so that you can use the `feedback` command to close the loop on a full-state feedback $u = -Kx$.

Limitation

Because `augstate` is only meaningful for state-space models, it cannot be used with TF, ZPK or FRD models.

See Also

`feedback`	Feedback connection
`parallel`	Parallel connection
`series`	Series connection

Purpose

Input/output balancing of state-space realizations

Syntax

```
sysb = balreal(sys)
[sysb,g,T,Ti] = balreal(sys)
```

Description

`sysb = balreal(sys)` produces a balanced realization `sysb` of the LTI model `sys` with equal and diagonal controllability and observability grammians (see `gram` for a definition of grammian). `balreal` handles both continuous and discrete systems. If `sys` is not a state-space model, it is first and automatically converted to state space using `ss`.

`[sysb,g,T,Ti] = balreal(sys)` also returns the vector `g` containing the diagonal of the balanced grammian, the state similarity transformation $x_b = Tx$ used to convert `sys` to `sysb`, and the inverse transformation `Ti` $= T^{-1}$.

If the system is normalized properly, the diagonal `g` of the joint grammian can be used to reduce the model order. Because `g` reflects the combined controllability and observability of individual states of the balanced model, you can delete those states with a small `g(i)` while retaining the most important input-output characteristics of the original system. Use `modred` to perform the state elimination.

Example

Consider the zero-pole-gain model

```
sys = zpk([-10 -20.01],[-5 -9.9 -20.1],1)

Zero/pole/gain:
   (s+10) (s+20.01)
----------------------
(s+5) (s+9.9) (s+20.1)
```

A state-space realization with balanced grammians is obtained by

```
[sysb,g] = balreal(sys)
```

The diagonal entries of the joint grammian are

```
g'

ans =
```

```
    1.0062e-01   6.8039e-05   1.0055e-05
```

which indicates that the last two states of sysb are weakly coupled to the input and output. You can then delete these states by

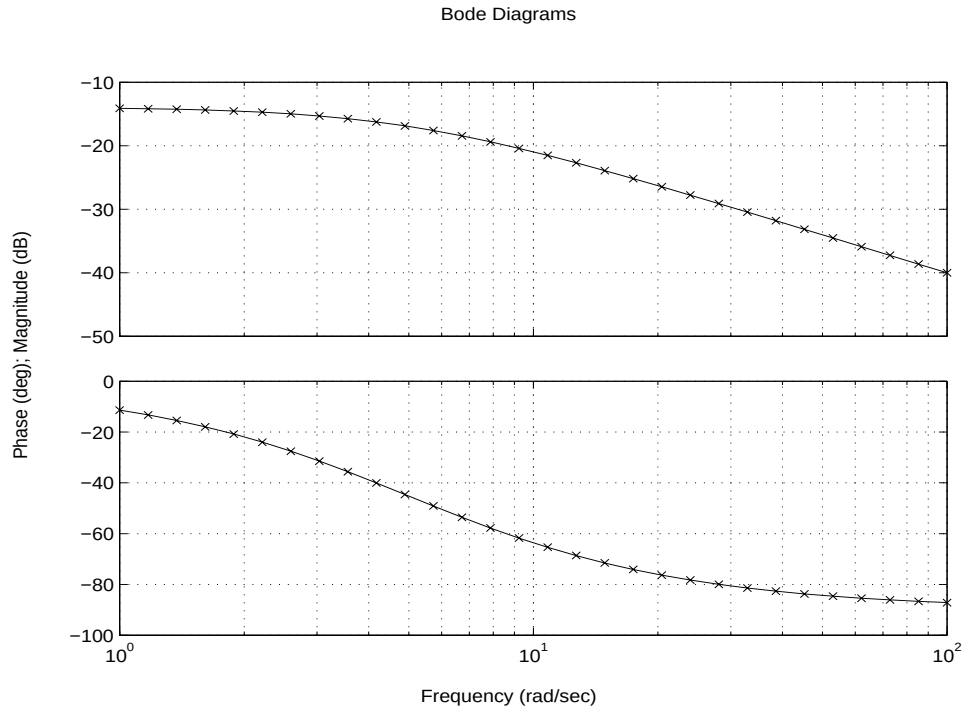
```
sysr = modred(sysb,[2 3],'del')
```

to obtain the following first-order approximation of the original system.

```
zpk(sysr)

Zero/pole/gain:
 1.0001
--------
(s+4.97)
```

Compare the Bode responses of the original and reduced-order models.

```
bode(sys,'-',sysr,'x')
```



Algorithm

Consider the model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

with controllability and observability grammians W_c and W_o. The state coordinate transformation $\bar{x} = Tx$ produces the equivalent model

$$\dot{\bar{x}} = TAT^{-1}\bar{x} + TBu$$
$$y = CT^{-1}\bar{x} + Du$$

and transforms the grammians to

$$\overline{W}_c = TW_cT^T, \qquad \overline{W}_o = T^{-T}W_oT^{-1}$$

The function `balreal` computes a particular similarity transformation T such that

$$\overline{W}_c = \overline{W}_o = diag(g)$$

See [1,2] for details on the algorithm.

Limitations

The LTI model `sys` must be stable. In addition, controllability and observability are required for state-space models.

See Also

`gram`	Controllability and observability grammians
`minreal`	Minimal realizations
`modred`	Model order reduction

References

[1] Laub, A.J., M.T. Heath, C.C. Paige, and R.C. Ward, "Computation of System Balancing Transformations and Other Applications of Simultaneous Diagonalization Algorithms," *IEEE Trans. Automatic Control*, AC-32 (1987), pp. 115–122.

[2] Moore, B., "Principal Component Analysis in Linear Systems: Controllability, Observability, and Model Reduction," *IEEE Transactions on Automatic Control*, AC-26 (1981), pp. 17–31.

[3] Laub, A.J., "Computation of Balancing Transformations," *Proc. ACC*, San Francisco, Vol.1, paper FA8-E, 1980.

Purpose

Compute the Bode frequency response of LTI models

Syntax

```
bode(sys)
bode(sys,w)

bode(sys1,sys2,...,sysN)
bode(sys1,sys2,...,sysN,w)
bode(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[mag,phase,w] = bode(sys)
```

Description

bode computes the magnitude and phase of the frequency response of LTI models. When invoked without left-hand arguments, bode produces a Bode plot on the screen. The magnitude is plotted in decibels (dB), and the phase in degrees. The decibel calculation for mag is computed as $20\log_{10}(|H(j\omega)|)$, where $|H(j\omega)|$ is the system's frequency response. Bode plots are used to analyze system properties such as the gain margin, phase margin, DC gain, bandwidth, disturbance rejection, and stability.

bode(sys) plots the Bode response of an arbitrary LTI model sys. This model can be continuous or discrete, and SISO or MIMO. In the MIMO case, bode produces an array of Bode plots, each plot showing the Bode response of one particular I/O channel. The frequency range is determined automatically based on the system poles and zeros.

bode(sys,w) explicitly specifies the frequency range or frequency points to be used for the plot. To focus on a particular frequency interval [wmin,wmax], set w = {wmin,wmax}. To use particular frequency points, set w to the vector of desired frequencies. Use logspace to generate logarithmically spaced frequency vectors. All frequencies should be specified in radians/sec.

bode(sys1,sys2,...,sysN) or bode(sys1,sys2,...,sysN,w) plots the Bode responses of several LTI models on a single figure. All systems must have the same number of inputs and outputs, but may otherwise be a mix of continuous and discrete systems. This syntax is useful to compare the Bode responses of multiple systems.

bode(sys1,'PlotStyle1',...,sysN,'PlotStyleN') specifies which color, linestyle, and/or marker should be used to plot each system. For example,

```
bode(sys1,'r--',sys2,'gx')
```

uses red dashed lines for the first system `sys1` and green `'x'` markers for the second system `sys2`.

When invoked with left-hand arguments

```
[mag,phase,w] = bode(sys)
[mag,phase] = bode(sys,w)
```

return the magnitude and phase (in degrees) of the frequency response at the frequencies `w` (in rad/sec). The outputs `mag` and `phase` are 3-D arrays with the frequency as the last dimension (see "Arguments" below for details). You can convert the magnitude to decibels by

```
magdb = 20*log10(mag)
```

Remark

If `sys` is an FRD model, `bode(sys,w)`, `w` can only include frequencies in `sys.frequency`.

Arguments

The output arguments `mag` and `phase` are 3-D arrays with dimensions

(number of outputs) × (number of inputs) × (length of `w`)

For SISO systems, `mag(1,1,k)` and `phase(1,1,k)` give the magnitude and phase of the response at the frequency ω_k = `w(k)`.

$$\text{mag}(1,1,k) = \left|h(j\omega_k)\right|$$
$$\text{phase}(1,1,k) = \angle h(j\omega_k)$$

MIMO systems are treated as arrays of SISO systems and the magnitudes and phases are computed for each SISO entry h_{ij} independently (h_{ij} is the transfer function from input j to output i). The values `mag(i,j,k)` and `phase(i,j,k)` then characterize the response of h_{ij} at the frequency `w(k)`.

$$\text{mag}(i,j,k) = \left|h_{ij}(j\omega_k)\right|$$
$$\text{phase}(i,j,k) = \angle h_{ij}(j\omega_k)$$

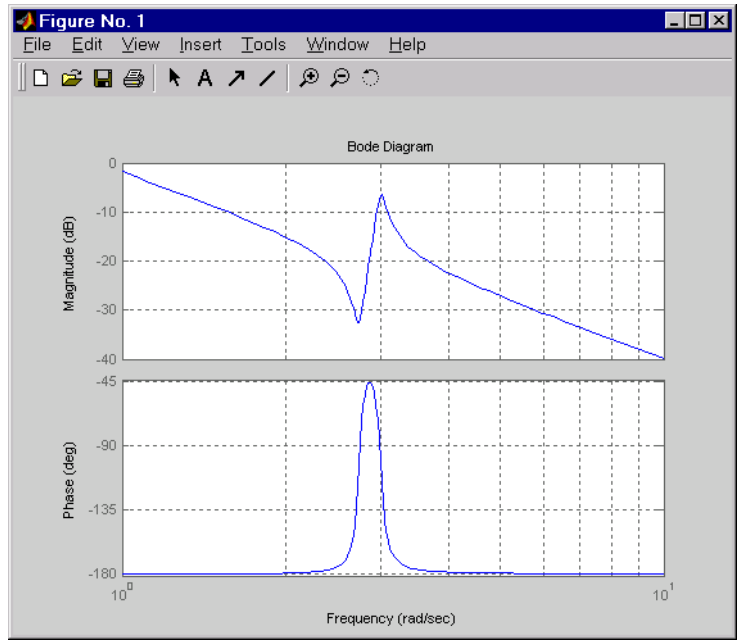
Example

You can plot the Bode response of the continuous SISO system

$$H(s) = \frac{s^2 + 0.1s + 7.5}{s^4 + 0.12s^3 + 9s^2}$$

by typing

```
g = tf([1 0.1 7.5],[1 0.12 9 0 0]);
bode(g)
```



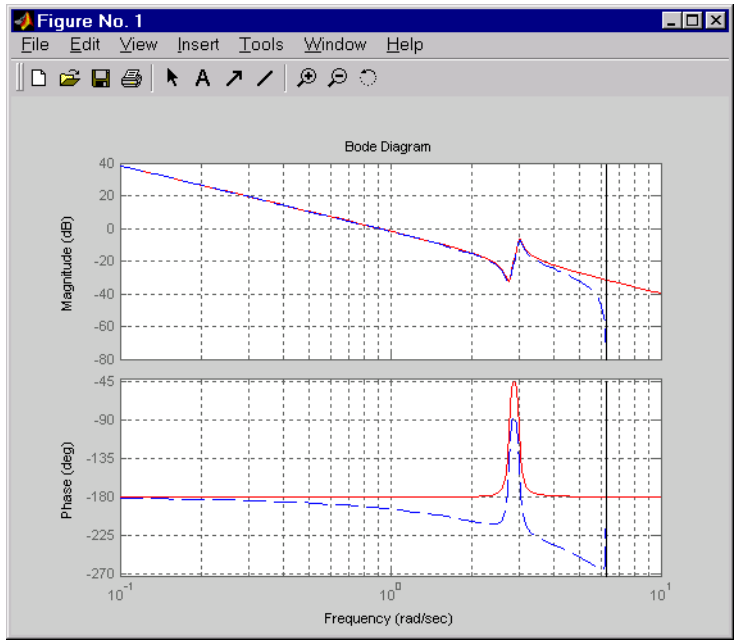
To plot the response on a wider frequency range, for example, from 0.1 to 100 rad/sec, type

```
bode(g,{0.1 , 100})
```

You can also discretize this system using zero-order hold and the sample time $T_s = 0.5$ second, and compare the continuous and discretized responses by typing

```
gd = c2d(g,0.5)
```

```
bode(g,'r',gd,'b--')
```



Algorithm

For continuous-time systems, bode computes the frequency response by evaluating the transfer function $H(s)$ on the imaginary axis $s = j\omega$. Only positive frequencies ω are considered. For state-space models, the frequency response is $D + C(j\omega - A)^{-1}B$, $\omega \geq 0$

When numerically safe, A is diagonalized for maximum speed. Otherwise, A is reduced to upper Hessenberg form and the linear equation $(j\omega - A)X = B$ is solved at each frequency point, taking advantage of the Hessenberg structure. The reduction to Hessenberg form provides a good compromise between efficiency and reliability. See [1] for more details on this technique.

For discrete-time systems, the frequency response is obtained by evaluating the transfer function $H(z)$ on the unit circle. To facilitate interpretation, the upper-half of the unit circle is parametrized as

$$z = e^{j\omega T_s}, \qquad 0 \leq \omega \leq \omega_N = \frac{\pi}{T_s}$$

where T_s is the sample time. ω_N is called the *Nyquist frequency*. The equivalent "continuous-time frequency" ω is then used as the x-axis variable. Because

$$H(e^{j\omega T_s})$$

is periodic with period $2\omega_N$, `bode` plots the response only up to the Nyquist frequency ω_N. If the sample time is unspecified, the default value $T_s = 1$ is assumed.

Diagnostics

If the system has a pole on the $j\omega$ axis (or unit circle in the discrete case) and `w` happens to contain this frequency point, the gain is infinite, $j\omega I - A$ is singular, and `bode` produces the warning message

```
Singularity in freq. response due to jw-axis or unit circle pole.
```

See Also

`evalfr`	Response at single complex frequency
`freqresp`	Frequency response computation
`ltiview`	LTI system viewer
`nichols`	Nichols plot
`nyquist`	Nyquist plot
`sigma`	Singular value plot

References

[1] Laub, A.J., "Efficient Multivariable Frequency Response Computations," *IEEE Transactions on Automatic Control*, AC-26 (1981), pp. 407–408.

bodemag

Purpose

Compute the Bode magnitude response of LTI models

Syntax

```
bodemag(sys)
bodemag(sys,{wmin,wmax})
bodemag(sys,w)

bodemag(sys1,sys2,...,sysN,w)
bodemag(sys1,'PlotStyle1',...,sysN,'PlotStyleN')
```

Description

`bodemag(sys)` plots the magnitude of the frequency response of the LTI model SYS (Bode plot without the phase diagram). The frequency range and number of points are chosen automatically.

`bodemag(sys,{wmin,wmax})` draws the magnitude plot for frequencies between `wmin` and `wmax` (in radians/second).

`bodemag(sys,w)` uses the user-supplied vector W of frequencies, in radians/second, at which the frequency response is to be evaluated.

`bodemag(sys1,sys2,...,sysN,w)` shows the frequency response magnitude of several LTI models `sys1,sys2,...,sysN` on a single plot. The frequency vector `w` is optional. You can also specify a color, line style, and marker for each model, as in

```
bodemag(sys1,'r',sys2,'y--',sys3,'gx').
```

See Also

`bode`	Compute the Bode frequency response of LTI models
`ltiview`	Open an LTI Viewer
`ltimodels`	Help on LTI models

Purpose

Discretize continuous-time systems

Syntax

```
sysd = c2d(sys,Ts)
sysd = c2d(sys,Ts,method)
[sysd,G] = c2d(sys,Ts,method)
```

Description

`sysd = c2d(sys,Ts)` discretizes the continuous-time LTI model sys using zero-order hold on the inputs and a sample time of `Ts` seconds.

`sysd = c2d(sys,Ts,`*`method`*`)` gives access to alternative discretization schemes. The string *method* selects the discretization method among the following:

`'zoh'`	Zero-order hold. The control inputs are assumed piecewise constant over the sampling period `Ts`.
`'foh'`	Triangle approximation (modified first-order hold, see [1], p. 151). The control inputs are assumed piecewise linear over the sampling period `Ts`.
`'tustin'`	Bilinear (Tustin) approximation.
`'prewarp'`	Tustin approximation with frequency prewarping.
`'matched'`	Matched pole-zero method. See [1], p. 147.

Refer to “Continuous/Discrete Conversions of LTI Models” in Chapter 3 for more detail on these discretization methods.

`c2d` supports MIMO systems (except for the `'matched'` method) as well as LTI models with delays with some restrictions for `'matched'` and `'tustin'` methods.

`[sysd,G] = c2d(sys,Ts,`*`method`*`)` returns a matrix G that maps the continuous initial conditions x_0 and u_0 to their discrete counterparts $x[0]$ and $u[0]$ according to

$$x[0] = G \cdot \begin{bmatrix} x_0 \\ u_0 \end{bmatrix}$$

$$u[0] = u_0$$

Example

Consider the system

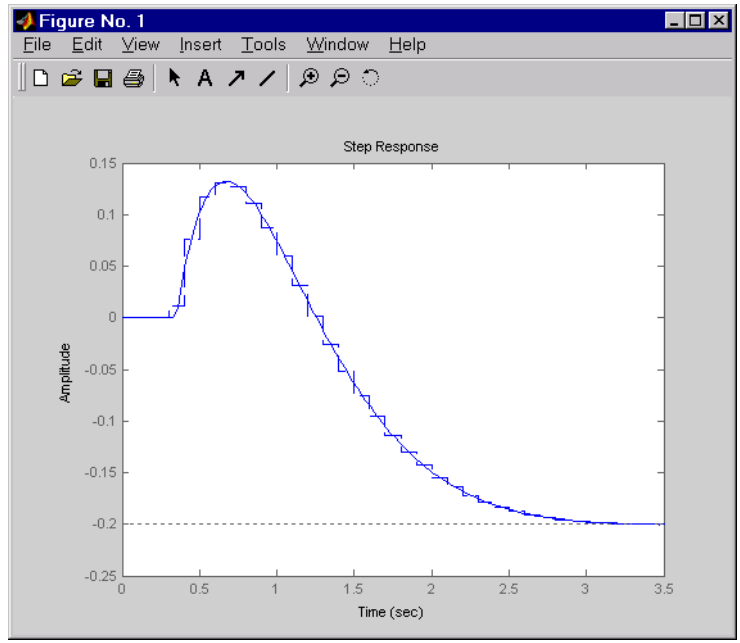
$$H(s) = \frac{s-1}{s^2+4s+5}$$

with input delay $T_d = 0.35$ second. To discretize this system using the triangle approximation with sample time $T_s = 0.1$ second, type

```
H = tf([1 -1],[1 4 5],'inputdelay',0.35)

Transfer function:
                    s - 1
exp(-0.35*s) * -------------
                 s^2 + 4 s + 5

Hd = c2d(H,0.1,'foh')

Transfer function:
0.0115 z^3 + 0.0456 z^2 - 0.0562 z - 0.009104
-----------------------------------------------
        z^6 - 1.629 z^5 + 0.6703 z^4

Sampling time: 0.1
```

The next command compares the continuous and discretized step responses.

```
step(H,'-',Hd,'--')
```



See Also

d2c	Discrete to continuous conversion
d2d	Resampling of discrete systems-

References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.

Purpose

Compute canonical state-space realizations

Syntax

```
csys = canon(sys,'type')
[csys,T] = canon(sys,'type')
```

Description

canon computes a canonical state-space model for the continuous or discrete LTI system sys. Two types of canonical forms are supported.

Modal Form

`csys = canon(sys,'modal')` returns a realization csys in modal form, that is, where the real eigenvalues appear on the diagonal of the A matrix and the complex conjugate eigenvalues appear in 2-by-2 blocks on the diagonal of A. For a system with eigenvalues $(\lambda_1, \sigma \pm j\omega, \lambda_2)$, the modal A matrix is of the form

$$\begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \sigma & \omega & 0 \\ 0 & -\omega & \sigma & 0 \\ 0 & 0 & 0 & \lambda_2 \end{bmatrix}$$

Companion Form

`csys = canon(sys,'companion')` produces a companion realization of sys where the characteristic polynomial of the system appears explicitly in the rightmost column of the A matrix. For a system with characteristic polynomial

$$p(s) = s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n$$

the corresponding companion A matrix is

$$A = \begin{bmatrix} 0 & 0 & .. & .. & 0 & -a_n \\ 1 & 0 & 0 & .. & 0 & -a_{n-1} \\ 0 & 1 & 0 & . & : & : \\ : & 0 & . & . & : & : \\ 0 & . & . & 1 & 0 & -a_2 \\ 0 & .. & .. & 0 & 1 & -a_1 \end{bmatrix}$$

For state-space models sys,

```
[csys,T] = canon(a,b,c,d,'type')
```

also returns the state coordinate transformation T relating the original state vector x and the canonical state vector x_c.

$$x_c = Tx$$

This syntax returns T=[] when sys is not a state-space model.

Algorithm

Transfer functions or zero-pole-gain models are first converted to state space using ss.

The transformation to modal form uses the matrix P of eigenvectors of the A matrix. The modal form is then obtained as

$$\dot{x_c} = P^{-1}APx_c + P^{-1}Bu$$
$$y = CPx_c + Du$$

The state transformation T returned is the inverse of P.

The reduction to companion form uses a state similarity transformation based on the controllability matrix [1].

Limitations

The modal transformation requires that the A matrix be diagonalizable. A sufficient condition for diagonalizability is that A has no repeated eigenvalues.

The companion transformation requires that the system be controllable from the first input. The companion form is often poorly conditioned for most state-space computations; avoid using it when possible.

See Also

ctrb	Controllability matrix
ctrbf	Controllability canonical form
ss2ss	State similarity transformation

References

[1] Kailath, T. *Linear Systems*, Prentice-Hall, 1980.

Purpose

Solve continuous-time algebraic Riccati equations (CARE)

Syntax

```
[X,L,G,rr] = care(A,B,Q)
[X,L,G,rr] = care(A,B,Q,R,S,E)

[X,L,G,report] = care(A,B,Q,...,'report')
[X1,X2,L,report] = care(A,B,Q,...,'implicit')
```

Description

`[X,L,G,rr] = care(A,B,Q)` computes the unique solution X of the algebraic Riccati equation

$$Ric(X) = A^T X + XA - XBB^T X + Q = 0$$

such that $A - BB^T X$ has all its eigenvalues in the open left-half plane. The matrix X is symmetric and called the *stabilizing* solution of $Ric(X) = 0$. `[X,L,G,rr] = care(A,B,Q)` also returns:

- The eigenvalues `L` of $A - BB^T X$
- The gain matrix $G = B^T X$
- The relative residual `rr` defined by $rr = \dfrac{\|Ric(X)\|_F}{\|X\|_F}$

`[X,L,G,rr] = care(A,B,Q,R,S,E)` solves the more general Riccati equation

$$Ric(X) = A^T XE + E^T XA - (E^T XB + S)R^{-1}(B^T XE + S^T) + Q = 0$$

Here the gain matrix is $G = R^{-1}(B^T XE + S^T)$ and the "closed-loop" eigenvalues are `L = eig(A-B*G,E)`.

Two additional syntaxes are provided to help develop applications such as H_∞-optimal control design.

`[X,L,G,report] = care(A,B,Q,...,'report')` turns off the error messages when the solution X fails to exist and returns a failure report instead.

The value of `report` is:

- `-1` when the associated Hamiltonian pencil has eigenvalues on or very near the imaginary axis (failure)
- `-2` when there is no finite solution, i.e., $X = X_2 X_1^{-1}$ with X_1 singular (failure)

- The relative residual rr defined above when the solution exists (success)

Alternatively, `[X1,X2,L,report] = care(A,B,Q,...,'implicit')` also turns off error messages but now returns X in implicit form.

$$X = X_2 X_1^{-1}$$

Note that this syntax returns `report = 0` when successful.

Examples

Example 1

Given

$$A = \begin{bmatrix} -3 & 2 \\ 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 \end{bmatrix} \qquad R = 3$$

you can solve the Riccati equation

$$A^T X + XA - XBR^{-1}B^T X + C^T C = 0$$

by

```
a = [-3 2;1 1]
b = [0 ; 1]
c = [1 -1]
r = 3
[x,l,g] = care(a,b,c'*c,r)
```

This yields the solution

```
x

x =
    0.5895    1.8216
    1.8216    8.8188
```

You can verify that this solution is indeed stabilizing by comparing the eigenvalues of `a` and `a-b*g`.

```
[eig(a)  eig(a-b*g)]

ans =
```

```
   -3.4495   -3.5026
    1.4495   -1.4370
```

Finally, note that the variable `l` contains the closed-loop eigenvalues `eig(a-b*g)`.

```
l

l =
   -3.5026
   -1.4370
```

Example 2

To solve the H_∞-like Riccati equation

$$A^T X + XA + X(\gamma^{-2} B_1 B_1^T - B_2 B_2^T)X + C^T C = 0$$

rewrite it in the `care` format as

$$A^T X + XA - X \underbrace{[B_1, B_2]}_{B} \underbrace{\begin{bmatrix} -\gamma^{-2} I & 0 \\ 0 & I \end{bmatrix}^{-1}}_{R} \begin{bmatrix} B_1^T \\ B_2^T \end{bmatrix} X + C^T C = 0$$

You can now compute the stabilizing solution X by

```
B = [B1 , B2]
m1 = size(B1,2)
m2 = size(B2,2)
R = [-g^2*eye(m1) zeros(m1,m2) ; zeros(m2,m1) eye(m2)]

X = care(A,B,C'*C,R)
```

Algorithm

`care` implements the algorithms described in [1]. It works with the Hamiltonian matrix when R is well-conditioned and $E = I$; otherwise it uses the extended Hamiltonian pencil and QZ algorithm.

Limitations

The (A, B) pair must be stabilizable (that is, all unstable modes are controllable). In addition, the associated Hamiltonian matrix or pencil must

have no eigenvalue on the imaginary axis. Sufficient conditions for this to hold are (Q, A) detectable when $S = 0$ and $R > 0$, or

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0$$

See Also

`dare`	Solve discrete-time Riccati equations
`lyap`	Solve continuous-time Lyapunov equations

References

[1] Arnold, W.F., III and A.J. Laub, "Generalized Eigenproblem Algorithms and Software for Algebraic Riccati Equations," *Proc. IEEE*, 72 (1984), pp. 1746–1754.

chgunits

Purpose

Convert the frequency units of an FRD model

Syntax

```
sys = chgunits(sys,units)
```

Description

`sys = chgunits(sys,units)` converts the units of the frequency points stored in an FRD model, `sys` to `units`, where `units` is either of the strings `'Hz'` or `'rad/s'`. This operation changes the assigned frequencies by applying the appropriate (`2*pi`) scaling factor, and the `'Units'` property is updated.

If the `'Units'` field already matches `units`, no conversion is made.

Example

```
w = logspace(1,2,2);
sys = rss(3,1,1);
sys = frd(sys,w)

From input 'input 1' to:

  Frequency(rad/s)        output 1
  ----------------        --------

           10         0.293773+0.001033i

          100         0.294404+0.000109i

Continuous-time frequency response data.

sys = chgunits(sys,'Hz')
sys.freq

ans =
    1.5915
   15.9155
```

See Also

`frd`	Create or convert to an FRD model
`get`	Get the properties of an LTI model
`set`	Set the properties of an LTI model

Purpose

Derive state-space model from block diagram description

Syntax

```
sysc = connect(sys,Q,inputs,outputs)
```

Description

Complex dynamical systems are often given in block diagram form. For systems of even moderate complexity, it can be quite difficult to find the state-space model required in order to bring certain analysis and design tools into use. Starting with a block diagram description, you can use `append` and `connect` to construct a state-space model of the system.

First, use

```
sys = append(sys1,sys2,...,sysN)
```

to specify each block `sysj` in the diagram and form a block-diagonal, *unconnected* LTI model `sys` of the diagram.

Next, use

```
sysc = connect(sys,Q,inputs,outputs)
```

to connect the blocks together and derive a state-space model `sysc` for the overall interconnection. The arguments `Q`, `inputs`, and `outputs` have the following purpose:

- The matrix `Q` indicates how the blocks on the diagram are connected. It has a row for each input of `sys`, where the first element of each row is the input number. The subsequent elements of each row specify where the block input gets its summing inputs; negative elements indicate minus inputs to the summing junction. For example, if input 7 gets its inputs from the outputs 2, 15, and 6, where the input from output 15 is negative, the corresponding row of `Q` is `[7 2 -15 6]`. Short rows can be padded with trailing zeros (see example below).
- Given `sys` and `Q`, `connect` computes a state-space model of the interconnection with the same inputs and outputs as `sys` (that is, the concatenation of all block inputs and outputs). The index vectors `inputs` and `outputs` then indicate which of the inputs and outputs in the large unconnected system are external inputs and outputs of the block diagram. For example, if the external inputs are inputs 1, 2, and 15 of `sys`, and the external outputs are outputs 2 and 7 of `sys`, then `inputs` and `outputs` should be set to

```
inputs = [1 2 15];
outputs = [2 7];
```

The final model sysc has these particular inputs and outputs.

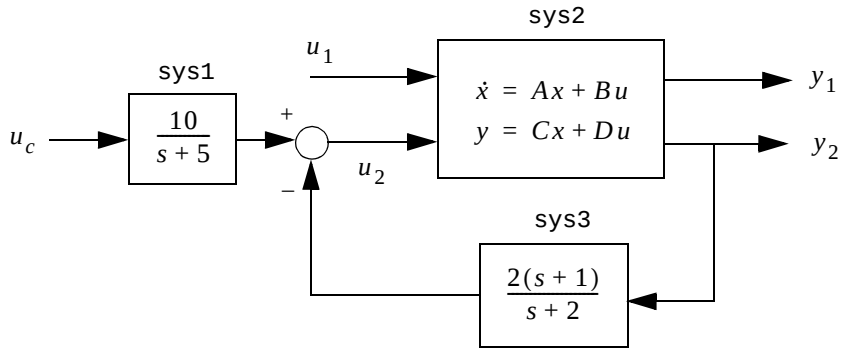
Since it is easy to make a mistake entering all the data required for a large model, be sure to verify your model in as many ways as you can. Here are some suggestions:

- Make sure the poles of the unconnected model sys match the poles of the various blocks in the diagram.
- Check that the final poles and DC gains are reasonable.
- Plot the step and bode responses of sysc and compare them with your expectations.

If you need to work extensively with block diagrams, Simulink is a much easier and more comprehensive tool for model building.

Example

Consider the following block diagram



Given the matrices of the state-space model sys2

```
A = [ -9.0201  17.7791
      -1.6943  3.2138 ];
B = [ -.5112  .5362
      -.002  -1.8470];
C = [ -3.2897  2.4544
```

```
      -13.5009  18.0745];
D = [-.5476  -.1410
     -.6459  .2958 ];
```

Define the three blocks as individual LTI models.

```
sys1 = tf(10,[1 5],'inputname','uc')
sys2 = ss(A,B,C,D,'inputname',{'u1' 'u2'},...
                  'outputname',{'y1' 'y2'})
sys3 = zpk(-1,-2,2)
```

Next append these blocks to form the unconnected model sys.

```
sys = append(sys1,sys2,sys3)
```

This produces the block-diagonal model

```
sys

a =
                    x1          x2          x3          x4
          x1        -5           0           0           0
          x2         0      -9.0201      17.779           0
          x3         0      -1.6943      3.2138           0
          x4         0           0           0          -2


b =
                    uc          u1          u2           ?
          x1         4           0           0           0
          x2         0      -0.5112      0.5362           0
          x3         0       -0.002      -1.847           0
          x4         0           0           0      1.4142


c =
                    x1          x2          x3          x4
           ?       2.5           0           0           0
          y1         0      -3.2897      2.4544           0
          y2         0      -13.501      18.075           0
           ?         0           0           0     -1.4142
```

```
d =
                    uc          u1          u2           ?
          ?          0           0           0           0
         y1          0     -0.5476      -0.141           0
         y2          0     -0.6459      0.2958           0
          ?          0           0           0           2

Continuous-time system.
```

Note that the ordering of the inputs and outputs is the same as the block ordering you chose. Unnamed inputs or outputs are denoted by ?.

To derive the overall block diagram model from sys, specify the interconnections and the external inputs and outputs. You need to connect outputs 1 and 4 into input 3 (u2), and output 3 (y2) into input 4. The interconnection matrix Q is therefore

```
Q = [3 1 -4
     4 3  0];
```

Note that the second row of Q has been padded with a trailing zero. The block diagram has two external inputs uc and u1 (inputs 1 and 2 of sys), and two external outputs y1 and y2 (outputs 2 and 3 of sys). Accordingly, set inputs and outputs as follows.

```
inputs = [1 2];
outputs = [2 3];
```

You can obtain a state-space model for the overall interconnection by typing

```
sysc = connect(sys,Q,inputs,outputs)

a =
                    x1          x2          x3          x4
          x1          -5           0           0           0
          x2     0.84223    0.076636      5.6007     0.47644
          x3     -2.9012      -33.029      45.164     -1.6411
          x4     0.65708      -11.996       16.06     -1.6283


b =
```

```
                uc          u1
      x1         4           0
      x2         0    -0.076001
      x3         0     -1.5011
      x4         0     -0.57391


c =
                x1          x2          x3          x4
      y1  -0.22148     -5.6818      5.6568    -0.12529
      y2   0.46463     -8.4826      11.356     0.26283


d =
                uc          u1
      y1         0    -0.66204
      y2         0    -0.40582

  Continuous-time system.
```

Note that the inputs and outputs are as desired.

See Also

`append`	Append LTI systems
`feedback`	Feedback connection
`minreal`	Minimal state-space realization
`parallel`	Parallel connection
`series`	Series connection

References

[1] Edwards, J.W., "A Fortran Program for the Analysis of Linear Continuous and Sampled-Data Systems," *NASA Report TM X56038*, Dryden Research Center, 1976.

covar

Purpose

Output and state covariance of a system driven by white noise

Syntax

```
[P,Q] = covar(sys,W)
```

Description

covar calculates the stationary covariance of the output y of an LTI model sys driven by Gaussian white noise inputs w. This function handles both continuous- and discrete-time cases.

`P = covar(sys,W)` returns the steady-state output response covariance

$$P = E(yy^T)$$

given the noise intensity

$$E(w(t)w(\tau)^T) = W\,\delta(t-\tau) \qquad \text{(continuous time)}$$
$$E(w[k]w[l]^T) = W\,\delta_{kl} \qquad \text{(discrete time)}$$

`[P,Q] = covar(sys,W)` also returns the steady-state state covariance

$$Q = E(xx^T)$$

when sys is a state-space model (otherwise `Q` is set to `[]`).

When applied to an `N`-dimensional LTI array sys, covar returns multi-dimensional arrays P, Q such that

`P(:,:,i1,...iN)` and `Q(:,:,i1,...iN)` are the covariance matrices for the model `sys(:,:,i1,...iN)`.

Example

Compute the output response covariance of the discrete SISO system

$$H(z) = \frac{2z+1}{z^2+0.2z+0.5}, \qquad T_s = 0.1$$

due to Gaussian white noise of intensity `W = 5`. Type

```
sys = tf([2 1],[1 0.2 0.5],0.1);
p = covar(sys,5)
```

and MATLAB returns

```
p =
    30.3167
```

You can compare this output of `covar` to simulation results.

```
randn('seed',0)
w = sqrt(5)*randn(1,1000);  % 1000 samples

% Simulate response to w with LSIM:
y = lsim(sys,w);

% Compute covariance of y values
psim = sum(y .* y)/length(w);
```

This yields

```
psim =
    32.6269
```

The two covariance values `p` and `psim` do not agree perfectly due to the finite simulation horizon.

Algorithm

Transfer functions and zero-pole-gain models are first converted to state space with `ss`.

For continuous-time state-space models

$$\dot{x} = Ax + Bw$$
$$y = Cx + Dw$$

Q is obtained by solving the Lyapunov equation

$$AQ + QA^T + BWB^T = 0$$

The output response covariance P is finite only when $D = 0$ and then $P = CQC^T$.

In discrete time, the state covariance solves the discrete Lyapunov equation

$$AQA^T - Q + BWB^T = 0$$

and P is given by $P = CQC^T + DWD^T$

Note that P is well defined for nonzero D in the discrete case.

Limitations

The state and output covariances are defined for *stable* systems only. For continuous systems, the output response covariance P is finite only when the D matrix is zero (strictly proper system).

See Also

`dlyap`	Solver for discrete-time Lyapunov equations
`lyap`	Solver for continuous-time Lyapunov equations

References

[1] Bryson, A.E. and Y.C. Ho, *Applied Optimal Control*, Hemisphere Publishing, 1975, pp. 458-459.

Purpose

Form the controllability matrix

Syntax

```
Co = ctrb(A,B)
Co = ctrb(sys)
```

Description

`ctrb` computes the controllability matrix for state-space systems. For an *n*-by-*n* matrix `A` and an *n*-by-*m* matrix `B`, `ctrb(A,B)` returns the controllability matrix

$$Co = \left[B \; AB \; A^2B \; \ldots \; A^{n-1}B\right] \qquad \textbf{(16-1)}$$

where Co has n rows and nm columns.

`Co = ctrb(sys)` calculates the controllability matrix of the state-space LTI object `sys`. This syntax is equivalent to executing

```
Co = ctrb(sys.A,sys.B)
```

The system is controllable if `Co` has full rank n.

Example

Check if the system with the following data

```
A =
     1     1
     4    -2

B =
     1    -1
     1    -1
```

is controllable. Type

```
Co=ctrb(A,B);

% Number of uncontrollable states
unco=length(A)-rank(Co)
```

and MATLAB returns

```
unco =
     1
```

Limitations

Estimating the rank of the controllability matrix is ill-conditioned; that is, it is very sensitive to round-off errors and errors in the data. An indication of this can be seen from this simple example.

$$A = \begin{bmatrix} 1 & \delta \\ 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ \delta \end{bmatrix}$$

This pair is controllable if $\delta \neq 0$ but if $\delta < \sqrt{eps}$, where *eps* is the relative machine precision. `ctrb(A,B)` returns

$$\begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \delta & \delta \end{bmatrix}$$

which is not full rank. For cases like these, it is better to determine the controllability of a system using `ctrbf`.

See Also

`ctrbf`	Compute the controllability staircase form
`obsv`	Compute the observability matrix

Purpose

Compute the controllability staircase form

Syntax

```
[Abar,Bbar,Cbar,T,k] = ctrbf(A,B,C)
[Abar,Bbar,Cbar,T,k] = ctrbf(A,B,C,tol)
```

Description

If the controllability matrix of (A, B) has rank $r \le n$, where n is the size of A, then there exists a similarity transformation such that

$$\overline{A} = TAT^T, \qquad \overline{B} = TB, \qquad \overline{C} = CT^T$$

where T is unitary, and the transformed system has a *staircase* form, in which the uncontrollable modes, if there are any, are in the upper left corner.

$$\overline{A} = \begin{bmatrix} A_{uc} & 0 \\ A_{21} & A_c \end{bmatrix}, \qquad \overline{B} = \begin{bmatrix} 0 \\ B_c \end{bmatrix}, \qquad \overline{C} = \begin{bmatrix} C_{nc} & C_c \end{bmatrix}$$

where (A_c, B_c) is controllable, all eigenvalues of A_{uc} are uncontrollable, and

$$C_c(sI - A_c)^{-1}B_c = C(sI - A)^{-1}B.$$

`[Abar,Bbar,Cbar,T,k] = ctrbf(A,B,C)` decomposes the state-space system represented by `A`, `B`, and `C` into the controllability staircase form, `Abar`, `Bbar`, and `Cbar`, described above. `T` is the similarity transformation matrix and `k` is a vector of length n, where n is the order of the system represented by `A`. Each entry of `k` represents the number of controllable states factored out during each step of the transformation matrix calculation. The number of nonzero elements in `k` indicates how many iterations were necessary to calculate `T`, and `sum(k)` is the number of states in A_c, the controllable portion of `Abar`.

`ctrbf(A,B,C,tol)` uses the tolerance `tol` when calculating the controllable/uncontrollable subspaces. When the tolerance is not specified, it defaults to `10*n*norm(A,1)*eps`.

Example

Compute the controllability staircase form for

```
A =
     1     1
     4    -2
```

```
B =
     1    -1
     1    -1

C =
     1     0
     0     1
```

and locate the uncontrollable mode.

```
[Abar,Bbar,Cbar,T,k]=ctrbf(A,B,C)

Abar =
   -3.0000         0
   -3.0000    2.0000

Bbar =
    0.0000    0.0000
    1.4142   -1.4142

Cbar =
   -0.7071    0.7071
    0.7071    0.7071

T =
   -0.7071    0.7071
    0.7071    0.7071
k =
     1     0
```

The decomposed system `Abar` shows an uncontrollable mode located at –3 and a controllable mode located at 2.

Algorithm

`ctrbf` is an M-file that implements the Staircase Algorithm of [1].

See Also

`ctrb`	Form the controllability matrix
`minreal`	Minimum realization and pole-zero cancellation

References

[1] Rosenbrock, M.M., *State-Space and Multivariable Theory*, John Wiley, 1970.

Purpose

Convert discrete-time LTI models to continuous time

Syntax

```
sysc = d2c(sysd)
sysc = d2c(sysd,method)
```

Description

`d2c` converts LTI models from discrete to continuous time using one of the following conversion methods:

`'zoh'`	Zero-order hold on the inputs. The control inputs are assumed piecewise constant over the sampling period.
`'tustin'`	Bilinear (Tustin) approximation to the derivative.
`'prewarp'`	Tustin approximation with frequency prewarping.
`'matched'`	Matched pole-zero method of [1] (for SISO systems only).

The string *method* specifies the conversion method. If *method* is omitted then zero-order hold (`'zoh'`) is assumed. See "Continuous/Discrete Conversions of LTI Models" in Chapter 3 of this manual and reference [1] for more details on the conversion methods.

Example

Consider the discrete-time model with transfer function

$$H(z) = \frac{z-1}{z^2 + z + 0.3}$$

and sample time $T_s = 0.1$ second. You can derive a continuous-time zero-order-hold equivalent model by typing

```
Hc = d2c(H)
```

Discretizing the resulting model `Hc` with the zero-order hold method (this is the default method) and sampling period $T_s = 0.1$ gives back the original discrete model $H(z)$. To see this, type

```
c2d(Hc,0.1)
```

To use the Tustin approximation instead of zero-order hold, type

```
Hc = d2c(H,'tustin')
```

As with zero-order hold, the inverse discretization operation

```
c2d(Hc,0.1,'tustin')
```

gives back the original $H(z)$.

Algorithm

The `'zoh'` conversion is performed in state space and relies on the matrix logarithm (see `logm` in *Using MATLAB*).

Limitations

The Tustin approximation is not defined for systems with poles at $z = -1$ and is ill-conditioned for systems with poles near $z = -1$.

The zero-order hold method cannot handle systems with poles at $z = 0$. In addition, the `'zoh'` conversion increases the model order for systems with negative real poles, [2]. This is necessary because the matrix logarithm maps real negative poles to complex poles. As a result, a discrete model with a single pole at $z = -0.5$ would be transformed to a continuous model with a single *complex* pole at $\log(-0.5) \approx -0.6931 + j\pi$. Such a model is not meaningful because of its complex time response.

To ensure that all complex poles of the continuous model come in conjugate pairs, `d2c` replaces negative real poles $z = -\alpha$ with a pair of complex conjugate poles near $-\alpha$. The conversion then yields a continuous model with higher order. For example, the discrete model with transfer function

$$H(z) = \frac{z + 0.2}{(z + 0.5)(z^2 + z + 0.4)}$$

and sample time 0.1 second is converted by typing

```
Ts = 0.1
H = zpk(-0.2,-0.5,1,Ts) * tf(1,[1 1 0.4],Ts)
Hc = d2c(H)
```

MATLAB responds with

```
Warning: System order was increased to handle real negative poles.

Zero/pole/gain:
  -33.6556 (s-6.273) (s^2 + 28.29s + 1041)
-----------------------------------------
(s^2 + 9.163s + 637.3) (s^2 + 13.86s + 1035)
```

Convert `Hc` back to discrete time by typing

```
c2d(Hc,Ts)
```

yielding

```
Zero/pole/gain:
     (z+0.5) (z+0.2)
------------------------
(z+0.5)^2 (z^2 + z + 0.4)

Sampling time: 0.1
```

This discrete model coincides with $H(z)$ after canceling the pole/zero pair at $z = -0.5$.

See Also

`c2d`	Continuous- to discrete-time conversion
`d2d`	Resampling of discrete models
`logm`	Matrix logarithm

References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.

[2] Kollár, I., G.F. Franklin, and R. Pintelon, "On the Equivalence of z-domain and s-domain Models in System Identification," *Proceedings of the IEEE Instrumentation and Measurement Technology Conference*, Brussels, Belgium, June, 1996, Vol. 1, pp. 14-19.

d2d

Purpose

Resample discrete-time LTI models or add input delays

Syntax

```
sys1 = d2d(sys,Ts)
```

Description

`sys1 = d2d(sys,Ts)` resamples the discrete-time LTI model `sys` to produce an equivalent discrete-time model `sys1` with the new sample time `Ts` (in seconds). The resampling assumes zero-order hold on the inputs and is equivalent to consecutive `d2c` and `c2d` conversions.

```
sys1 = c2d(d2c(sys),Ts)
```

Example

Consider the zero-pole-gain model

$$H(z) = \frac{z - 0.7}{z - 0.5}$$

with sample time 0.1 second. You can resample this model at 0.05 second by typing

```
H = zpk(0.7,0.5,1,0.1)
H2 = d2d(H,0.05)
```

```
Zero/pole/gain:
(z-0.8243)
----------
(z-0.7071)

Sampling time: 0.05
```

Note that the inverse resampling operation, performed by typing `d2d(H2,0.1)`, yields back the initial model $H(z)$.

```
Zero/pole/gain:
(z-0.7)
-------
(z-0.5)

Sampling time: 0.1
```

See Also

`c2d`	Continuous- to discrete-time conversion
`d2c`	Discrete- to continuous-time conversion

Purpose

Compute damping factors and natural frequencies

Syntax

```
[Wn,Z] = damp(sys)
[Wn,Z,P] = damp(sys)
```

Description

`damp` calculates the damping factor and natural frequencies of the poles of an LTI model `sys`. When invoked without lefthand arguments, a table of the eigenvalues in increasing frequency, along with their damping factors and natural frequencies, is displayed on the screen.

`[Wn,Z] = damp(sys)` returns column vectors `Wn` and `Z` containing the natural frequencies ω_n and damping factors ζ of the poles of `sys`. For discrete-time systems with poles z and sample time T_s, `damp` computes "equivalent" continuous-time poles s by solving

$$z = e^{sT_s}$$

The values `Wn` and `Z` are then relative to the continuous-time poles s. Both `Wn` and `Z` are empty if the sample time is unspecified.

`[Wn,Z,P] = damp(sys)` returns an additional vector `P` containing the (true) poles of `sys`. Note that `P` returns the same values as `pole(sys)` (up to reordering).

Example

Compute and display the eigenvalues, natural frequencies, and damping factors of the continuous transfer function

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

Type

```
H = tf([2 5 1],[1 2 3])
```

```
Transfer function:
2 s^2 + 5 s + 1
---------------
 s^2 + 2 s + 3
```

Type

```
damp(H)
```

and MATLAB returns

```
Eigenvalue            Damping     Freq. (rad/s)

-1.00e+000 + 1.41e+000i     5.77e-001      1.73e+000
-1.00e+000 - 1.41e+000i     5.77e-001      1.73e+000
```

See Also

`eig`	Calculate eigenvalues and eigenvectors
`esort,dsort`	Sort system poles
`pole`	Compute system poles
`pzmap`	Pole-zero map
`zero`	Compute (transmission) zeros

Purpose

Solve discrete-time algebraic Riccati equations (DARE)

Syntax

```
[X,L,G,rr] = dare(A,B,Q,R)
[X,L,G,rr] = dare(A,B,Q,R,S,E)

[X,L,G,report] = dare(A,B,Q,...,'report')
[X1,X2,L,report] = dare(A,B,Q,...,'implicit')
```

Description

`[X,L,G,rr] = dare(A,B,Q,R)` computes the unique solution X of the discrete-time algebraic Riccati equation

$$Ric(X) = A^T XA - X - A^T XB(B^T XB + R)^{-1} B^T XA + Q = 0$$

such that the "closed-loop" matrix

$$A_{cl} = A - B(B^T XB + R)^{-1} B^T XA$$

has all its eigenvalues inside the unit disk. The matrix X is symmetric and called the *stabilizing* solution of $Ric(X) = 0$. `[X,L,G,rr] = dare(A,B,Q,R)` also returns:

- The eigenvalues `L` of A_{cl}
- The gain matrix

$$G = (B^T XB + R)^{-1} B^T XA$$

- The relative residual `rr` defined by

$$rr = \frac{\|Ric(X)\|_F}{\|X\|_F}$$

`[X,L,G,rr] = dare(A,B,Q,R,S,E)` solves the more general DARE:

$$A^T XA - E^T XE - (A^T XB + S)(B^T XB + R)^{-1}(B^T XA + S^T) + Q = 0$$

The corresponding gain matrix and closed-loop eigenvalues are

$$G = (B^T XB + R)^{-1}(B^T XA + S^T)$$

and `L = eig(A-B*G,E)`.

Two additional syntaxes are provided to help develop applications such as H_∞-optimal control design.

`[X,L,G,report] = dare(A,B,Q,...,'report')` turns off the error messages when the solution X fails to exist and returns a failure report instead. The value of `report` is:

- `-1` when the associated symplectic pencil has eigenvalues on or very near the unit circle (failure)
- `-2` when there is no finite solution, that is, $X = X_2 X_1^{-1}$ with X_1 singular (failure)
- The relative residual rr defined above when the solution exists (success)

Alternatively, `[X1,X2,L,report] = dare(A,B,Q,...,'implicit')` also turns off error messages but now returns X in implicit form as

$$X = X_2 X_1^{-1}$$

Note that this syntax returns `report = 0` when successful.

Algorithm

`dare` implements the algorithms described in [1]. It uses the QZ algorithm to deflate the extended symplectic pencil and compute its stable invariant subspace.

Limitations

The (A, B) pair must be stabilizable (that is, all eigenvalues of A outside the unit disk must be controllable). In addition, the associated symplectic pencil must have no eigenvalue on the unit circle. Sufficient conditions for this to hold are (Q, A) detectable when $S = 0$ and $R > 0$, or

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0$$

See Also

`care`	Solve continuous-time Riccati equations
`dlyap`	Solve discrete-time Lyapunov equations

References

[1] Arnold, W.F., III and A.J. Laub, “Generalized Eigenproblem Algorithms and Software for Algebraic Riccati Equations,” *Proc. IEEE*, 72 (1984), pp. 1746–1754.

dcgain

Purpose

Compute low frequency (DC) gain of LTI system

Syntax

```
k = dcgain(sys)
```

Description

`k = dcgain(sys)` computes the DC gain `k` of the LTI model `sys`.

Continuous Time

The continuous-time DC gain is the transfer function value at the frequency $s = 0$. For state-space models with matrices (A, B, C, D), this value is

$$K = D - CA^{-1}B$$

Discrete Time

The discrete-time DC gain is the transfer function value at $z = 1$. For state-space models with matrices (A, B, C, D), this value is

$$K = D + C(I - A)^{-1}B$$

Remark

The DC gain is infinite for systems with integrators.

Example

To compute the DC gain of the MIMO transfer function

$$H(s) = \begin{bmatrix} 1 & \dfrac{s-1}{s^2+s+3} \\ \dfrac{1}{s+1} & \dfrac{s+2}{s-3} \end{bmatrix}$$

type

```
H = [1 tf([1 -1],[1 1 3]) ; tf(1,[1 1]) tf([1 2],[1 -3])]
dcgain(H)
```

```
ans =
    1.0000   -0.3333
    1.0000   -0.6667
```

See Also

`evalfr`	Evaluates frequency response at single frequency
`norm`	LTI system norms

Purpose

Replace delays of discrete-time TF, SS, or ZPK models by poles at z=0, or replace delays of FRD models by a phase shift

Syntax

```
sys = delay2z(sys)
```

Description

`sys = delay2z(sys)` maps all time delays to poles at z=0 for discrete-time TF, ZPK, or SS models `sys`. Specifically, a delay of `k` sampling periods is replaced by `(1/z)^k` in the transfer function corresponding to the model.

For FRD models, `delay2z` absorbs all time delays into the frequency response data, and is applicable to both continuous- and discrete-time FRDs.

Example

```
z=tf('z',-1);
sys=(-.4*z -.1)/(z^2 + 1.05*z + .08)

Transfer function:

-0.4 z - 0.1
-------------------
z^2 + 1.05 z + 0.08

Sampling time: unspecified

sys.InputDelay = 1;
sys = delay2z(sys)

Transfer function:

     -0.4 z - 0.1
-----------------------
z^3 + 1.05 z^2 + 0.08 z

Sampling time: unspecified
```

See Also

`hasdelay`	True for LTI models with delays
`pade`	Pade approximation of time delays
`totaldelay`	Combine delays for an LTI model

dlqr

Purpose

Design linear-quadratic (LQ) state-feedback regulator for discrete-time plant

Syntax

```
[K,S,e] = dlqr(a,b,Q,R)
[K,S,e] = dlqr(a,b,Q,R,N)
```

Description

`[K,S,e] = dlqr(a,b,Q,R,N)` calculates the optimal gain matrix `K` such that the state-feedback law

$$u[n] = -Kx[n]$$

minimizes the quadratic cost function

$$J(u) = \sum_{n=1}^{\infty} (x[n]^T Q x[n] + u[n]^T R u[n] + 2x[n]^T N u[n])$$

for the discrete-time state-space mode

$$\mathrm{l}x[n+1] = Ax[n] + Bu[n]$$

The default value `N=0` is assumed when `N` is omitted.

In addition to the state-feedback gain `K`, `dlqr` returns the infinite horizon solution S of the associated discrete-time Riccati equation

$$A^T SA - S - (A^T SB + N)(B^T SB + R)^{-1}(B^T SA + N^T) + Q = 0$$

and the closed-loop eigenvalues `e = eig(a-b*K)`. Note that K is derived from S by

$$K = (B^T SB + R)^{-1}(B^T SA + N^T)$$

Limitations

The problem data must satisfy:

- The pair (A, B) is stabilizable.
- $R > 0$ and $Q - NR^{-1}N^T \geq 0$.
- $(Q - NR^{-1}N^T, A - BR^{-1}N^T)$ has no unobservable mode on the unit circle.

See Also

`dare`	Solve discrete Riccati equations
`lqgreg`	LQG regulator

`lqr`	State-feedback LQ regulator for continuous plant
`lqrd`	Discrete LQ regulator for continuous plant
`lqry`	State-feedback LQ regulator with output weighting

dlyap

Purpose

Solve discrete-time Lyapunov equations

Syntax

```
X = dlyap(A,Q)
```

Description

`dlyap` solves the discrete-time Lyapunov equation

$$A^T X A - X + Q = 0$$

where A and Q are n-by-n matrices.

The solution X is symmetric when Q is symmetric, and positive definite when Q is positive definite and A has all its eigenvalues inside the unit disk.

Diagnostics

The discrete-time Lyapunov equation has a (unique) solution if the eigenvalues $\alpha_1, \alpha_2, ..., \alpha_n$ of A satisfy $\alpha_i \alpha_j \neq 1$ for all (i, j).

If this condition is violated, `dlyap` produces the error message

```
Solution does not exist or is not unique.
```

See Also

`covar`	Covariance of system response to white noise
`lyap`	Solve continuous Lyapunov equations

Purpose

Generate stable random discrete test models

Syntax

```
sys = drss(n)
sys = drss(n,p)
sys = drss(n,p,m)
sys = drss(n,p,m,s1,...sn)
```

Description

`sys = drss(n)` produces a random *n*-th order stable model with one input and one output, and returns the model in the state-space object `sys`.

`drss(n,p)` produces a random *n*-th order stable model with one input and `p` outputs.

`drss(n,m,p)` generates a random *n*-th order stable model with `m` inputs and `p` outputs.

`drss(n,p,m,s1,...sn)` generates a *s1*–by–*sn* array of random *n*-th order stable model with `m` inputs and `p` outputs.

In all cases, the discrete-time state-space model or array returned by `drss` has an unspecified sampling time. To generate transfer function or zero-pole-gain systems, convert `sys` using `tf` or `zpk`.

Example

Generate a random discrete LTI system with three states, two inputs, and two outputs.

```
sys = drss(3,2,2)

a =
                    x1          x2          x3
          x1     0.38630    -0.21458    -0.09914
          x2    -0.23390    -0.15220    -0.06572
          x3    -0.03412     0.11394    -0.22618

b =
                    u1          u2
          x1     0.98833     0.51551
          x2           0     0.33395
          x3     0.42350     0.43291
```

```
c =
                x1          x2          x3
y1         0.22595     0.76037           0
y2               0           0           0

d =
                u1          u2
y1               0     0.68085
y2         0.78333     0.46110

Sampling time: unspecified
Discrete-time system.
```

See Also

`rss`	Generate stable random continuous test models
`tf`	Convert LTI systems to transfer functions form
`zpk`	Convert LTI systems to zero-pole-gain form

Purpose

Sort discrete-time poles by magnitude

Syntax

```
s = dsort(p)
[s,ndx] = dsort(p)
```

Description

`dsort` sorts the discrete-time poles contained in the vector `p` in descending order by magnitude. Unstable poles appear first.

When called with one lefthand argument, `dsort` returns the sorted poles in `s`.

`[s,ndx] = dsort(p)` also returns the vector `ndx` containing the indices used in the sort.

Example

Sort the following discrete poles.

```
p =
  -0.2410 + 0.5573i
  -0.2410 - 0.5573i
   0.1503
  -0.0972
  -0.2590

s = dsort(p)

s =
  -0.2410 + 0.5573i
  -0.2410 - 0.5573i
  -0.2590
   0.1503
  -0.0972
```

Limitations

The poles in the vector `p` must appear in complex conjugate pairs.

See Also

`eig`	Calculate eigenvalues and eigenvectors
`esort`, `sort`	Sort system poles
`pole`	Compute system poles
`pzmap`	Pole-zero map
`zero`	Compute (transmission) zeros

dss

Purpose

Specify descriptor state-space models

Syntax

```
sys = dss(a,b,c,d,e)
sys = dss(a,b,c,d,e,Ts)
sys = dss(a,b,c,d,e,ltisys)

sys = dss(a,b,c,d,e,'Property1',Value1,...,'PropertyN',ValueN)
sys = dss(a,b,c,d,e,Ts,'Property1',Value1,...,'PropertyN',ValueN)
```

Description

`sys = dss(a,b,c,d,e)` creates the continuous-time descriptor state-space model

$$E\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

The E matrix must be nonsingular. The output `sys` is an SS model storing the model data (see "LTI Objects" on page 2-3). Note that `ss` produces the same type of object. If the matrix $D = 0$, do can simply set `d` to the scalar `0` (zero).

`sys = dss(a,b,c,d,e,Ts)` creates the discrete-time descriptor model

$$Ex[n+1] = Ax[n] + Bu[n]$$
$$y[n] = Cx[n] + Du[n]$$

with sample time `Ts` (in seconds).

`sys = dss(a,b,c,d,e,ltisys)` creates a descriptor model with generic LTI properties inherited from the LTI model `ltisys` (including the sample time). See "LTI Properties" on page 2-26 for an overview of generic LTI properties.

Any of the previous syntaxes can be followed by property name/property value pairs

```
'Property',Value
```

Each pair specifies a particular LTI property of the model, for example, the input names or some notes on the model history. See `set` and the example below for details.

Example

The command

```
sys = dss(1,2,3,4,5,'td',0.1,'inputname','voltage',...
                                'notes','Just an example')
```

creates the model

$$5\dot{x} = x + 2u$$
$$y = 3x + 4u$$

with a 0.1 second input delay. The input is labeled `'voltage'`, and a note is attached to tell you that this is just an example.

See Also

`dssdata`	Retrieve A, B, C, D, E matrices of descriptor model
`get`	Get properties of LTI models
`set`	Set properties of LTI models
`ss`	Specify (regular) state-space models

dssdata

Purpose

Quick access to descriptor state-space data

Syntax

```
[a,b,c,d,e] = dssdata(sys)
[a,b,c,d,e,Ts] = dssdata(sys)
```

Description

`[a,b,c,d,e] = dssdata(sys)` extracts the descriptor matrix data (A, B, C, D, E) from the state-space model `sys`. If `sys` is a transfer function or zero-pole-gain model, it is first converted to state space. Note that `dssdata` is then equivalent to `ssdata` because it always returns $E = I$.

`[a,b,c,d,e,Ts] = dssdata(sys)` also returns the sample time `Ts`.

You can access the remaining LTI properties of `sys` with `get` or by direct referencing, for example,

```
sys.notes
```

See Also

`dss`	Specify descriptor state-space models
`get`	Get properties of LTI models
`ssdata`	Quick access to state-space data
`tfdata`	Quick access to transfer function data
`zpkdata`	Quick access to zero-pole-gain data

Purpose

Sort continuous-time poles by real part

Syntax

```
s = esort(p)
[s,ndx] = esort(p)
```

Description

`esort` sorts the continuous-time poles contained in the vector `p` by real part. Unstable eigenvalues appear first and the remaining poles are ordered by decreasing real parts.

When called with one left-hand argument, `s = esort(p)` returns the sorted eigenvalues in `s`.

`[s,ndx] = esort(p)` returns the additional argument `ndx`, a vector containing the indices used in the sort.

Example

Sort the following continuous eigenvalues.

```
p
p =
  -0.2410+ 0.5573i
  -0.2410- 0.5573i
   0.1503
  -0.0972
  -0.2590

esort(p)

ans =
   0.1503
  -0.0972
  -0.2410+ 0.5573i
  -0.2410- 0.5573i
  -0.2590
```

Limitations

The eigenvalues in the vector `p` must appear in complex conjugate pairs.

See Also

`dsort`, `sort`	Sort system poles
`eig`	Calculate eigenvalues and eigenvectors
`pole`	Compute system poles
`pzmap`	Pole-zero map

`zero`	Compute (transmission) zeros

Purpose

Form state estimator given estimator gain

Syntax

```
est = estim(sys,L)
est = estim(sys,L,sensors,known)
```

Description

`est = estim(sys,L)` produces a state/output estimator `est` given the plant state-space model `sys` and the estimator gain `L`. All inputs w of `sys` are assumed stochastic (process and/or measurement noise), and all outputs y are measured. The estimator `est` is returned in state-space form (SS object). For a continuous-time plant `sys` with equations

$$\dot{x} = Ax + Bw$$
$$y = Cx + Dw$$

`estim` generates plant output and state estimates $\hat{y}$ and $\hat{x}$ as given by the following model.

$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x})$$
$$\begin{bmatrix}\hat{y}\\ \hat{x}\end{bmatrix} = \begin{bmatrix}C\\ I\end{bmatrix}\hat{x}$$

The discrete-time estimator has similar equations.

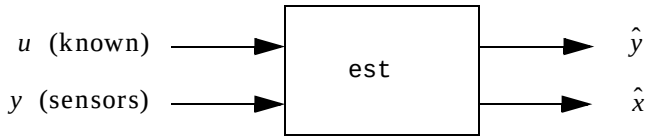
`est = estim(sys,L,sensors,known)` handles more general plants `sys` with both known inputs u and stochastic inputs w, and both measured outputs y and nonmeasured outputs z.

$$\dot{x} = Ax + B_1 w + B_2 u$$
$$\begin{bmatrix}z\\ y\end{bmatrix} = \begin{bmatrix}C_1\\ C_2\end{bmatrix}x + \begin{bmatrix}D_{11}\\ D_{21}\end{bmatrix}w + \begin{bmatrix}D_{12}\\ D_{22}\end{bmatrix}u$$

The index vectors `sensors` and `known` specify which outputs y are measured and which inputs u are known. The resulting estimator `est` uses both u and y to produce the output and state estimates.

$$\dot{\hat{x}} = A\hat{x} + B_2 u + L(y - C_2\hat{x} - D_{22}u)$$

$$\begin{bmatrix} \hat{y} \\ \hat{x} \end{bmatrix} = \begin{bmatrix} C_2 \\ I \end{bmatrix}\hat{x} + \begin{bmatrix} D_{22} \\ 0 \end{bmatrix} u$$



`estim` handles both continuous- and discrete-time cases. You can use the functions `place` (pole placement) or `kalman` (Kalman filtering) to design an adequate estimator gain L. Note that the estimator poles (eigenvalues of $A - LC$) should be faster than the plant dynamics (eigenvalues of A) to ensure accurate estimation.

Example

Consider a state-space model `sys` with seven outputs and four inputs. Suppose you designed a Kalman gain matrix L using outputs 4, 7, and 1 of the plant as sensor measurements, and inputs 1,4, and 3 of the plant as known (deterministic) inputs. You can then form the Kalman estimator by

```
sensors = [4,7,1];
known = [1,4,3];
est = estim(sys,L,sensors,known)
```

See the function `kalman` for direct Kalman estimator design.

See Also

`kalman`	Design Kalman estimator
`place`	Pole placement
`reg`	Form regulator given state-feedback and estimator gains

Purpose

Evaluate frequency response at a single (complex) frequency

Syntax

```
frsp = evalfr(sys,f)
```

Description

`frsp = evalfr(sys,f)` evaluates the transfer function of the TF, SS, or ZPK model `sys` at the complex number `f`. For state-space models with data (A, B, C, D), the result is

$$H(f) = D + C(fI - A)^{-1}B$$

`evalfr` is a simplified version of `freqresp` meant for quick evaluation of the response at a single point. Use `freqresp` to compute the frequency response over a set of frequencies.

Example

To evaluate the discrete-time transfer function

$$H(z) = \frac{z-1}{z^2+z+1}$$

at $z = 1 + j$, type

```
H = tf([1 -1],[1 1 1],-1)
z = 1+j
evalfr(H,z)

ans =
  2.3077e-01 +  1.5385e-01i
```

Limitations

The response is not finite when `f` is a pole of `sys`.

See Also

`bode`	Bode frequency response
`freqresp`	Frequency response over a set of frequencies
`sigma`	Singular value response

Purpose

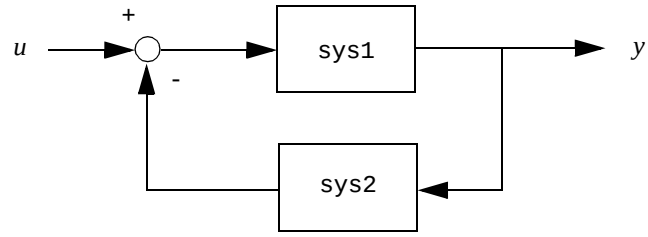
Feedback connection of two LTI models

Syntax

```
sys = feedback(sys1,sys2)
sys = feedback(sys1,sys2,sign)
sys = feedback(sys1,sys2,feedin,feedout,sign)
```

Description

`sys = feedback(sys1,sys2)` returns an LTI model `sys` for the negative feedback interconnection.



The closed-loop model `sys` has *u* as input vector and *y* as output vector. The LTI models `sys1` and `sys2` must be both continuous or both discrete with identical sample times. Precedence rules are used to determine the resulting model type (see Precedence Rules).

To apply positive feedback, use the syntax

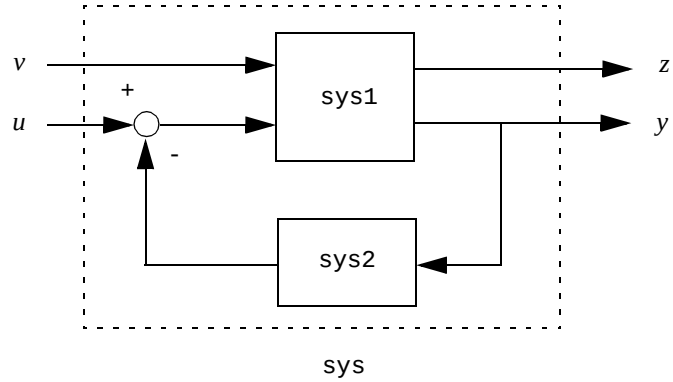
```
sys = feedback(sys1,sys2,+1)
```

By default, `feedback(sys1,sys2)` assumes negative feedback and is equivalent to `feedback(sys1,sys2,-1)`.

Finally,

```
sys = feedback(sys1,sys2,feedin,feedout)
```

computes a closed-loop model sys for the more general feedback loop.



The vector feedin contains indices into the input vector of sys1 and specifies which inputs *u* are involved in the feedback loop. Similarly, feedout specifies which outputs *y* of sys1 are used for feedback. The resulting LTI model sys has the same inputs and outputs as sys1 (with their order preserved). As before, negative feedback is applied by default and you must use

```
sys = feedback(sys1,sys2,feedin,feedout,+1)
```

to apply positive feedback.

For more complicated feedback structures, use append and connect.

Remark

You can specify static gains as regular matrices, for example,

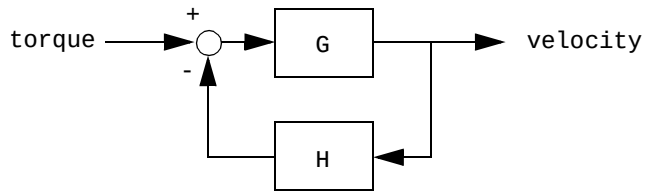
```
sys = feedback(sys1,2)
```

However, at least one of the two arguments sys1 and sys2 should be an LTI object. For feedback loops involving two static gains k1 and k2, use the syntax

```
sys = feedback(tf(k1),k2)
```

Examples

Example 1



To connect the plant

$$G(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

with the controller

$$H(s) = \frac{5(s + 2)}{s + 10}$$

using negative feedback, type

```
G = tf([2 5 1],[1 2 3],'inputname','torque',...
                       'outputname','velocity');
H = zpk(-2,-10,5)
Cloop = feedback(G,H)
```

and MATLAB returns

```
Zero/pole/gain from input "torque" to output "velocity":
0.18182 (s+10) (s+2.281) (s+0.2192)
-----------------------------------
 (s+3.419) (s^2 + 1.763s + 1.064)
```

The result is a zero-pole-gain model as expected from the precedence rules. Note that `Cloop` inherited the input and output names from `G`.

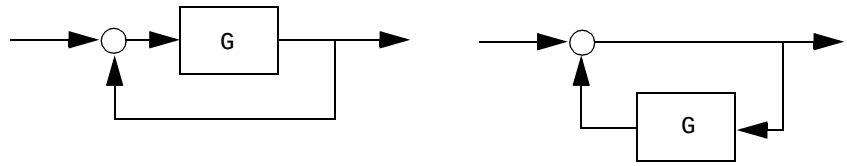
Example 2

Consider a state-space plant `P` with five inputs and four outputs and a state-space feedback controller `K` with three inputs and two outputs. To connect outputs 1, 3, and 4 of the plant to the controller inputs, and the controller outputs to inputs 4 and 2 of the plant, use

```
feedin = [4 2];
feedout = [1 3 4];
Cloop = feedback(P,K,feedin,feedout)
```

Example 3

You can form the following negative-feedback loops



by

```
Cloop = feedback(G,1)    % left diagram
Cloop = feedback(1,G)    % right diagram
```

Limitations

The feedback connection should be free of algebraic loop. If D_1 and D_2 are the feedthrough matrices of `sys1` and `sys2`, this condition is equivalent to:

- $I + D_1 D_2$ nonsingular when using negative feedback
- $I - D_1 D_2$ nonsingular when using positive feedback.

See Also

`series`	Series connection
`parallel`	Parallel connection
`connect`	Derive state-space model for block diagram interconnection

filt

Purpose

Specify discrete transfer functions in DSP format

Syntax

```
sys = filt(num,den)
sys = filt(num,den,Ts)
sys = filt(M)

sys = filt(num,den,'Property1',Value1,...,'PropertyN',ValueN)
sys = filt(num,den,Ts,'Property1',Value1,...,'PropertyN',ValueN)
```

Description

In digital signal processing (DSP), it is customary to write transfer functions as rational expressions in z^{-1} and to order the numerator and denominator terms in *ascending* powers of z^{-1}, for example,

$$H(z^{-1}) = \frac{2 + z^{-1}}{1 + 0.4z^{-1} + 2z^{-2}}$$

The function `filt` is provided to facilitate the specification of transfer functions in DSP format.

`sys = filt(num,den)` creates a discrete-time transfer function `sys` with numerator(s) `num` and denominator(s) `den`. The sample time is left unspecified (`sys.Ts = -1`) and the output `sys` is a TF object.

`sys = filt(num,den,Ts)` further specifies the sample time `Ts` (in seconds).

`sys = filt(M)` specifies a static filter with gain matrix `M`.

Any of the previous syntaxes can be followed by property name/property value pairs of the form

```
'Property',Value
```

Each pair specifies a particular LTI property of the model, for example, the input names or the transfer function variable. See LTI Properties and the `set` entry for additional information on LTI properties and admissible property values.

Arguments

For SISO transfer functions, `num` and `den` are row vectors containing the numerator and denominator coefficients ordered in ascending powers of z^{-1}. For example, `den = [1 0.4 2]` represents the polynomial $1 + 0.4z^{-1} + 2z^{-2}$.

MIMO transfer functions are regarded as arrays of SISO transfer functions (one per I/O channel), each of which is characterized by its numerator and denominator. The input arguments `num` and `den` are then cell arrays of row vectors such that:

- `num` and `den` have as many rows as outputs and as many columns as inputs.
- Their (i,j) entries `num{i,j}` and `den{i,j}` specify the numerator and denominator of the transfer function from input `j` to output `i`.

If all SISO entries have the same denominator, you can also set `den` to the row vector representation of this common denominator. See also MIMO Transfer Function Models for alternative ways to specify MIMO transfer functions.

Remark

`filt` behaves as `tf` with the `Variable` property set to `'z^-1'` or `'q'`. See `tf` entry below for details.

Example

Typing the commands

```
num = {1 , [1 0.3]}
den = {[1 1 2] ,[5 2]}
H = filt(num,den,'inputname',{'channel1' 'channel2'})
```

creates the two-input digital filter

$$H(z^{-1}) = \left[\begin{array}{cc} \dfrac{1}{1+z^{-1}+2z^{-2}} & \dfrac{1+0.3z^{-1}}{5+2z^{-1}} \end{array}\right]$$

with unspecified sample time and input names `'channel1'` and `'channel2'`.

See Also

`tf`	Create transfer functions
`zpk`	Create zero-pole-gain models
`ss`	Create state-space models

frd

Purpose

Create a frequency response data (FRD) object or convert another model type to an FRD model

Syntax

```
sys = frd(response,frequency)
sys = frd(response,frequency,Ts)
sys = frd
sys = frd(response,frequency,ltisys)

sysfrd = frd(sys,frequency)
sysfrd = frd(sys,frequency,'Units',units)
```

Description

`sys = frd(response,frequency)` creates an FRD model `sys` from the frequency response data stored in the multidimensional array `response`. The vector `frequency` represents the underlying frequencies for the frequency response data. See Table 16-16, Data Format for the Argument response in FRD Models.

`sys = frd(response,frequency,Ts)` creates a discrete-time FRD model `sys` with scalar sample time `Ts`. Set `Ts = -1` to create a discrete-time FRD model without specifying the sample time.

`sys = frd` creates an empty FRD model.

The input argument list for any of these syntaxes can be followed by property name/property value pairs of the form

```
'PropertyName',PropertyValue
```

You can use these extra arguments to set the various properties of FRD models (see the `set` command, or LTI Properties and Model-Specific Properties). These properties include `'Units'`. The default units for FRD models are in `'rad/s'`.

To force an FRD model `sys` to inherit all of its generic LTI properties from any existing LTI model `refsys`, use the syntax

```
sys = frd(response,frequency,ltisys)
```

`sysfrd = frd(sys,frequency)` converts a TF, SS, or ZPK model to an FRD model. The frequency response is computed at the frequencies provided by the vector `frequency`.

`sysfrd = frd(sys,frequency,'Units',units)` converts an FRD model from a TF, SS, or ZPK model while specifying the units for `frequency` to be `units` (`'rad/s'` or `'Hz'`).

Arguments

When you specify a SISO or MIMO FRD model, or an array of FRD models, the input argument `frequency` is always a vector of length `Nf`, where `Nf` is the number of frequency data points in the FRD. The specification of the input argument `response` is summarized in the following table.

Table 16-16: Data Format for the Argument response in FRD Models

Model Form	Response Data Format
SISO model	Vector of length `Nf` for which `response(i)` is the frequency response at the frequency `frequency(i)`
MIMO model with `Ny` outputs and `Nu` inputs	`Ny`-by-`Nu`-by-`Nf` multidimensional array for which `response(i,j,k)` specifies the frequency response from input `j` to output `i` at frequency `frequency(k)`
`S1`-by-...-by-`Sn` array of models with `Ny` outputs and `Nu` inputs	Multidimensional array of size `[Ny Nu S1 ... Sn]` for which `response(i,j,k,:)` specifies the array of frequency response data from input `j` to output `i` at frequency `frequency(k)`

Remarks

See Frequency Response Data (FRD) Models for more information on single FRD models, and Creating LTI Models for information on building arrays of FRD models.

Example

Type the commands

```
freq = logspace(1,2);
resp = .05*(freq).*exp(i*2*freq);
sys = frd(resp,freq)
```

to create a SISO FRD model.

See Also

`chgunits`	Change units for an FRD model
`frdata`	Quick access to data for an FRD model
`set`	Set the properties for an LTI model
`ss`	Create state-space models

`tf`	Create transfer functions
`zpk`	Create zero-pole-gain models

Purpose

Quick access to data for a frequency response data object

Syntax

```
[response,freq] = frdata(sys)
[response,freq,Ts] = frdata(sys)
[response,freq] = frdata(sys,'v')
```

Description

`[response,freq] = frdata(sys)` returns the response data and frequency samples of the FRD model `sys`. For an FRD model with `Ny` outputs and `Nu` inputs at `Nf` frequencies:

- `response` is an `Ny`-by-`Nu`-by-`Nf` multidimensional array where the `(i,j)` entry specifies the response from input `j` to output `i`.
- `freq` is a column vector of length `Nf` that contains the frequency samples of the FRD model.

See Table 11-14, “Data Format for the Argument response in FRD Models,” on page 80 for more information on the data format for FRD response data.

For SISO FRD models, the syntax

```
[response,freq] = frdata(sys,'v')
```

forces `frdata` to return the response data and frequencies directly as column vectors rather than as cell arrays (see example below).

`[response,freq,Ts] = frdata(sys)` also returns the sample time `Ts`.

Other properties of `sys` can be accessed with `get` or by direct structure-like referencing (e.g., `sys.Units`).

Arguments

The input argument `sys` to `frdata` must be an FRD model.

Example

Typing the commands

```
freq = logspace(1,2,2);
resp = .05*(freq).*exp(i*2*freq);
sys = frd(resp,freq);
[resp,freq] = frdata(sys,'v')
```

returns the FRD model data

```
resp =
   0.2040 + 0.4565i
```

```
      2.4359 - 4.3665i

   freq =
       10
      100
```

See Also

`frd`	Create or convert to FRD models
`get`	Get the properties for an LTI model
`set`	Set model properties

Purpose

Compute frequency response over grid of frequencies

Syntax

```
H = freqresp(sys,w)
```

Description

`H = freqresp(sys,w)` computes the frequency response of the LTI model sys at the real frequency points specified by the vector `w`. The frequencies must be in radians/sec. For single LTI Models, `freqresp(sys,w)` returns a 3-D array `H` with the frequency as the last dimension (see "Arguments" below). For LTI arrays of size `[Ny Nu S1 ... Sn]`, `freqresp(sys,w)` returns a `[Ny-by-Nu-by-S1-by-...-by-Sn]` length (w) array.

In continuous time, the response at a frequency ω is the transfer function value at $s = j\omega$. For state-space models, this value is given by

$$H(j\omega) = D + C(j\omega I - A)^{-1}B$$

In discrete time, the real frequencies `w(1)`,...,`w(N)` are mapped to points on the unit circle using the transformation $z = e^{j\omega T_s}$

where T_s is the sample time. The transfer function is then evaluated at the resulting z values. The default $T_s = 1$ is used for models with unspecified sample time.

Remark

If sys is an FRD model, `freqresp(sys,w)`, `w` can only include frequencies in `sys.frequency`. Interpolation and extrapolation are not supported. To interpolate an FRD model, use `interp`.

Arguments

The output argument `H` is a 3-D array with dimensions

(number of outputs) × (number of inputs) × (length of w)

For SISO systems, `H(1,1,k)` gives the scalar response at the frequency `w(k)`. For MIMO systems, the frequency response at `w(k)` is `H(:,:,k)`, a matrix with as many rows as outputs and as many columns as inputs.

Example

Compute the frequency response of

$$P(s) = \begin{bmatrix} 0 & \frac{1}{s+1} \\ \frac{s-1}{s+2} & 1 \end{bmatrix}$$

at the frequencies $\omega = 1, 10, 100$. Type

```
w = [1 10 100]
H = freqresp(P,w)
```

```
H(:,:,1) =

         0            0.5000- 0.5000i
  -0.2000+ 0.6000i    1.0000


H(:,:,2) =

         0            0.0099- 0.0990i
   0.9423+ 0.2885i    1.0000


H(:,:,3) =

         0            0.0001- 0.0100i
   0.9994+ 0.0300i    1.0000
```

The three displayed matrices are the values of $P(j\omega)$ for

$\omega = 1, \quad \omega = 10, \quad \omega = 100$

The third index in the 3-D array `H` is relative to the frequency vector `w`, so you can extract the frequency response at $\omega = 10$ rad/sec by

```
H(:,:,w==10)

ans =
         0            0.0099- 0.0990i
   0.9423+ 0.2885i    1.0000
```

Algorithm

For transfer functions or zero-pole-gain models, `freqresp` evaluates the numerator(s) and denominator(s) at the specified frequency points. For continuous-time state-space models (A, B, C, D), the frequency response is

$$D + C(j\omega - A)^{-1}B, \qquad \omega = \omega_1, ..., \omega_N$$

For efficiency, A is reduced to upper Hessenberg form and the linear equation $(j\omega - A)X = B$ is solved at each frequency point, taking advantage of the Hessenberg structure. The reduction to Hessenberg form provides a good compromise between efficiency and reliability. See [1] for more details on this technique.

Diagnostics

If the system has a pole on the $j\omega$ axis (or unit circle in the discrete-time case) and `w` happens to contain this frequency point, the gain is infinite, $j\omega I - A$ is singular, and `freqresp` produces the following warning message.

```
Singularity in freq. response due to jw-axis or unit circle pole.
```

See Also

`evalfr`	Response at single complex frequency
`bode`	Bode plot
`nyquist`	Nyquist plot
`nichols`	Nichols plot
`sigma`	Singular value plot
`ltiview`	LTI system viewer
`interp`	Interpolate FRD model between frequency points

References

[1] Laub, A.J., "Efficient Multivariable Frequency Response Computations," *IEEE Transactions on Automatic Control*, AC-26 (1981), pp. 407-408.

gensig

Purpose

Generate test input signals for `lsim`

Syntax

```
[u,t] = gensig(type,tau)
[u,t] = gensig(type,tau,Tf,Ts)
```

Description

`[u,t] = gensig(type,tau)` generates a scalar signal `u` of class `type` and with period `tau` (in seconds). The following types of signals are available.

`type = 'sin'`	Sine wave.
`type = 'square'`	Square wave.
`type = 'pulse'`	Periodic pulse.

`gensig` returns a vector `t` of time samples and the vector `u` of signal values at these samples. All generated signals have unit amplitude.

`[u,t] = gensig(type,tau,Tf,Ts)` also specifies the time duration `Tf` of the signal and the spacing `Ts` between the time samples `t`.

You can feed the outputs `u` and `t` directly to `lsim` and simulate the response of a single-input linear system to the specified signal. Since `t` is uniquely determined by `Tf` and `Ts`, you can also generate inputs for multi-input systems by repeated calls to `gensig`.

Example

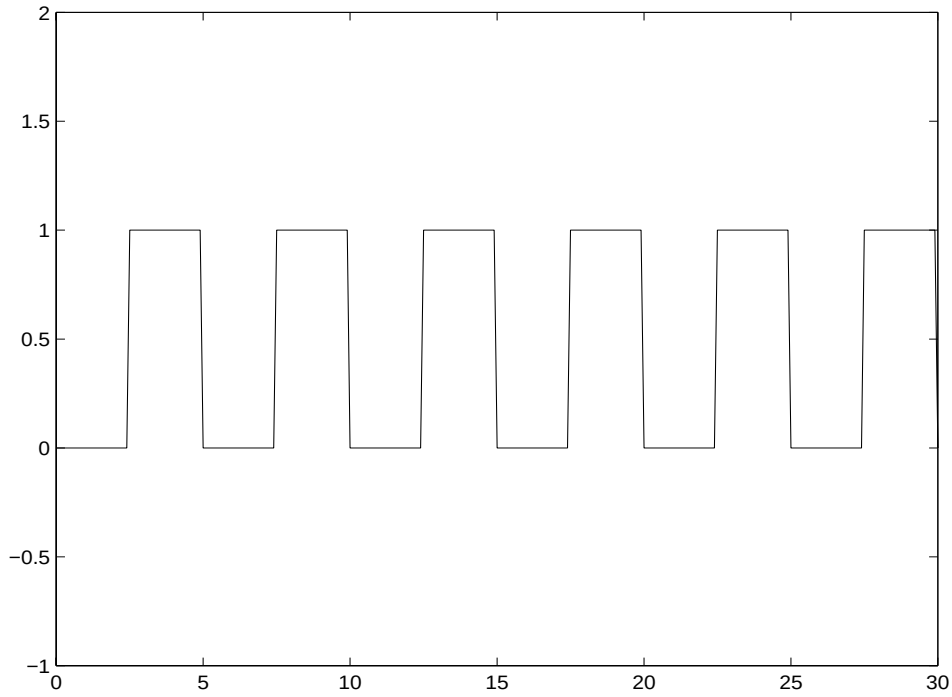
Generate a square wave with period 5 seconds, duration 30 seconds, and sampling every 0.1 seconds.

```
[u,t] = gensig('square',5,30,0.1)
```

Plot the resulting signal.

```
plot(t,u)
```

```
axis([0 30 -1 2])
```



See Also `lsim` Simulate response to arbitrary inputs

get

Purpose

Access/query LTI property values

Syntax

```
Value = get(sys,'PropertyName')
get(sys)
Struct = get(sys)
```

Description

`Value = get(sys,'PropertyName')` returns the current value of the property *PropertyName* of the LTI model sys. The string '*PropertyName*' can be the full property name (for example, `'UserData'`) or any unambiguous case-insensitive abbreviation (for example, `'user'`). You can specify any generic LTI property, or any property specific to the model sys (see "LTI Properties" for details on generic and model-specific LTI properties).

`Struct = get(sys)` converts the TF, SS, or ZPK object sys into a standard MATLAB structure with the property names as field names and the property values as field values.

Without left-hand argument,

```
get(sys)
```

displays all properties of sys and their values.

Example

Consider the discrete-time SISO transfer function defined by

```
h = tf(1,[1 2],0.1,'inputname','voltage','user','hello')
```

You can display all LTI properties of h with

```
get(h)
       num = {[0 1]}
       den = {[1 2]}
        Variable = 'z'
        Ts = 0.1
        InputDelay = 0
        OutputDelay = 0
        ioDelay = 0
        InputName = {'voltage'}
        OutputName = {''}
        InputGroup = {0x2 cell}
        OutputGroup = {0x2 cell}
```

```
          Notes = {}
          UserData = 'hello'
```

or query only about the numerator and sample time values by

```
get(h,'num')

ans =
    [1x2 double]
```

and

```
get(h,'ts')

ans =
    0.1000
```

Because the numerator data (`num` property) is always stored as a cell array, the first command evaluates to a cell array containing the row vector `[0 1]`.

Remark

An alternative to the syntax

```
Value = get(sys,'PropertyName')
```

is the structure-like referencing

```
Value = sys.PropertyName
```

For example,

```
sys.Ts
sys.a
sys.user
```

return the values of the sample time, A matrix, and `UserData` property of the (state-space) model `sys`.

See Also

`frdata`	Quick access to frequency response data
`set`	Set/modify LTI properties
`ssdata`	Quick access to state-space data
`tfdata`	Quick access to transfer function data
`zpkdata`	Quick access to zero-pole-gain data

Purpose

Compute controllability and observability state-space models:descriptor;state-space models:quick data retrievalgrammians

Syntax

```
Wc = gram(sys,'c')
Wo = gram(sys,'o')
```

Description

`gram` calculates controllability and observability grammians. You can use grammians to study the controllability and observability properties of state-space models and for model reduction [1,2]. They have better numerical properties than the controllability and observability matrices formed by `ctrb` and `obsv`.

Given the continuous-time state-space model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

the controllability grammian is defined by

$$W_c = \int_0^\infty e^{A\tau} B B^T e^{A^T\tau} d\tau$$

and the observability grammian by

$$W_o = \int_0^\infty e^{A^T\tau} C^T C e^{A\tau} d\tau$$

The discrete-time counterparts are

$$W_c = \sum_{k=0}^{\infty} A^k B B^T (A^T)^k, \qquad W_o = \sum_{k=0}^{\infty} (A^T)^k C^T C A^k$$

The controllability grammian is positive definite if and only if (A, B) is controllable. Similarly, the observability grammian is positive definite if and only if (C, A) is observable.

Use the commands

```
Wc = gram(sys,'c')   % controllability grammian
Wo = gram(sys,'o')   % observability grammian
```

to compute the grammians of a continuous or discrete system. The LTI model sys must be in state-space form.

Algorithm

The controllability grammian W_c is obtained by solving the continuous-time Lyapunov equation

$$AW_c + W_cA^T + BB^T = 0$$

or its discrete-time counterpart

$$AW_cA^T - W_c + BB^T = 0$$

Similarly, the observability grammian W_o solves the Lyapunov equation

$$A^TW_o + W_oA + C^TC = 0$$

in continuous time, and the Lyapunov equation

$$A^TW_oA - W_o + C^TC = 0$$

in discrete time.

Limitations

The A matrix must be stable (all eigenvalues have negative real part in continuous time, and magnitude strictly less than one in discrete time).

See Also

`balreal`	Grammian-based balancing of state-space realizations
`ctrb`	Controllability matrix
`lyap, dlyap`	Lyapunov equation solvers
`obsv`	Observability matrix

References

[1] Kailath, T., *Linear Systems*, Prentice-Hall, 1980.

hasdelay

Purpose

Test if an LTI model has time delays

Syntax

```
hasdelay(sys)
```

Description

`hasdelay(sys)` returns 1 (true) if the LTI model `sys` has input delays, output delays, or I/O delays, and 0 (false) otherwise.

See Also

`delay2z`	Changes transfer functions of discrete-time LTI models with delays to rational functions or absorbs FRD delays into the frequency response phase information
`totaldelay`	Combines delays for an LTI model

Purpose

Compute the impulse response of LTI models

Syntax

```
impulse(sys)
impulse(sys,t)

impulse(sys1,sys2,...,sysN)
impulse(sys1,sys2,...,sysN,t)
impulse(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[y,t,x] = impulse(sys)
```

Description

`impulse` calculates the unit impulse response of a linear system. The impulse response is the response to a Dirac input $\delta(t)$ for continuous-time systems and to a unit pulse at $t = 0$ for discrete-time systems. Zero initial state is assumed in the state-space case. When invoked without left-hand arguments, this function plots the impulse response on the screen.

`impulse(sys)` plots the impulse response of an arbitrary LTI model `sys`. This model can be continuous or discrete, and SISO or MIMO. The impulse response of multi-input systems is the collection of impulse responses for each input channel. The duration of simulation is determined automatically to display the transient behavior of the response.

`impulse(sys,t)` sets the simulation horizon explicitly. You can specify either a final time `t = Tfinal` (in seconds), or a vector of evenly spaced time samples of the form

```
t = 0:dt:Tfinal
```

For discrete systems, the spacing `dt` should match the sample period. For continuous systems, `dt` becomes the sample time of the discretized simulation model (see "Algorithm"), so make sure to choose `dt` small enough to capture transient phenomena.

To plot the impulse responses of several LTI models `sys1`,..., `sysN` on a single figure, use

```
impulse(sys1,sys2,...,sysN)
impulse(sys1,sys2,...,sysN,t)
```

As with bode or plot, you can specify a particular color, linestyle, and/or marker for each system, for example,

```
impulse(sys1,'y:',sys2,'g--')
```

See "Plotting and Comparing Multiple Systems" on and the bode entry in this chapter for more details.

When invoked with lefthand arguments,

```
[y,t] = impulse(sys)
[y,t,x] = impulse(sys)       % for state-space models only
y = impulse(sys,t)
```

return the output response y, the time vector t used for simulation, and the state trajectories x (for state-space models only). No plot is drawn on the screen. For single-input systems, y has as many rows as time samples (length of t), and as many columns as outputs. In the multi-input case, the impulse responses of each input channel are stacked up along the third dimension of y. The dimensions of y are then

$$(\text{length of } t) \times (\text{number of outputs}) \times (\text{number of inputs})$$

and y(:,:,j) gives the response to an impulse disturbance entering the jth input channel. Similarly, the dimensions of x are

$$(\text{length of } t) \times (\text{number of states}) \times (\text{number of inputs})$$

Example

To plot the impulse response of the second-order state-space model

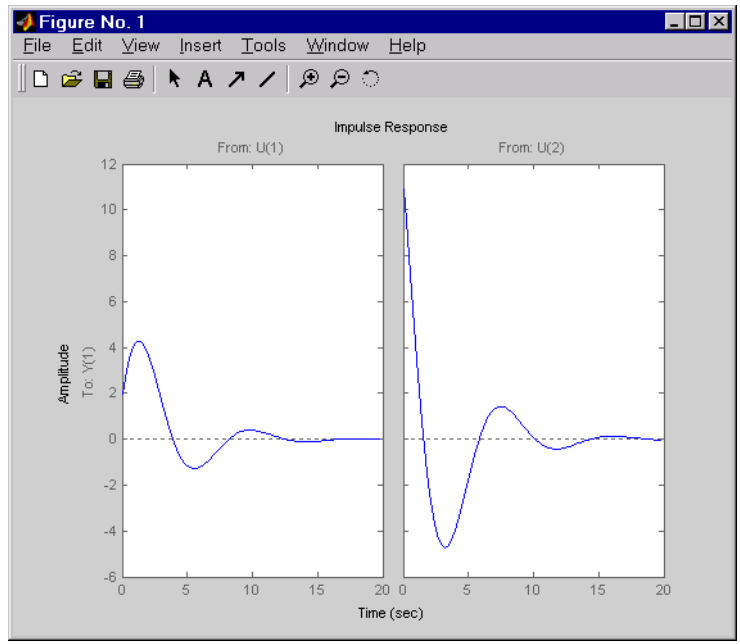
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.5572 & -0.7814 \\ 0.7814 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1.9691 & 6.4493 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

use the following commands.

```
a = [-0.5572 -0.7814;0.7814  0];
b = [1 -1;0 2];
c = [1.9691  6.4493];
```

```
sys = ss(a,b,c,0);
impulse(sys)
```



The left plot shows the impulse response of the first input channel, and the right plot shows the impulse response of the second input channel.

You can store the impulse response data in MATLAB arrays by

```
[y,t] = impulse(sys)
```

Because this system has two inputs, y is a 3-D array with dimensions

```
size(y)

ans =
   101     1     2
```

(the first dimension is the length of t). The impulse response of the first input channel is then accessed by

```
y(:,:,1)
```

Algorithm

Continuous-time models are first converted to state space. The impulse response of a single-input state-space model

$$\dot{x} = Ax + bu$$
$$y = Cx$$

is equivalent to the following unforced response with initial state b.

$$\dot{x} = Ax, \qquad x(0) = b$$
$$y = Cx$$

To simulate this response, the system is discretized using zero-order hold on the inputs. The sampling period is chosen automatically based on the system dynamics, except when a time vector `t = 0:dt:Tf` is supplied (`dt` is then used as sampling period).

Limitations

The impulse response of a continuous system with nonzero D matrix is infinite at $t = 0$. `impulse` ignores this discontinuity and returns the lower continuity value Cb at $t = 0$.

See Also

`ltiview`	LTI system viewer
`step`	Step response
`initial`	Free response to initial condition
`lsim`	Simulate response to arbitrary inputs

Purpose

Compute the initial condition response of state-space models

Syntax

```
initial(sys,x0)
initial(sys,x0,t)

initial(sys1,sys2,...,sysN,x0)
initial(sys1,sys2,...,sysN,x0,t)
initial(sys1,'PlotStyle1',...,sysN,'PlotStyleN',x0)

[y,t,x] = initial(sys,x0)
```

Description

`initial` calculates the unforced response of a state-space model with an initial condition on the states.

$$\dot{x} = Ax\,, \qquad x(0) = x_0$$
$$y = Cx$$

This function is applicable to either continuous- or discrete-time models. When invoked without lefthand arguments, `initial` plots the initial condition response on the screen.

`initial(sys,x0)` plots the response of `sys` to an initial condition `x0` on the states. `sys` can be any *state-space* model (continuous or discrete, SISO or MIMO, with or without inputs). The duration of simulation is determined automatically to reflect adequately the response transients.

`initial(sys,x0,t)` explicitly sets the simulation horizon. You can specify either a final time `t = Tfinal` (in seconds), or a vector of evenly spaced time samples of the form

```
t = 0:dt:Tfinal
```

For discrete systems, the spacing `dt` should match the sample period. For continuous systems, `dt` becomes the sample time of the discretized simulation model (see `impulse`), so make sure to choose `dt` small enough to capture transient phenomena.

To plot the initial condition responses of several LTI models on a single figure, use

```
initial(sys1,sys2,...,sysN,x0)
initial(sys1,sys2,...,sysN,x0,t)
```

(see `impulse` for details).

When invoked with lefthand arguments,

```
[y,t,x] = initial(sys,x0)
[y,t,x] = initial(sys,x0,t)
```

return the output response `y`, the time vector `t` used for simulation, and the state trajectories `x`. No plot is drawn on the screen. The array `y` has as many rows as time samples (length of `t`) and as many columns as outputs. Similarly, `x` has `length(t)` rows and as many columns as states.

Example

Plot the response of the state-space model

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-0.5572 & -0.7814\\ 0.7814 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

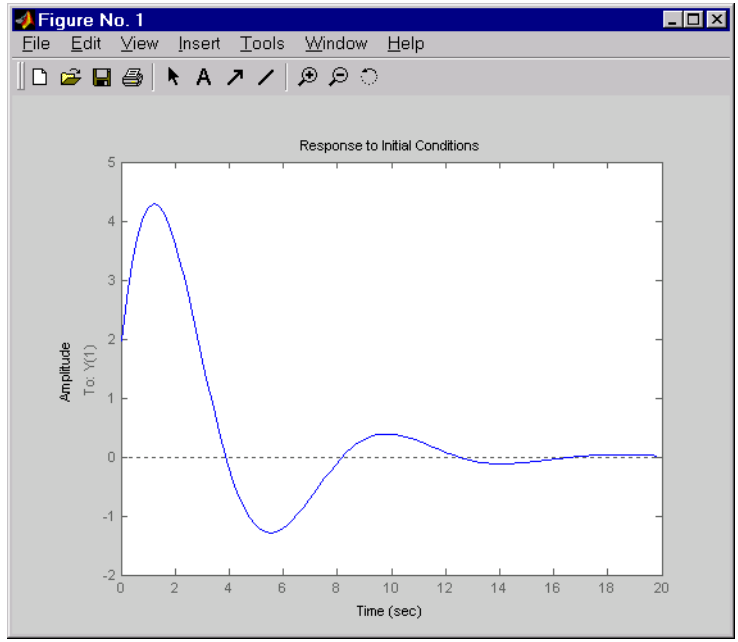
$$y = \begin{bmatrix}1.9691 & 6.4493\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

to the initial condition

$$x(0) = \begin{bmatrix}1\\ 0\end{bmatrix}$$

```
a = [-0.5572   -0.7814;0.7814  0];
c = [1.9691  6.4493];
x0 = [1 ; 0]

sys = ss(a,[],c,[]);
```

```
initial(sys,x0)
```



See Also

`impulse`	Impulse response
`lsim`	Simulate response to arbitrary inputs
`ltiview`	LTI system viewer
`step`	Step response

interp

Purpose

Interpolate an FRD model between frequency points

Syntax

`isys = interp(sys,freqs)` interpolates the frequency response data contained in the FRD model `sys` at the frequencies `freqs`. `interp`, which is an overloaded version of the MATLAB function `interp`, uses linear interpolation and returns an FRD model `isys` containing the interpolated data at the new frequencies `freqs`.

You should express the frequency values `freqs` in the same units as `sys.frequency`. The frequency values must lie between the smallest and largest frequency points in `sys` (extrapolation is not supported).

freqresp	Frequency response of LTI models
ltimodels	Help on LTI models

Purpose

Invert LTI systems

Syntax

```
isys = inv(sys)
```

Description

inv inverts the input/output relation

$$y = G(s)u$$

to produce the LTI system with the transfer matrix $H(s) = G(s)^{-1}$.

$$u = H(s)y$$

This operation is defined only for square systems (same number of inputs and outputs) with an invertible feedthrough matrix D. inv handles both continuous- and discrete-time systems.

Example

Consider

$$H(s) = \begin{bmatrix} 1 & \frac{1}{s+1} \\ 0 & 1 \end{bmatrix}$$

At the MATLAB prompt, type

```
H = [1 tf(1,[1 1]);0 1]
Hi = inv(H)
```

to invert it. MATLAB returns

```
Transfer function from input 1 to output...
 #1:  1

 #2:  0

Transfer function from input 2 to output...
       -1
 #1:  -----
      s + 1

 #2:  1
```

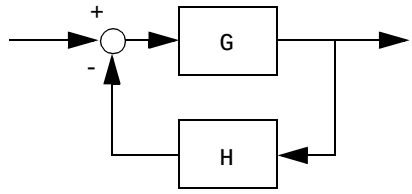
You can verify that

```
H * Hi
```

is the identity transfer function (static gain `I`).

Limitations

Do not use `inv` to model feedback connections such as



While it seems reasonable to evaluate the corresponding closed-loop transfer function $(I + GH)^{-1}G$ as

```
inv(1+g*h) * g
```

this typically leads to nonminimal closed-loop models. For example,

```
g = zpk([],1,1)
h = tf([2 1],[1 0])
cloop = inv(1+g*h) * g
```

yields a third-order closed-loop model with an unstable pole-zero cancellation at `s = 1`.

```
cloop

Zero/pole/gain:
      s (s-1)
-------------------
(s-1) (s^2 + s + 1)
```

Use `feedback` to avoid such pitfalls.

```
cloop = feedback(g,h)

Zero/pole/gain:
      s
-------------
(s^2 + s + 1)
```

Purpose

Determine whether an LTI model is continuous or discrete

Syntax

```
boo = isct(sys)
boo = isdt(sys)
```

Description

`boo = isct(sys)` returns `1` (true) if the LTI model `sys` is continuous and `0` (false) otherwise. `sys` is continuous if its sample time is zero, that is, `sys.Ts=0`.

`boo = isdt(sys)` returns `1` (true) if `sys` is discrete and `0` (false) otherwise. Discrete-time LTI models have a nonzero sample time, except for empty models and static gains, which are regarded as either continuous or discrete as long as their sample time is not explicitly set to a nonzero value. Thus both

```
isct(tf(10))
isdt(tf(10))
```

are true. However, if you explicitly label a gain as discrete, for example, by typing

```
g = tf(10,'ts',0.01)
```

`isct(g)` now returns false and only `isdt(g)` is true.

See Also

`isa`	Determine LTI model type
`isempty`	True for empty LTI models
`isproper`	True for proper LTI models

isempty

Purpose

Test if an LTI model is empty

Syntax

```
boo = isempty(sys)
```

Description

`isempty(sys)` returns `1` (true) if the LTI model sys has no input or no output, and `0` (false) otherwise.

Example

Both commands

```
isempty(tf)  % tf by itself returns an empty transfer function
isempty(ss(1,2,[],[]))
```

return `1` (true) while

```
isempty(ss(1,2,3,4))
```

returns `0` (false).

See Also

`issiso`	True for SISO systems
`size`	I/O dimensions and array dimensions of LTI models

Purpose

Test if an LTI model is proper

Syntax

```
boo = isproper(sys)
```

Description

`isproper(sys)` returns `1` (true) if the LTI model `sys` is proper and `0` (false) otherwise.

State-space models are always proper. SISO transfer functions or zero-pole-gain models are proper if the degree of their numerator is less than or equal to the degree of their denominator. MIMO transfer functions are proper if all their SISO entries are proper.

Example

The following commands

```
isproper(tf([1 0],1))        % transfer function s
isproper(tf([1 0],[1 1]))    % transfer function s/(s+1)
```

return false and true, respectively.

issiso

Purpose

Test if an LTI model is single-input/single-output (SISO)

Syntax

```
boo = issiso(sys)
```

Description

`issiso(sys)` returns `1` (true) if the LTI model `sys` is SISO and `0` (false) otherwise.

See Also

`isempty`	True for empty LTI models
`size`	I/O dimensions and array dimensions of LTI models

Purpose

Design continuous- or discrete-time Kalman estimator

Syntax

```
[kest,L,P] = kalman(sys,Qn,Rn,Nn)
[kest,L,P,M,Z] = kalman(sys,Qn,Rn,Nn)   % discrete time only
[kest,L,P] = kalman(sys,Qn,Rn,Nn,sensors,known)
```

Description

`kalman` designs a Kalman state estimator given a state-space model of the plant and the process and measurement noise covariance data. The Kalman estimator is the optimal solution to the following continuous or discrete estimation problems.

Continuous-Time Estimation

Given the continuous plant

$$\dot{x} = Ax + Bu + Gw \qquad \text{(state equation)}$$
$$y_v = Cx + Du + Hw + v \qquad \text{(measurement equation)}$$

with known inputs u and process and measurement white noise w, v satisfying

$$E(w) = E(v) = 0, \quad E(ww^T) = Q, \quad E(vv^T) = R, \quad E(wv^T) = N$$

construct a state estimate $\hat{x}(t)$ that minimizes the steady-state error covariance

$$P = \lim_{t \to \infty} E(\{x - \hat{x}\}\{x - \hat{x}\}^T)$$

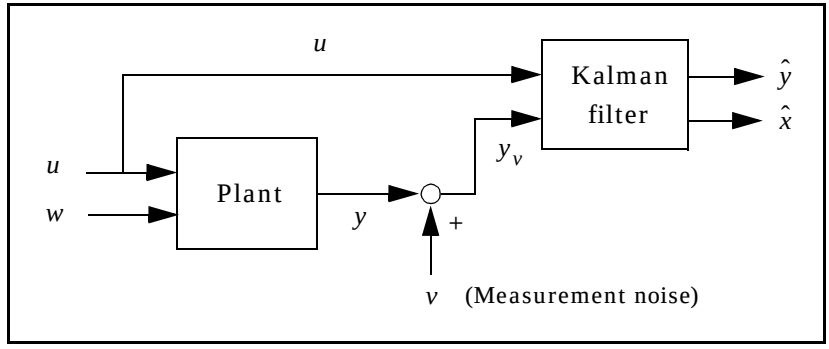
The optimal solution is the Kalman filter with equations

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y_v - C\hat{x} - Du)$$
$$\begin{bmatrix} \hat{y} \\ \hat{x} \end{bmatrix} = \begin{bmatrix} C \\ I \end{bmatrix} \hat{x} + \begin{bmatrix} D \\ 0 \end{bmatrix} u$$

where the filter gain L is determined by solving an algebraic Riccati equation. This estimator uses the known inputs u and the measurements y_v to generate

the output and state estimates y and x. Note that y estimates the true plant output

$$y = Cx + Du + Hw$$



Kalman estimator

Discrete-Time Estimation

Given the discrete plant

$$x[n+1] = Ax[n] + Bu[n] + Gw[n]$$
$$y_v[n] = Cx[n] + Du[n] + Hw[n] + v[n]$$

and the noise covariance data

$$E(w[n]w[n]^T) = Q, \quad E(v[n]v[n]^T) = R, \quad E(w[n]v[n]^T) = N$$

the Kalman estimator has equations

$$\hat{x}[n+1|n] = A\hat{x}[n|n-1] + Bu[n] + L(y_v[n] - C\hat{x}[n|n-1] - Du[n])$$

$$\begin{bmatrix}\hat{y}[n|n]\\ \hat{x}[n|n]\end{bmatrix} = \begin{bmatrix}C(I-MC)\\ I-MC\end{bmatrix}\hat{x}[n|n-1] + \begin{bmatrix}(I-CM)D & CM\\ -MD & M\end{bmatrix}\begin{bmatrix}u[n]\\ y_v[n]\end{bmatrix}$$

and generates optimal "current" output and state estimates $y[n|n]$ and $x[n|n]$ using all available measurements including $y_v[n]$. The gain matrices L and M are derived by solving a discrete Riccati equation. The *innovation gain* M is used to update the prediction $\hat{x}[n|n-1]$ using the new measurement $y_v[n]$.

$$\hat{x}[n|n] = \hat{x}[n|n-1] + M(\underbrace{y_v[n] - C\hat{x}[n|n-1] - Du[n]}_{\text{innovation}})$$

Usage

`[kest,L,P] = kalman(sys,Qn,Rn,Nn)` returns a state-space model `kest` of the Kalman estimator given the plant model `sys` and the noise covariance data `Qn`, `Rn`, `Nn` (matrices Q, R, N above). `sys` must be a state-space model with matrices

$$A, \begin{bmatrix} B & G \end{bmatrix}, C, \begin{bmatrix} D & H \end{bmatrix}$$

The resulting estimator `kest` has $[u\ ;\ y_v]$ as inputs and $[\hat{y}\ ;\ \hat{x}]$ (or their discrete-time counterparts) as outputs. You can omit the last input argument `Nn` when $N = 0$.

The function `kalman` handles both continuous and discrete problems and produces a continuous estimator when `sys` is continuous, and a discrete estimator otherwise. In continuous time, `kalman` also returns the Kalman gain `L` and the steady-state error covariance matrix `P`. Note that `P` is the solution of the associated Riccati equation. In discrete time, the syntax

```
[kest,L,P,M,Z] = kalman(sys,Qn,Rn,Nn)
```

returns the filter gain L and innovations gain M, as well as the steady-state error covariances

$$P = \lim_{n\to\infty} E(e[n|n-1]e[n|n-1]^T), \qquad e[n|n-1] = x[n] - x[n|n-1]$$

$$Z = \lim_{n\to\infty} E(e[n|n]e[n|n]^T), \qquad e[n|n] = x[n] - x[n|n]$$

Finally, use the syntaxes

```
[kest,L,P] = kalman(sys,Qn,Rn,Nn,sensors,known)
[kest,L,P,M,Z] = kalman(sys,Qn,Rn,Nn,sensors,known)
```

for more general plants sys where the known inputs u and stochastic inputs w are mixed together, and not all outputs are measured. The index vectors sensors and known then specify which outputs y of sys are measured and which inputs u are known. All other inputs are assumed stochastic.

Example

See LQG Design for the x-Axis and Kalman Filtering for examples that use the kalman function.

Limitations

The plant and noise data must satisfy:

- (C, A) detectable
- $\overline{R} > 0$ and $\overline{Q} - \overline{N}\overline{R}^{-1}\overline{N}^T \geq 0$
- $(A - \overline{N}\overline{R}^{-1}C, \overline{Q} - \overline{N}\overline{R}^{-1}\overline{N}^T)$ has no uncontrollable mode on the imaginary axis (or unit circle in discrete time)

with the notation

$$\overline{Q} = GQG^T$$
$$\overline{R} = R + HN + N^T H^T + HQH^T$$
$$\overline{N} = G(QH^T + N)$$

See Also

care	Solve continuous-time Riccati equations
dare	Solve discrete-time Riccati equations
estim	Form estimator given estimator gain
kalmd	Discrete Kalman estimator for continuous plant
lqgreg	Assemble LQG regulator
lqr	Design state-feedback LQ regulator

References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.

Purpose

Design discrete Kalman estimator for continuous plant

Syntax

```
[kest,L,P,M,Z] = kalmd(sys,Qn,Rn,Ts)
```

Description

`kalmd` designs a discrete-time Kalman estimator that has response characteristics similar to a continuous-time estimator designed with `kalman`. This command is useful to derive a discrete estimator for digital implementation after a satisfactory continuous estimator has been designed.

`[kest,L,P,M,Z] = kalmd(sys,Qn,Rn,Ts)` produces a discrete Kalman estimator `kest` with sample time `Ts` for the continuous-time plant

$$\dot{x} = Ax + Bu + Gw \qquad \text{(state equation)}$$
$$y_v = Cx + Du + v \qquad \text{(measurement equation)}$$

with process noise w and measurement noise v satisfying

$$E(w) = E(v) = 0\,, \quad E(ww^T) = Q_n\,, \quad E(vv^T) = R_n\,, \quad E(wv^T) = 0$$

The estimator `kest` is derived as follows. The continuous plant `sys` is first discretized using zero-order hold with sample time `Ts` (see `c2d` entry), and the continuous noise covariance matrices Q_n and R_n are replaced by their discrete equivalents

$$Q_d = \int_0^{T_s} e^{A\tau} G Q G^T e^{A^T\tau} d\tau$$
$$R_d = R / T_s$$

The integral is computed using the matrix exponential formulas in [2]. A discrete-time estimator is then designed for the discretized plant and noise. See `kalman` for details on discrete-time Kalman estimation.

`kalmd` also returns the estimator gains `L` and `M`, and the discrete error covariance matrices `P` and `Z` (see `kalman` for details).

Limitations

The discretized problem data should satisfy the requirements for `kalman`.

See Also

`kalman` Design Kalman estimator

`lqgreg`	Assemble LQG regulator
`lqrd`	Discrete LQ-optimal gain for continuous plant

References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.

[2] Van Loan, C.F., "Computing Integrals Involving the Matrix Exponential," *IEEE Trans. Automatic Control*, AC-15, October 1970.

Purpose

Redheffer star product (linear fractional transformation) of two LTI models

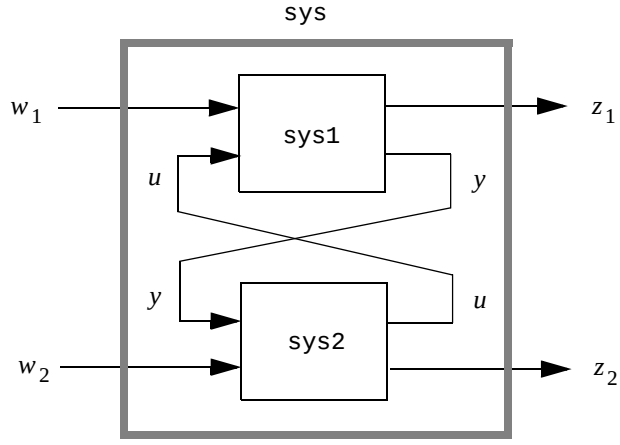
Syntax

```
sys = lft(sys1,sys2)
sys = lft(sys1,sys2,nu,ny)
```

Description

`lft` forms the star product or linear fractional transformation (LFT) of two LTI models or LTI arrays. Such interconnections are widely used in robust control techniques.

`sys = lft(sys1,sys2,nu,ny)` forms the star product `sys` of the two LTI models (or LTI arrays) `sys1` and `sys2`. The star product amounts to the following feedback connection for single LTI models (or for each model in an LTI array).



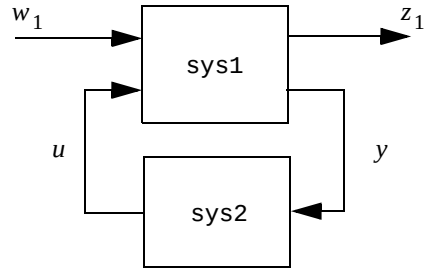
This feedback loop connects the first `nu` outputs of `sys2` to the last `nu` inputs of `sys1` (signals u), and the last `ny` outputs of `sys1` to the first `ny` inputs of `sys2` (signals y). The resulting system `sys` maps the input vector $[w_1 ; w_2]$ to the output vector $[z_1 ; z_2]$.

The abbreviated syntax

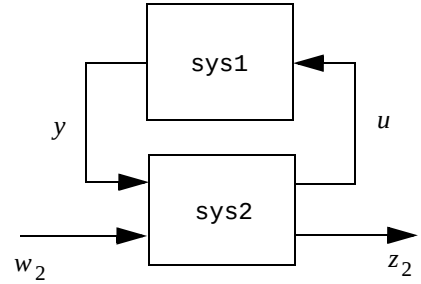
```
sys = lft(sys1,sys2)
```

produces:

- The lower LFT of `sys1` and `sys2` if `sys2` has fewer inputs and outputs than `sys1`. This amounts to deleting w_2 and z_2 in the above diagram.
- The upper LFT of `sys1` and `sys2` if `sys1` has fewer inputs and outputs than `sys2`. This amounts to deleting w_1 and z_1 in the above diagram.



Lower LFT connection



Upper LFT connection

Algorithm

The closed-loop model is derived by elementary state-space manipulations.

Limitations

There should be no algebraic loop in the feedback connection.

See Also

`connect`	Derive state-space model for block diagram interconnection
`feedback`	Feedback connection

Purpose

Form LQG regulator given state-feedback gain and Kalman estimator

Syntax

```
rlqg = lqgreg(kest,k)
rlqg = lqgreg(kest,k,'current')   % discrete-time only

rlqg = lqgreg(kest,k,controls)
```

Description

`lqgreg` forms the LQG regulator by connecting the Kalman estimator designed with `kalman` and the optimal state-feedback gain designed with `lqr`, `dlqr`, or `lqry`. The LQG regulator minimizes some quadratic cost function that trades off regulation performance and control effort. This regulator is dynamic and relies on noisy output measurements to generate the regulating commands.

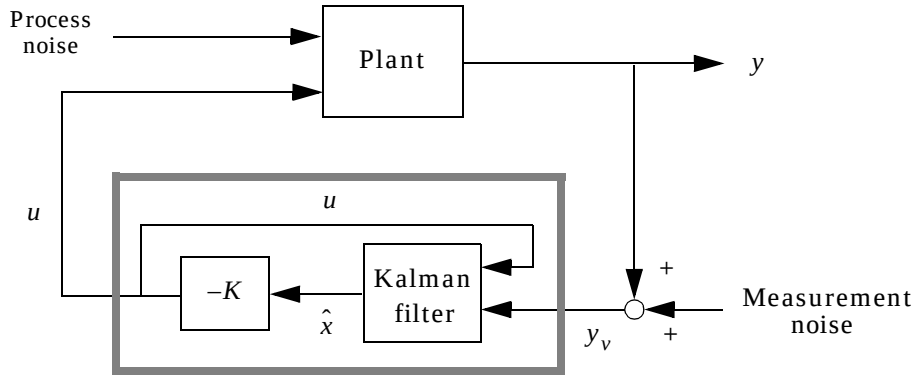
In continuous time, the LQG regulator generates the commands

$$u = -K\hat{x}$$

where $\hat{x}$ is the Kalman state estimate. The regulator state-space equations are

$$\dot{\hat{x}} = \left[A - LC - (B - LD)K\right]\hat{x} + Ly_v$$
$$u = -K\hat{x}$$

where y_v is the vector of plant output measurements (see `kalman` for background and notation). The diagram below shows this dynamic regulator in relation to the plant.



LQG regulator

In discrete time, you can form the LQG regulator using either the prediction $\hat{x}[n|n-1]$ of $x[n]$ based on measurements up to $y_v[n-1]$, or the current state estimate $\hat{x}[n|n]$ based on all available measurements including $y_v[n]$. While the regulator

$$u[n] = -K\hat{x}[n|n-1]$$

is always well-defined, the *current regulator*

$$u[n] = -K\hat{x}[n|n]$$

is causal only when $I - KMD$ is invertible (see `kalman` for the notation). In addition, practical implementations of the current regulator should allow for the processing time required to compute $u[n]$ once the measurements $y_v[n]$ become available (this amounts to a time delay in the feedback loop).

Usage

`rlqg = lqgreg(kest,k)` returns the LQG regulator `rlqg` (a state-space model) given the Kalman estimator `kest` and the state-feedback gain matrix `k`. The same function handles both continuous- and discrete-time cases. Use consistent tools to design `kest` and `k`:

- Continuous regulator for continuous plant: use `lqr` or `lqry` and `kalman`.
- Discrete regulator for discrete plant: use `dlqr` or `lqry` and `kalman`.

• Discrete regulator for continuous plant: use `lqrd` and `kalmd`.

In discrete time, `lqgreg` produces the regulator

$$u[n] = -K\hat{x}[n|n-1]$$

by default (see "Description"). To form the "current" LQG regulator instead, use

$$u[n] = -K\hat{x}[n|n]$$

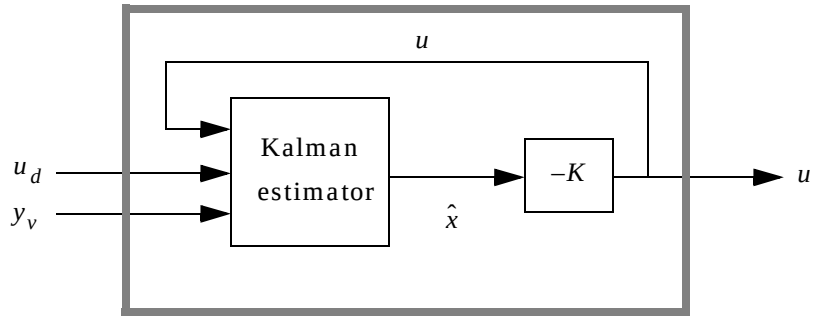
the syntax

```
rlqg = lqgreg(kest,k,'current')
```

This syntax is meaningful only for discrete-time problems.

`rlqg = lqgreg(kest,k,controls)` handles estimators that have access to additional known plant inputs u_d. The index vector `controls` then specifies which estimator inputs are the controls u, and the resulting LQG regulator `rlqg` has u_d and y_v as inputs (see figure below).

Note: Always use *positive* feedback to connect the LQG regulator to the plant.



LQG regulator

Example

See the example LQG Regulation.

See Also

`kalman`	Kalman estimator design
`kalmd`	Discrete Kalman estimator for continuous plant
`lqr, dlqr`	State-feedback LQ regulator
`lqrd`	Discrete LQ regulator for continuous plant
`lqry`	LQ regulator with output weighting
`reg`	Form regulator given state-feedback and estimator gains

Purpose

Design linear-quadratic (LQ) state-feedback regulator for continuous plant

Syntax

```
[K,S,e] = lqr(A,B,Q,R)
[K,S,e] = lqr(A,B,Q,R,N)
```

Description

`[K,S,e] = lqr(A,B,Q,R,N)` calculates the optimal gain matrix `K` such that the state-feedback law $u = -Kx$

minimizes the quadratic cost function

$$J(u) = \int_0^\infty (x^T Q x + u^T R u + 2x^T N u)\,dt$$

for the continuous-time state-space model $\dot{x} = Ax + Bu$

The default value `N=0` is assumed when `N` is omitted.

In addition to the state-feedback gain `K`, `lqr` returns the solution `S` of the associated Riccati equation

$$A^T S + SA - (SB + N)R^{-1}(B^T S + N^T) + Q = 0$$

and the closed-loop eigenvalues `e = eig(A-B*K)`. Note that K is derived from S by

$$K = R^{-1}(B^T S + N^T)$$

Limitations

The problem data must satisfy:

- The pair (A, B) is stabilizable.
- $R > 0$ and $Q - NR^{-1}N^T \geq 0$.
- $(Q - NR^{-1}N^T, A - BR^{-1}N^T)$ has no unobservable mode on the imaginary axis.

See Also

`care`	Solve continuous Riccati equations
`dlqr`	State-feedback LQ regulator for discrete plant
`lqgreg`	Form LQG regulator
`lqrd`	Discrete LQ regulator for continuous plant
`lqry`	State-feedback LQ regulator with output weighting

lqrd

Purpose

Design discrete LQ regulator for continuous plant

Syntax

```
[Kd,S,e] = lqrd(A,B,Q,R,Ts)
[Kd,S,e] = lqrd(A,B,Q,R,N,Ts)
```

Description

`lqrd` designs a discrete full-state-feedback regulator that has response characteristics similar to a continuous state-feedback regulator designed using `lqr`. This command is useful to design a gain matrix for digital implementation after a satisfactory continuous state-feedback gain has been designed.

`[Kd,S,e] = lqrd(A,B,Q,R,Ts)` calculates the discrete state-feedback law

$$u[n] = -K_d x[n]$$

that minimizes a discrete cost function equivalent to the continuous cost function

$$J = \int_0^\infty (x^T Q x + u^T R u)\,dt$$

The matrices `A` and `B` specify the continuous plant dynamics

$$\dot{x} = Ax + Bu$$

and `Ts` specifies the sample time of the discrete regulator. Also returned are the solution `S` of the discrete Riccati equation for the discretized problem and the discrete closed-loop eigenvalues `e = eig(Ad-Bd*Kd)`.

`[Kd,S,e] = lqrd(A,B,Q,R,N,Ts)` solves the more general problem with a cross-coupling term in the cost function.

$$J = \int_0^\infty (x^T Q x + u^T R u + 2x^T N u)\,dt$$

Algorithm

The equivalent discrete gain matrix `Kd` is determined by discretizing the continuous plant and weighting matrices using the sample time `Ts` and the zero-order hold approximation.

With the notation

$$\Phi(\tau) = e^{A\tau}, \qquad A_d = \Phi(T_s)$$

$$\Gamma(\tau) = \int_0^{\tau} e^{A\eta} B\, d\eta, \qquad B_d = \Gamma(T_s)$$

the discretized plant has equations

$$x[n+1] = A_d x[n] + B_d u[n]$$

and the weighting matrices for the equivalent discrete cost function are

$$\begin{bmatrix} Q_d & N_d \\ N_d^T & R_d \end{bmatrix} = \int_0^{T_s} \begin{bmatrix} \Phi^T(\tau) & 0 \\ \Gamma^T(\tau) & I \end{bmatrix} \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{bmatrix} \Phi(\tau) & \Gamma(\tau) \\ 0 & I \end{bmatrix} d\tau$$

The integrals are computed using matrix exponential formulas due to Van Loan (see [2]). The plant is discretized using `c2d` and the gain matrix is computed from the discretized data using `dlqr`.

Limitations

The discretized problem data should meet the requirements for `dlqr`.

See Also

`c2d`	Discretization of LTI model
`dlqr`	State-feedback LQ regulator for discrete plant
`kalmd`	Discrete Kalman estimator for continuous plant
`lqr`	State-feedback LQ regulator for continuous plant

References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1980, pp. 439–440

[2] Van Loan, C.F., "Computing Integrals Involving the Matrix Exponential," *IEEE Trans. Automatic Control*, AC-15, October 1970.

lqry

Purpose

Linear-quadratic (LQ) state-feedback regulator with output weighting

Syntax

```
[K,S,e] = lqry(sys,Q,R)
[K,S,e] = lqry(sys,Q,R,N)
```

Description

Given the plant

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

or its discrete-time counterpart, `lqry` designs a state-feedback control

$$u = -Kx$$

that minimizes the quadratic cost function with output weighting

$$J(u) = \int_0^\infty (y^T Q y + u^T R u + 2y^T N u)\,dt$$

(or its discrete-time counterpart). The function `lqry` is equivalent to `lqr` or `dlqr` with weighting matrices:

$$\begin{bmatrix} \overline{Q} & \overline{N} \\ \overline{N}^T & \overline{R} \end{bmatrix} = \begin{bmatrix} C^T & 0 \\ D^T & I \end{bmatrix} \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}$$

`[K,S,e] = lqry(sys,Q,R,N)` returns the optimal gain matrix `K`, the Riccati solution `S`, and the closed-loop eigenvalues `e = eig(A-B*K)`. The state-space model `sys` specifies the continuous- or discrete-time plant data (A, B, C, D). The default value `N=0` is assumed when `N` is omitted.

Example

See LQG Design for the x-Axis for an example.

Limitations

The data $A, B, \overline{Q}, \overline{R}, \overline{N}$ must satisfy the requirements for `lqr` or `dlqr`.

See Also

`lqr`	State-feedback LQ regulator for continuous plant
`dlqr`	State-feedback LQ regulator for discrete plant
`kalman`	Kalman estimator design
`lqgreg`	Form LQG regulator

Purpose

Simulate LTI model response to arbitrary inputs

Syntax

```
lsim(sys,u,t)
lsim(sys,u,t,x0)
lsim(sys,u,t,x0,'zoh')
lsim(sys,u,t,x0,'foh')

lsim(sys1,sys2,...,sysN,u,t)
lsim(sys1,sys2,...,sysN,u,t,x0)
lsim(sys1,'PlotStyle1',...,sysN,'PlotStyleN',u,t)

[y,t,x] = lsim(sys,u,t,x0)
```

Description

`lsim` simulates the (time) response of continuous or discrete linear systems to arbitrary inputs. When invoked without left-hand arguments, `lsim` plots the response on the screen.

`lsim(sys,u,t)` produces a plot of the time response of the LTI model `sys` to the input time history `t,u`. The vector `t` specifies the time samples for the simulation and consists of regularly spaced time samples.

```
t = 0:dt:Tfinal
```

The matrix `u` must have as many rows as time samples (`length(t)`) and as many columns as system inputs. Each row `u(i,:)` specifies the input value(s) at the time sample `t(i)`.

The LTI model `sys` can be continuous or discrete, SISO or MIMO. In discrete time, `u` must be sampled at the same rate as the system (`t` is then redundant and can be omitted or set to the empty matrix). In continuous time, the time sampling `dt=t(2)-t(1)` is used to discretize the continuous model. If `dt` is too large (undersampling), `lsim` issues a warning suggesting that you use a more appropriate sample time, but will use the specified sample time. See Algorithm on page 116 for a discussion of sample times.

`lsim(sys,u,t,x0)` further specifies an initial condition `x0` for the system states. This syntax applies only to state-space models.

`lsim(sys,u,t,x0,'zoh')` or `lsim(sys,u,t,x0,'foh')` explicitly specifies how the input values should be interpolated between samples (zero-order hold or

linear interpolation). By default, `lsim` selects the interpolation method automatically based on the smoothness of the signal U.

Finally,

```
lsim(sys1,sys2,...,sysN,u,t)
```

simulates the responses of several LTI models to the same input history `t,u` and plots these responses on a single figure. As with `bode` or `plot`, you can specify a particular color, linestyle, and/or marker for each system, for example,

```
lsim(sys1,'y:',sys2,'g--',u,t,x0)
```

The multisystem behavior is similar to that of `bode` or `step`.

When invoked with left-hand arguments,

```
[y,t] = lsim(sys,u,t)
[y,t,x] = lsim(sys,u,t)       % for state-space models only
[y,t,x] = lsim(sys,u,t,x0)    % with initial state
```

return the output response `y`, the time vector `t` used for simulation, and the state trajectories `x` (for state-space models only). No plot is drawn on the screen. The matrix `y` has as many rows as time samples (`length(t)`) and as many columns as system outputs. The same holds for `x` with "outputs" replaced by states. Note that the output `t` may differ from the specified time vector when the input data is undersampled (see "Algorithm").

Example

Simulate and plot the response of the system

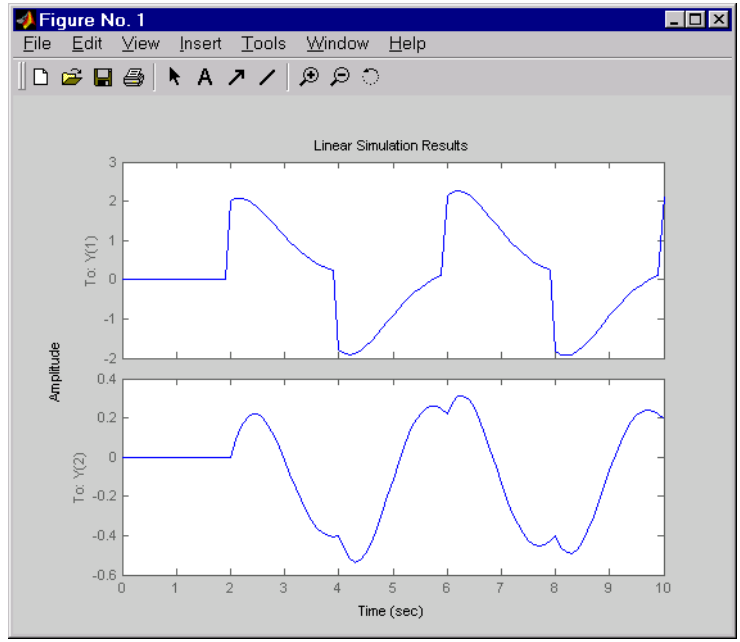
$$H(s) = \begin{bmatrix} \dfrac{2s^2 + 5s + 1}{s^2 + 2s + 3} \\ \dfrac{s - 1}{s^2 + s + 5} \end{bmatrix}$$

to a square wave with period of four seconds. First generate the square wave with `gensig`. Sample every 0.1 second during 10 seconds:

```
[u,t] = gensig('square',4,10,0.1);
```

Then simulate with `lsim`.

```
H = [tf([2 5 1],[1 2 3]) ; tf([1 -1],[1 1 5])]
lsim(H,u,t)
```



Algorithm

Discrete-time systems are simulated with `ltitr` (state space) or `filter` (transfer function and zero-pole-gain).

Continuous-time systems are discretized with `c2d` using either the `'zoh'` or `'foh'` method (`'foh'` is used for smooth input signals and `'zoh'` for discontinuous signals such as pulses or square waves). The sampling period is set to the spacing `dt` between the user-supplied time samples `t`.

The choice of sampling period can drastically affect simulation results. To illustrate why, consider the second-order model

$$H(s) = \frac{\omega^2}{s^2 + 2s + \omega^2}, \qquad \omega = 62.83$$

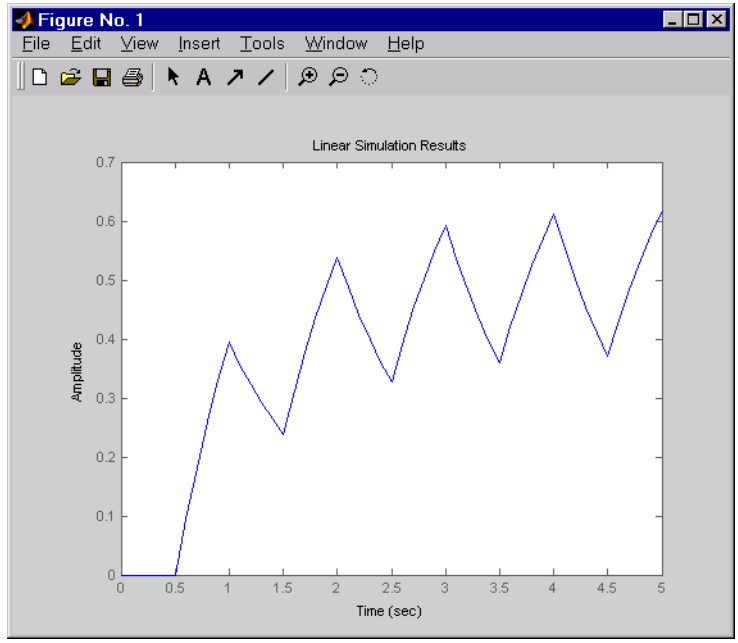
To simulate its response to a square wave with period 1 second, you can proceed as follows:

```
w2 = 62.83^2
h = tf(w2,[1 2 w2])
t = 0:0.1:5;           % vector of time samples
u = (rem(t,1)>=0.5);   % square wave values
lsim(h,u,t)
```

`lsim` evaluates the specified sample time, gives this warning

```
Warning: Input signal is undersampled. Sample every 0.016 sec or
faster.
```

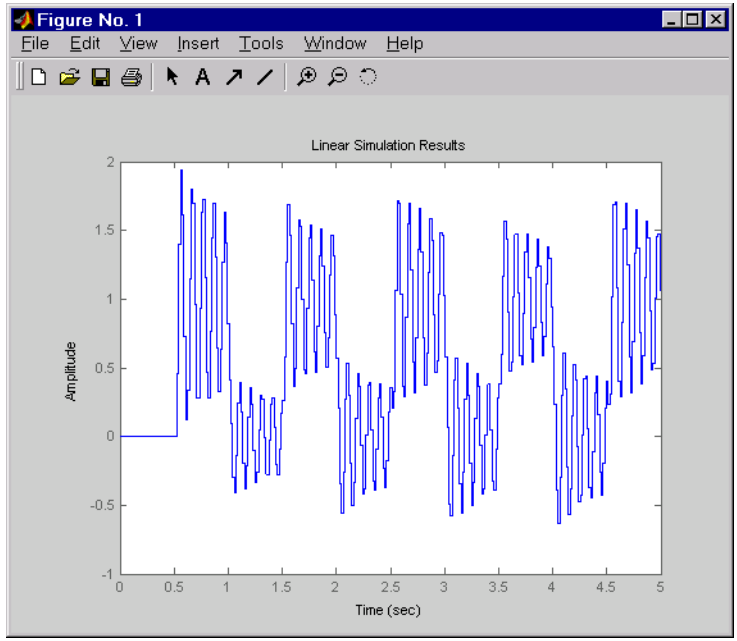
and produces this plot.



To improve on this response, discretize $H(s)$ using the recommended sampling period:

```
dt=0.016;
ts=0:dt:5;
us = (rem(ts,1)>=0.5)
hd = c2d(h,dt)
```

```
lsim(hd,us,ts)
```



This response exhibits strong oscillatory behavior hidden from the undersampled version.

See Also

`gensig`	Generate test input signals for `lsim`
`impulse`	Impulse response
`initial`	Free response to initial condition
`ltiview`	LTI system viewer
`step`	Step response

ltimodels

Purpose

Help on LTI models

Syntax

```
ltimodels
ltimodels(model type)
```

Description

`ltimodels` displays general information on the various types of LTI models supported in the Control System Toolbox.

`ltimodels(`*model type*`)` gives additional details and examples for each type of LTI model. The string *model type* selects the model type among the following:

- `tf` — Transfer functions (TF objects)
- `zpk` — Zero-pole-gain models (ZPK objects)
- `ss` — State-space models (SS objects)
- `frd` — Frequency response data models (FRD objects).

Note that you can type

```
ltimodels zpk
```

as a shorthand for

```
ltimodels('zpk')
```

See Also

`frd`	Create or convert to FRD models
`ltiprops`	Help on LTI model properties
`ss`	Create or convert to a state-space model
`tf`	Create or convert to a transfer function model
`zpk`	Create or convert to a zero/pole/gain model

Purpose

Help on LTI model properties

Syntax

```
ltimodels
ltimodels(model type)
```

Description

`ltiprops` displays details on the generic properties of LTI models.

`ltiprops(`*model type*`)` gives details on the properties specific to the various types of LTI models. The string *model type* selects the model type among the following:

- tf' — transfer functions (TF objects)
- zpk — zero-pole-gain models (ZPK objects)
- ss — state-space models (SS objects)
- frd — frequency response data (FRD objects).

Note that you can type

```
ltiprops tf
```

as a shorthand for

```
ltiprops('tf')
```

See also

`get`	Get the properties for an LTI model
`ltimodels`	Help on LTI models
`set`	Set or modify LTI model properties

ltiview

Purpose

Initialize an LTI Viewer for LTI system response analysis

Syntax

```
ltiview
ltiview(sys1,sys2,...,sysn)
ltiview('plottype',sys1,sys2,...,sysn)
ltiview('plottype',sys,extras)
ltiview('clear',viewers)
ltiview('current',sys1,sys2,...,sysn,viewers)
```

Description

`ltiview` when invoked without input arguments, initializes a new LTI Viewer for LTI system response analysis.

`ltiview(sys1,sys2,...,sysn)` opens an LTI Viewer containing the step response of the LTI models `sys1,sys2,...,sysn`. You can specify a distinctive color, line style, and marker for each system, as in

```
sys1 = rss(3,2,2);
sys2 = rss(4,2,2);
ltiview(sys1,'r-*',sys2,'m--');
```

`ltiview('plottype',sys)` initializes an LTI Viewer containing the LTI response type indicated by *plottype* for the LTI model `sys`. The string *plottype* can be any one of the following:

```
'step'
'impulse'
'initial'
'lsim'
'pzmap'
'bode'
'nyquist'
'nichols'
'sigma'
```

or,

plottype can be a cell vector containing up to six of these plot types. For example,

```
ltiview({'step';'nyquist'},sys)
```

displays the plots of both of these response types for a given system `sys`.

`ltiview(`*plottype*`,sys,extras)` allows the additional input arguments supported by the various LTI model response functions to be passed to the `ltiview` command.

extras is one or more input arguments as specified by the function named in *plottype*. These arguments may be required or optional, depending on the type of LTI response. For example, if *plottype* is `'step'` then `extras` may be the desired final time, `Tfinal`, as shown below.

```
ltiview('step',sys,Tfinal)
```

However, if *plottype* is `'initial'`, the `extras` arguments must contain the initial conditions `x0` and may contain other arguments, such as `Tfinal`.

```
ltiview('initial',sys,x0,Tfinal)
```

See the individual references pages of each possible *plottype* commands for a list of appropriate arguments for `extras`.

`ltiview('clear',viewers)` clears the plots and data from the LTI Viewers with handles `viewers`.

`ltiview('current',sys1,sys2,...,sysn,viewers)` adds the responses of the systems `sys1,sys2,...,sysn` to the LTI Viewers with handles `viewers`. If these new systems do not have the same I/O dimensions as those currently in the LTI Viewer, the LTI Viewer is first cleared and only the new responses are shown.

Finally,

```
ltiview(plottype,sys1,sys2,...sysN)
ltiview(plottype,sys1,PlotStyle1,sys2,PlotStyle2,...)
ltiview(plottype,sys1,sys2,...sysN,extras)
```

initializes an LTI Viewer containing the responses of multiple LTI models, using the plot styles in `PlotStyle`, when applicable. See the individual reference pages of the LTI response functions for more information on specifying plot styles.

See Also

`bode`	Bode response
`impulse`	Impulse response
`initial`	Response to initial condition
`lsim`	Simulate LTI model response to arbitrary inputs

`nichols`	Nichols response
`nyquist`	Nyquist response
`pzmap`	Pole/zero map
`sigma`	Singular value response
`step`	Step response

Purpose

Solve continuous-time Lyapunov equations

Syntax

```
X = lyap(A,Q)
X = lyap(A,B,C)
```

Description

`lyap` solves the special and general forms of the Lyapunov matrix equation. Lyapunov equations arise in several areas of control, including stability theory and the study of the RMS behavior of systems.

`X = lyap(A,Q)` solves the Lyapunov equation

$$AX + XA^T + Q = 0$$

where A and Q are square matrices of identical sizes. The solution `X` is a symmetric matrix if Q is.

`X = lyap(A,B,C)` solves the generalized Lyapunov equation (also called Sylvester equation).

$$AX + XB + C = 0$$

The matrices A, B, C must have compatible dimensions but need not be square.

Algorithm

`lyap` transforms the A and B matrices to complex Schur form, computes the solution of the resulting triangular system, and transforms this solution back [1].

Limitations

The continuous Lyapunov equation has a (unique) solution if the eigenvalues $\alpha_1, \alpha_2, ..., \alpha_n$ of A and $\beta_1, \beta_2, ..., \beta_n$ of B satisfy

$$\alpha_i + \beta_j \neq 0 \qquad \text{for all pairs } (i, j)$$

If this condition is violated, `lyap` produces the error message

```
Solution does not exist or is not unique.
```

See Also

`covar`	Covariance of system response to white noise
`dlyap`	Solve discrete Lyapunov equations

References

[1] Bartels, R.H. and G.W. Stewart, "Solution of the Matrix Equation AX + XB = C," *Comm. of the ACM*, Vol. 15, No. 9, 1972.

[2] Bryson, A.E. and Y.C. Ho, *Applied Optimal Control*, Hemisphere Publishing, 1975. pp. 328–338.

Purpose

Compute gain and phase margins and associated crossover frequencies

Syntax

```
[Gm,Pm,Wcg,Wcp] = margin(sys)
[Gm,Pm,Wcg,Wcp] = margin(mag,phase,w)
margin(sys)
```

Description

`margin` calculates the minimum gain margin, phase margin, and associated crossover frequencies of SISO open-loop models. The gain and phase margins indicate the relative stability of the control system when the loop is closed. When invoked without left-hand arguments, `margin` produces a Bode plot and displays the margins on this plot.

The gain margin is the amount of gain increase required to make the loop gain unity at the frequency where the phase angle is –180°. In other words, the gain margin is $1/g$ if g is the gain at the –180° phase frequency. Similarly, the phase margin is the difference between the phase of the response and –180° when the loop gain is 1.0. The frequency at which the magnitude is 1.0 is called the unity-gain frequency or crossover frequency. It is generally found that gain margins of three or more combined with phase margins between 30 and 60 degrees result in reasonable trade-offs between bandwidth and stability.

`[Gm,Pm,Wcg,Wcp] = margin(sys)` computes the gain margin `Gm`, the phase margin `Pm`, and the corresponding crossover frequencies `Wcg` and `Wcp`, given the SISO open-loop model `sys`. This function handles both continuous- and discrete-time cases. When faced with several crossover frequencies, `margin` returns the smallest gain and phase margins.

`[Gm,Pm,Wcg,Wcp] = margin(mag,phase,w)` derives the gain and phase margins from the Bode frequency response data (magnitude, phase, and frequency vector). Interpolation is performed between the frequency points to estimate the margin values. This approach is generally less accurate.

When invoked without left-hand argument,

```
margin(sys)
```

plots the open-loop Bode response with the gain and phase margins marked by vertical lines.

Example

You can compute the gain and phase margins of the open-loop discrete-time transfer function. Type

```
hd = tf([0.04798 0.0464],[1 -1.81 0.9048],0.1)
```

MATLAB responds with

```
Transfer function:
 0.04798 z + 0.0464
--------------------
z^2 - 1.81 z + 0.9048

Sampling time: 0.1
```

Type

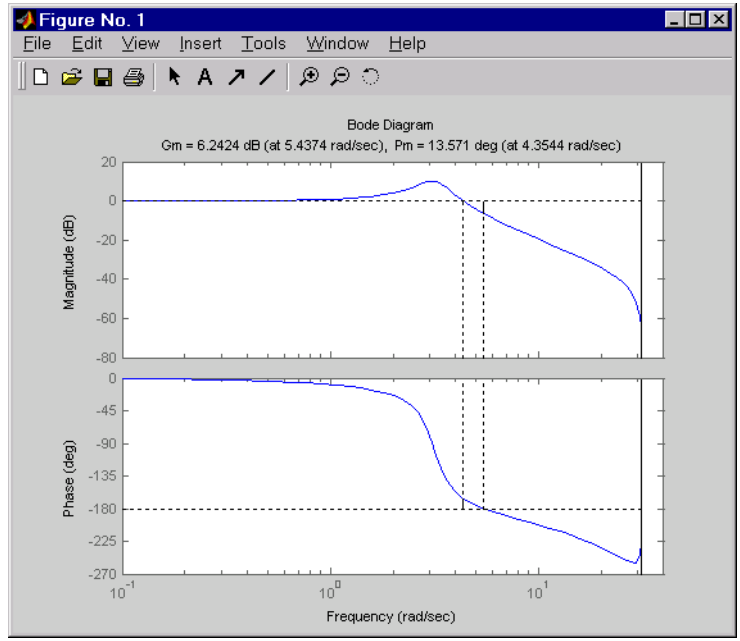
```
[Gm,Pm,Wcg,Wcp] = margin(hd);
[Gm,Pm,Wcg,Wcp]
```

and MATLAB returns

```
ans =
    2.0517   13.5711    5.4374    4.3544
```

You can also display these margins graphically.

```
margin(hd)
```



Algorithm

The phase margin is computed using H_∞ theory, and the gain margin by solving $H(j\omega) = \overline{H(j\omega)}$ for the frequency ω.

See Also

`bode`	Bode frequency response
`ltiview`	LTI system viewer

minreal

Purpose

Minimal realization or pole-zero cancellation

Syntax

```
sysr = minreal(sys)
sysr = minreal(sys,tol)
[sysr,u] = minreal(sys,tol)
```

Description

`sysr = minreal(sys)` eliminates uncontrollable or unobservable state in state-space models, or cancels pole-zero pairs in transfer functions or zero-pole-gain models. The output `sysr` has minimal order and the same response characteristics as the original model `sys`.

`sysr = minreal(sys,tol)` specifies the tolerance used for state elimination or pole-zero cancellation. The default value is `tol = sqrt(eps)` and increasing this tolerance forces additional cancellations.

`[sysr,u] = minreal(sys,tol)` returns, for state-space model `sys`, an orthogonal matrix `U` such that `(U*A*U',U*B,C*U')` is a Kalman decomposition of `(A,B,C)`

Example

The commands

```
g = zpk([],1,1)
h = tf([2 1],[1 0])
cloop = inv(1+g*h) * g
```

produce the nonminimal zero-pole-gain model by typing `cloop`.

```
Zero/pole/gain:
      s (s-1)
-------------------
(s-1) (s^2 + s + 1)
```

To cancel the pole-zero pair at $s = 1$, type

```
cloop = minreal(cloop)
```

and MATLAB returns

```
Zero/pole/gain:
      s
-------------
(s^2 + s + 1)
```

Algorithm

Pole-zero cancellation is a straightforward search through the poles and zeros looking for matches that are within tolerance. Transfer functions are first converted to zero-pole-gain form.

See Also

`balreal`	Grammian-based input/output balancing
`modred`	Model order reduction
`sminreal`	Structured model reduction

modred

Purpose

Model order reduction

Syntax

```
rsys = modred(sys,elim)
rsys = modred(sys,elim,'mdc')
rsys = modred(sys,elim,'del')
```

Description

`modred` reduces the order of a continuous or discrete state-space model sys. This function is usually used in conjunction with `balreal`. Two order reduction techniques are available:

- `rsys = modred(sys,elim)` or `rsys = modred(sys,elim,'mdc')` produces a reduced-order model `rsys` with matching DC gain (or equivalently, matching steady state in the step response). The index vector `elim` specifies the states to be eliminated. The resulting model `rsys` has `length(elim)` fewer states. This technique consists of setting the derivative of the eliminated states to zero and solving for the remaining states.
- `rsys = modred(sys,elim,'del')`simply deletes the states specified by `elim`. While this method does not guarantee matching DC gains, it tends to produce better approximations in the frequency domain (see example below).

If the state-space model sys has been balanced with `balreal` and the grammians have m small diagonal entries, you can reduce the model order by eliminating the last m states with `modred`.

Example

Consider the continuous fourth-order model

$$h(s) = \frac{s^3 + 11s^2 + 36s + 26}{s^4 + 14.6s^3 + 74.96s^2 + 153.7s + 99.65}$$

To reduce its order, first compute a balanced state-space realization with `balreal` by typing

```
h = tf([1 11 36 26],[1 14.6 74.96 153.7 99.65])
[hb,g] = balreal(h)
g'
```

MATLAB returns

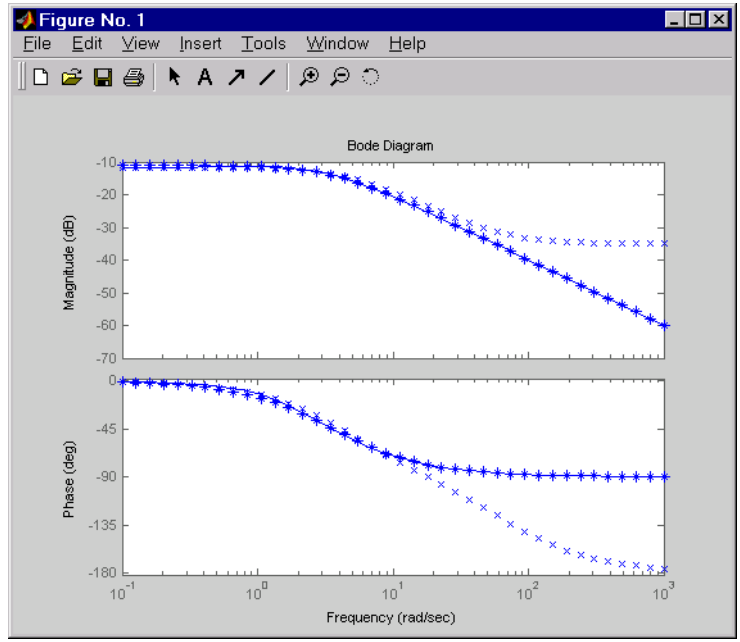
```
ans =
   1.3938e-01   9.5482e-03   6.2712e-04   7.3245e-06
```

The last three diagonal entries of the balanced grammians are small, so eliminate the last three states with `modred` using both matched DC gain and direct deletion methods.

```
hmdc = modred(hb,2:4,'mdc')
hdel = modred(hb,2:4,'del')
```

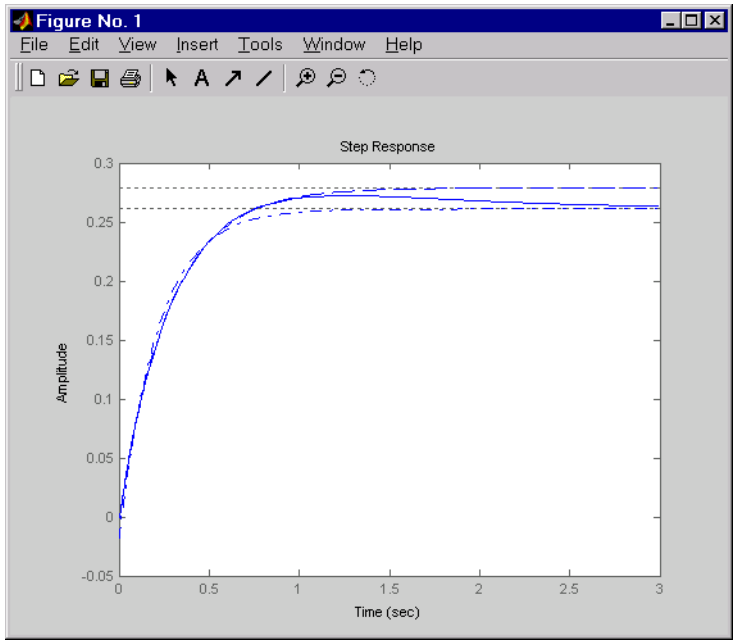
Both `hmdc` and `hdel` are first-order models. Compare their Bode responses against that of the original model $h(s)$.

```
bode(h,'-',hmdc,'x',hdel,'*')
```



The reduced-order model `hdel` is clearly a better frequency-domain approximation of $h(s)$. Now compare the step responses.

```
step(h,'-',hmdc,'-.',hdel,'--')
```



While `hdel` accurately reflects the transient behavior, only `hmdc` gives the true steady-state response.

Algorithm

The algorithm for the matched DC gain method is as follows. For continuous-time models

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

the state vector is partitioned into x_1, to be kept, and x_2, to be eliminated.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$
$$y = \begin{bmatrix} C_1 & C_2 \end{bmatrix} x + Du$$

Next, the derivative of x_2 is set to zero and the resulting equation is solved for x_1. The reduced-order model is given by

$$\dot{x}_1 = [A_{11} - A_{12}A_{22}^{-1}A_{21}]x_1 + [B_1 - A_{12}A_{22}^{-1}B_2]u$$

$$y = [C_1 - C_2A_{22}^{-1}A_{21}]x + [D - C_2A_{22}^{-1}B_2]u$$

The discrete-time case is treated similarly by setting

$$x_2[n+1] = x_2[n]$$

Limitations

With the matched DC gain method, A_{22} must be invertible in continuous time, and $I - A_{22}$ must be invertible in discrete time.

See Also

`balreal`	Input/output balancing of state-space models
`minreal`	Minimal state-space realizations

ndims

Purpose

Provide the number of the dimensions of an LTI model or LTI array

Syntax

```
n = ndims(sys)
```

Description

`n = ndims(sys)` is the number of dimensions of an LTI model or an array of LTI models sys. A single LTI model has two dimensions (one for outputs, and one for inputs). An LTI array has $2+p$ dimensions, where $p \geq 2$ is the number of array dimensions. For example, a 2-by-3-by-4 array of models has 2+3=5 dimensions.

```
ndims(sys) = length(size(sys))
```

Example

```
sys = rss(3,1,1,3);
ndims(sys)

ans =
     4
```

`ndims` returns 4 for this 3-by-1 array of SISO models.

See Also

`size`	Returns a vector containing the lengths of the dimensions of an LTI array or model

Purpose

Superimpose a Nichols chart on a Nichols plot

Syntax

```
ngrid
```

Description

`ngrid` superimposes Nichols chart grid lines over the Nichols frequency response of a SISO LTI system. The range of the Nichols grid lines is set to encompass the entire Nichols frequency response.

The chart relates the complex number $H/(1+H)$ to H, where H is any complex number. For SISO systems, when H is a point on the open-loop frequency response, then

$$\frac{H}{1+H}$$

is the corresponding value of the closed-loop frequency response assuming unit negative feedback.

If the current axis is empty, `ngrid` generates a new Nichols chart grid in the region –40 dB to 40 dB in magnitude and –360 degrees to 0 degrees in phase. If the current axis does not contain a SISO Nichols frequency response, `ngrid` returns a warning.

Example

Plot the Nichols response with Nichols grid lines for the system.

$$H(s) = \frac{-4s^4 + 48s^3 - 18s^2 + 250s + 600}{s^4 + 30s^3 + 282s^2 + 525s + 60}$$

Type

```
H = tf([-4 48 -18 250 600],[1 30 282 525 60])
```

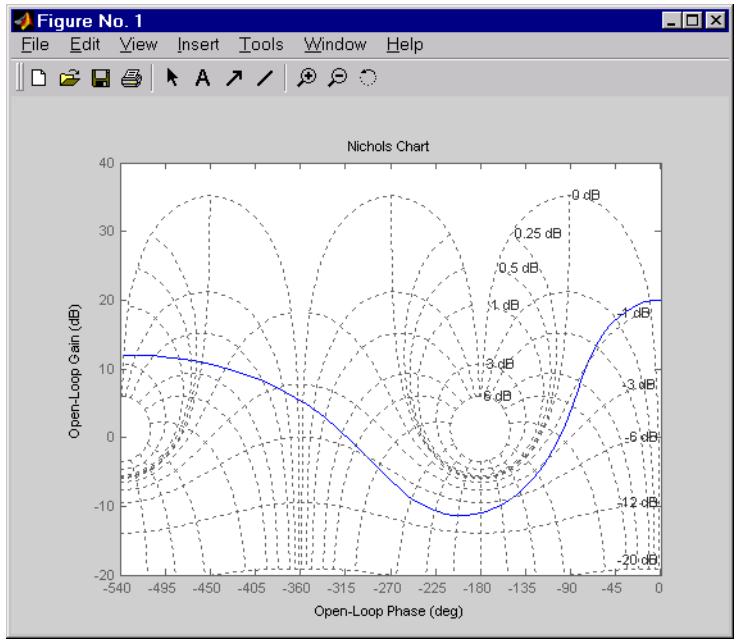
MATLAB returns

```
Transfer function:
- 4 s^4 + 48 s^3 - 18 s^2 + 250 s + 600
---------------------------------------
  s^4 + 30 s^3 + 282 s^2 + 525 s + 60
```

Type

```
nichols(H)
```

```
ngrid
```



See Also `nichols` Nichols plots

Purpose

Compute Nichols frequency response of LTI models

Syntax

```
nichols(sys)
nichols(sys,w)

nichols(sys1,sys2,...,sysN)
nichols(sys1,sys2,...,sysN,w)
nichols(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[mag,phase,w] = nichols(sys)
[mag,phase] = nichols(sys,w)
```

Description

`nichols` computes the frequency response of an LTI model and plots it in the Nichols coordinates. Nichols plots are useful to analyze open- and closed-loop properties of SISO systems, but offer little insight into MIMO control loops. Use `ngrid` to superimpose a Nichols chart on an existing SISO Nichols plot.

`nichols(sys)` produces a Nichols plot of the LTI model `sys`. This model can be continuous or discrete, SISO or MIMO. In the MIMO case, `nichols` produces an array of Nichols plots, each plot showing the response of one particular I/O channel. The frequency range and gridding are determined automatically based on the system poles and zeros.

`nichols(sys,w)` explicitly specifies the frequency range or frequency points to be used for the plot. To focus on a particular frequency interval `[wmin,wmax]`, set `w = {wmin,wmax}`. To use particular frequency points, set `w` to the vector of desired frequencies. Use `logspace` to generate logarithmically spaced frequency vectors. Frequencies should be specified in radians/sec.

`nichols(sys1,sys2,...,sysN)` or `nichols(sys1,sys2,...,sysN,w)` superimposes the Nichols plots of several LTI models on a single figure. All systems must have the same number of inputs and outputs, but may otherwise be a mix of continuous- and discrete-time systems. You can also specify a distinctive color, linestyle, and/or marker for each system plot with the syntax

```
nichols(sys1,'PlotStyle1',...,sysN,'PlotStyleN')
```

See `bode` for an example.

When invoked with left-hand arguments,

```
[mag,phase,w] = nichols(sys)
[mag,phase] = nichols(sys,w)
```

return the magnitude and phase (in degrees) of the frequency response at the frequencies `w` (in rad/sec). The outputs `mag` and `phase` are 3-D arrays similar to those produced by `bode` (see the `bode` reference page). They have dimensions

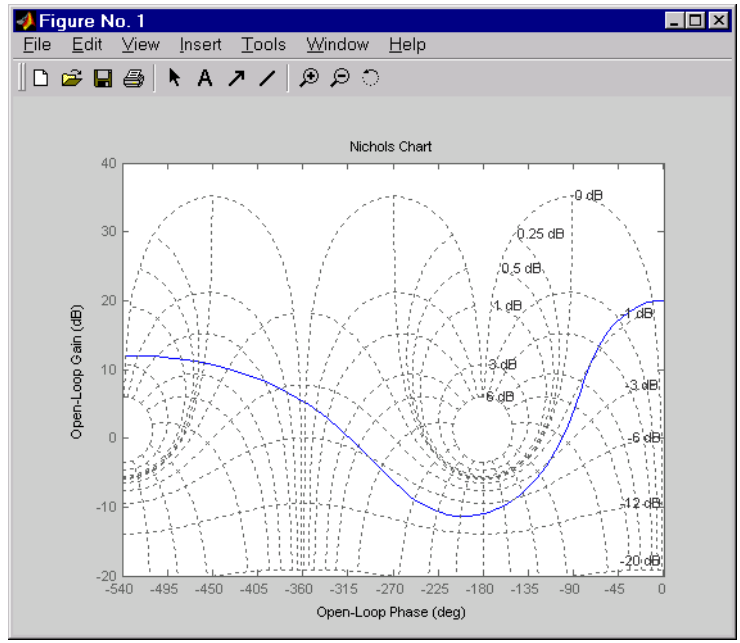
(number of outputs) × (number of inputs) × (length of `w`)

Example

Plot the Nichols response of the system

$$H(s) = \frac{-4s^4 + 48s^3 - 18s^2 + 250s + 600}{s^4 + 30s^3 + 282s^2 + 525s + 60}$$

```
num = [-4 48 -18 250 600];
den = [1 30 282 525 60];
H = tf(num,den)

nichols(H); ngrid
```



The right-click menu for Nichols plots includes the **Tight** option under **Zoom**. You can use this to clip unbounded branches of the Nichols plot.

Algorithm

See bode.

See Also

`bode`	Bode plot
`evalfr`	Response at single complex frequency
`freqresp`	Frequency response computation
`ltiview`	LTI system viewer
`ngrid`	Grid on Nichols plot
`nyquist`	Nyquist plot
`sigma`	Singular value plot

norm

Purpose

Compute LTI model norms

Syntax

```
norm(sys)
norm(sys,2)

norm(sys,inf)
norm(sys,inf,tol)
[ninf,fpeak] = norm(sys)
```

Description

`norm` computes the H_2 or L_∞ norm of a continuous- or discrete-time LTI model.

H_2 Norm

The H_2 norm of a stable continuous system with transfer function $H(s)$, is the root-mean-square of its impulse response, or equivalently

$$\|H\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{Trace}(H(j\omega)^H H(j\omega))\, d\omega}$$

This norm measures the steady-state covariance (or power) of the output response $y = Hw$ to unit white noise inputs w.

$$\|H\|_2^2 = \lim_{t\to\infty} E\{y(t)^T y(t)\}\ , \qquad E(w(t)w(\tau)^T) = \delta(t-\tau)I$$

Infinity Norm

The infinity norm is the peak gain of the frequency response, that is,

$$\|H(s)\|_\infty = \max_\omega |H(j\omega)| \qquad \text{(SISO case)}$$

$$\|H(s)\|_\infty = \max_\omega \sigma_{\max}(H(j\omega)) \qquad \text{(MIMO case)}$$

where $\sigma_{\max}(.)$ denotes the largest singular value of a matrix.

The discrete-time counterpart is

$$\|H(z)\|_\infty = \max_{\theta \in [0,\pi]} \sigma_{max}(H(e^{j\theta}))$$

Usage

`norm(sys)` or `norm(sys,2)` both return the H_2 norm of the TF, SS, or ZPK model sys. This norm is infinite in the following cases:

- sys is unstable.
- sys is continuous and has a nonzero feedthrough (that is, nonzero gain at the frequency $\omega = \infty$).

Note that `norm(sys)` produces the same result as

```
sqrt(trace(covar(sys,1)))
```

`norm(sys,inf)` computes the infinity norm of any type of LTI model sys. This norm is infinite if sys has poles on the imaginary axis in continuous time, or on the unit circle in discrete time.

`norm(sys,inf,tol)` sets the desired relative accuracy on the computed infinity norm (the default value is `tol=1e-2`).

`[ninf,fpeak] = norm(sys,inf)` also returns the frequency `fpeak` where the gain achieves its peak value.

Example

Consider the discrete-time transfer function

$$H(z) = \frac{z^3 - 2.841z^2 + 2.875z - 1.004}{z^3 - 2.417z^2 + 2.003z - 0.5488}$$

with sample time 0.1 second. Compute its H_2 norm by typing

```
H = tf([1 -2.841 2.875 -1.004],[1 -2.417 2.003 -0.5488],0.1)
norm(H)

ans =
    1.2438
```

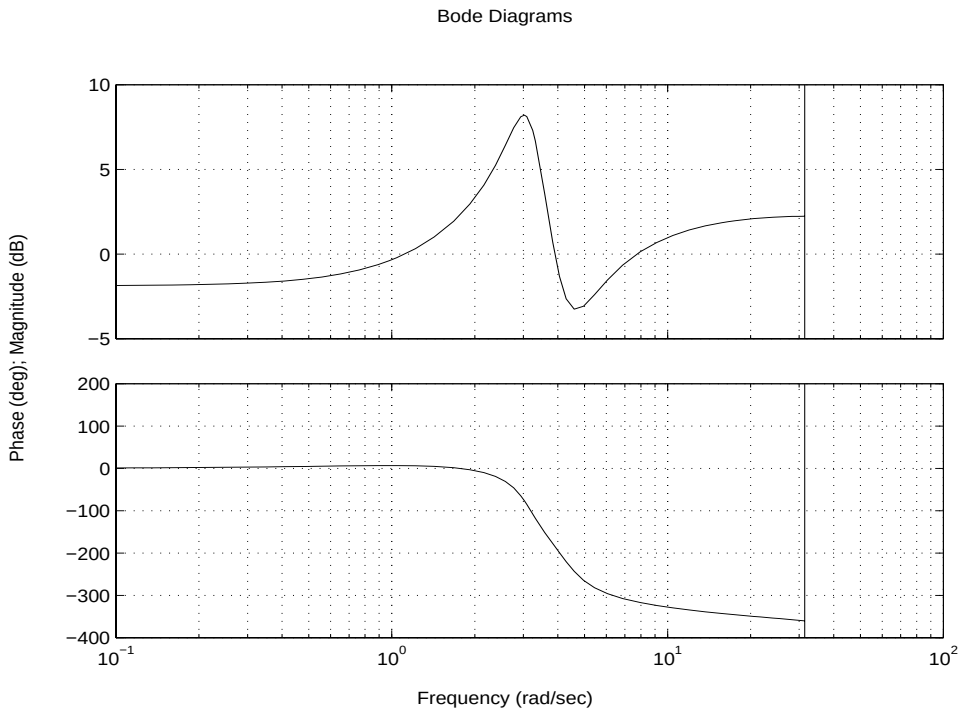
Compute its infinity norm by typing

```
[ninf,fpeak] = norm(H,inf)
```

```
ninf =
    2.5488

fpeak =
    3.0844
```

These values are confirmed by the Bode plot of $H(z)$.

```
bode(H)
```



The gain indeed peaks at approximately 3 rad/sec and its peak value in dB is found by typing

```
20*log10(ninf)
```

MATLAB returns

```
ans =
```

```
8.1268
```

Algorithm

`norm` uses the same algorithm as `covar` for the H_2 norm, and the algorithm of [1] for the infinity norm. `sys` is first converted to state space.

See Also

`bode`	Bode plot
`freqresp`	Frequency response computation
`sigma`	Singular value plot

References

[1] Bruisma, N.A. and M. Steinbuch, "A Fast Algorithm to Compute the H_∞-Norm of a Transfer Function Matrix," *System Control Letters*, 14 (1990), pp. 287–293.

nyquist

Purpose

Compute Nyquist frequency response of LTI models

Syntax

```
nyquist(sys)
nyquist(sys,w)

nyquist(sys1,sys2,...,sysN)
nyquist(sys1,sys2,...,sysN,w)
nyquist(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[re,im,w] = nyquist(sys)
[re,im] = nyquist(sys,w)
```

Description

`nyquist` calculates the Nyquist frequency response of LTI models. When invoked without left-hand arguments, `nyquist` produces a Nyquist plot on the screen. Nyquist plots are used to analyze system properties including gain margin, phase margin, and stability.

`nyquist(sys)` plots the Nyquist response of an arbitrary LTI model `sys`. This model can be continuous or discrete, and SISO or MIMO. In the MIMO case, `nyquist` produces an array of Nyquist plots, each plot showing the response of one particular I/O channel. The frequency points are chosen automatically based on the system poles and zeros.

`nyquist(sys,w)` explicitly specifies the frequency range or frequency points to be used for the plot. To focus on a particular frequency interval `[wmin,wmax]`, set `w = {wmin,wmax}`. To use particular frequency points, set `w` to the vector of desired frequencies. Use `logspace` to generate logarithmically spaced frequency vectors. Frequencies should be specified in rad/sec.

`nyquist(sys1,sys2,...,sysN)` or `nyquist(sys1,sys2,...,sysN,w)` superimposes the Nyquist plots of several LTI models on a single figure. All systems must have the same number of inputs and outputs, but may otherwise be a mix of continuous- and discrete-time systems. You can also specify a distinctive color, linestyle, and/or marker for each system plot with the syntax

```
nyquist(sys1,'PlotStyle1',...,sysN,'PlotStyleN')
```

See `bode` for an example.

When invoked with left-hand arguments

```
[re,im,w] = nyquist(sys)
[re,im] = nyquist(sys,w)
```

return the real and imaginary parts of the frequency response at the frequencies `w` (in rad/sec). `re` and `im` are 3-D arrays with the frequency as last dimension (see "Arguments" below for details).

Arguments

The output arguments `re` and `im` are 3-D arrays with dimensions

(number of outputs) × (number of inputs) × (length of w)

For SISO systems, the scalars `re(1,1,k)` and `im(1,1,k)` are the real and imaginary parts of the response at the frequency ω_k = w(k).

$$\text{re}(1,1,k) = \text{Re}(h(j\omega_k))$$
$$\text{im}(1,1,k) = \text{Im}(h(j\omega_k))$$

For MIMO systems with transfer function $H(s)$, `re(:,:,k)` and `im(:,:,k)` give the real and imaginary parts of $H(j\omega_k)$ (both arrays with as many rows as outputs and as many columns as inputs). Thus,

$$\text{re}(i,j,k) = \text{Re}(h_{ij}(j\omega_k))$$
$$\text{im}(i,j,k) = \text{Im}(h_{ij}(j\omega_k))$$

where h_{ij} is the transfer function from input j to output i.

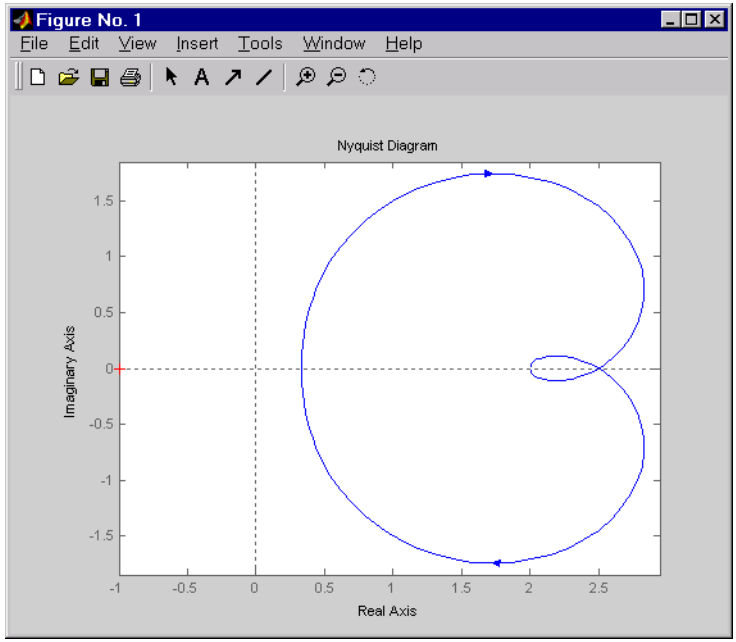
Example

Plot the Nyquist response of the system

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

```
H = tf([2 5 1],[1 2 3])
```

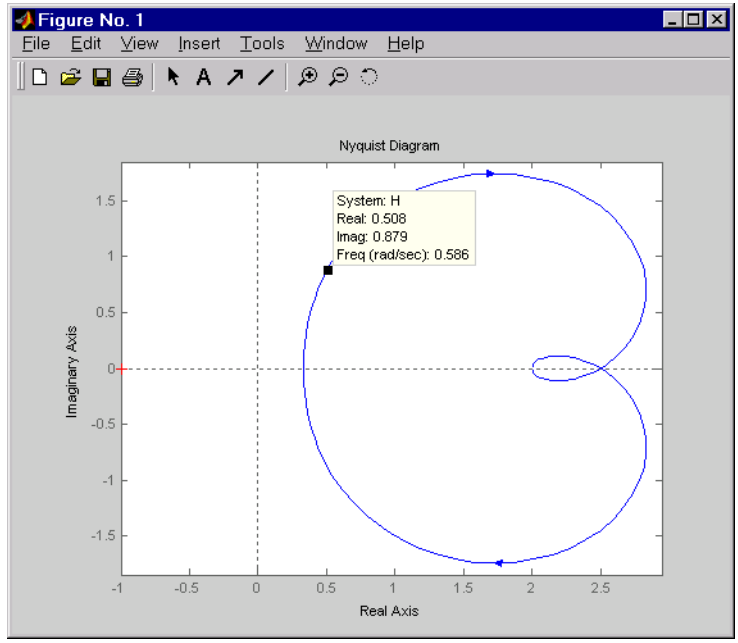
```
nyquist(H)
```



You have two zoom options available from the right-click menu that apply specifically to Nyquist plots:

- **Tight** —Clips unbounded branches of the Nyquist plot, but still includes the critical point (-1, 0)
- **On (-1,0)** — Zooms around the critical point (-1,0)

Also, click anywhere on the curve to activate data markers that display the real and imaginary values at a given frequency. This figure shows the nyquist plot with a data marker.



See Also

`bode`	Bode plot
`evalfr`	Response at single complex frequency
`freqresp`	Frequency response computation
`ltiview`	LTI system viewer
`nichols`	Nichols plot
`sigma`	Singular value plot

obsv

Purpose

Form the observability matrix

Syntax

```
Ob = obsv(A,B)
Ob = obsv(sys)
```

Description

obsv computes the observability matrix for state-space systems. For an *n*-by-*n* matrix A and a *p*-by-*n* matrix C, obsv(A,C) returns the observability matrix

$$Ob = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

with *n* columns and *np* rows.

Ob = obsv(sys) calculates the observability matrix of the state-space model sys. This syntax is equivalent to executing

```
Ob = obsv(sys.A,sys.C)
```

The model is observable if Ob has full rank *n*.

Example

Determine if the pair

```
A =
     1     1
     4    -2

C =
     1     0
     0     1
```

is observable. Type

```
Ob = obsv(A,C);

% Number of unobservable states
unob = length(A)-rank(Ob)
```

MATLAB responds with

```
unob =
     0
```

See Also

`obsvf` Compute the observability staircase form

obsvf

Purpose

Compute the observability staircase form

Syntax

```
[Abar,Bbar,Cbar,T,k] = obsvf(A,B,C)
[Abar,Bbar,Cbar,T,k] = obsvf(A,B,C,tol)
```

Description

If the observability matrix of `(A,C)` has rank $r \le n$, where n is the size of `A`, then there exists a similarity transformation such that

$$\overline{A} = TAT^T, \qquad \overline{B} = TB, \qquad \overline{C} = CT^T$$

where T is unitary and the transformed system has a *staircase* form with the unobservable modes, if any, in the upper left corner.

$$\overline{A} = \begin{bmatrix} A_{no} & A_{12} \\ 0 & A_o \end{bmatrix}, \qquad \overline{B} = \begin{bmatrix} B_{no} \\ B_o \end{bmatrix}, \qquad \overline{C} = \begin{bmatrix} 0 & C_o \end{bmatrix}$$

where (C_o, A_o) is observable, and the eigenvalues of A_{no} are the unobservable modes.

`[Abar,Bbar,Cbar,T,k] = obsvf(A,B,C)` decomposes the state-space system with matrices `A`, `B`, and `C` into the observability staircase form `Abar`, `Bbar`, and `Cbar`, as described above. `T` is the similarity transformation matrix and `k` is a vector of length n, where n is the number of states in `A`. Each entry of `k` represents the number of observable states factored out during each step of the transformation matrix calculation [1]. The number of nonzero elements in `k` indicates how many iterations were necessary to calculate `T`, and `sum(k)` is the number of states in A_o, the observable portion of `Abar`.

`obsvf(A,B,C,tol)` uses the tolerance `tol` when calculating the observable/unobservable subspaces. When the tolerance is not specified, it defaults to `10*n*norm(a,1)*eps`.

Example

Form the observability staircase form of

```
A =
     1     1
     4    -2

B =
```

```
     1    -1
     1    -1

C =
     1     0
     0     1
```

by typing

```
[Abar,Bbar,Cbar,T,k] = obsvf(A,B,C)
```

```
Abar =
     1     1
     4    -2
Bbar =
     1     1
     1    -1
Cbar =
     1     0
     0     1
T =
     1     0
     0     1
k =
     2     0
```

Algorithm

obsvf is an M-file that implements the Staircase Algorithm of [1] by calling ctrbf and using duality.

See Also

ctrbf	Compute the controllability staircase form
obsv	Calculate the observability matrix

References

[1] Rosenbrock, M.M., *State-Space and Multivariable Theory*, John Wiley, 1970.

ord2

Purpose

Generate continuous second-order systems

Syntax

```
[A,B,C,D] = ord2(wn,z)
[num,den] = ord2(wn,z)
```

Description

`[A,B,C,D] = ord2(wn,z)` generates the state-space description `(A,B,C,D)` of the second-order system

$$h(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n{}^2}$$

given the natural frequency `wn` (ω_n) and damping factor `z` (ζ). Use `ss` to turn this description into a state-space object.

`[num,den] = ord2(wn,z)` returns the numerator and denominator of the second-order transfer function. Use `tf` to form the corresponding transfer function object.

Example

To generate an LTI model of the second-order transfer function with damping factor $\zeta = 0.4$ and natural frequency $\omega_n = 2.4$ rad/sec. , type

```
[num,den] = ord2(2.4,0.4)

num =
     1
den =
    1.0000    1.9200    5.7600

sys = tf(num,den)

Transfer function:
          1
-------------------
s^2 + 1.92 s + 5.76
```

See Also

`rss`	Generate random stable continuous models
`ss`	Create a state-space LTI model
`tf`	Create a transfer function LTI model

Purpose

Compute the Padé approximation of models with time delays

Syntax

```
[num,den] = pade(T,N)
pade(T,N)

sysx = pade(sys,N)
sysx = pade(sys,NI,NO,Nio)
```

Description

pade approximates time delays by rational LTI models. Such approximations are useful to model time delay effects such as transport and computation delays within the context of continuous-time systems. The Laplace transform of an time delay of T seconds is $\exp(-sT)$. This exponential transfer function is approximated by a rational transfer function using the Padé approximation formulas [1].

`[num,den] = pade(T,N)` returns the `N`th-order (diagonal) Padé approximation of the continuous-time I/O delay $\exp(-sT)$ in transfer function form. The row vectors `num` and `den` contain the numerator and denominator coefficients in descending powers of s. Both are `N`th-order polynomials.

When invoked without output arguments,

```
pade(T,N)
```

plots the step and phase responses of the `N`th-order Padé approximation and compares them with the exact responses of the model with I/O delay `T`. Note that the Padé approximation has unit gain at all frequencies.

`sysx = pade(sys,N)` produces a delay-free approximation `sysx` of the continuous delay system `sys`. All delays are replaced by their `N`th-order Padé approximation. See Time Delays for details on LTI models with delays.

`sysx = pade(sys,NI,NO,Nio)` specifies independent approximation orders for each input, output, and I/O delay. These approximation orders are given by the arrays of integers `NI`, `NO`, and `Nio`, such that:

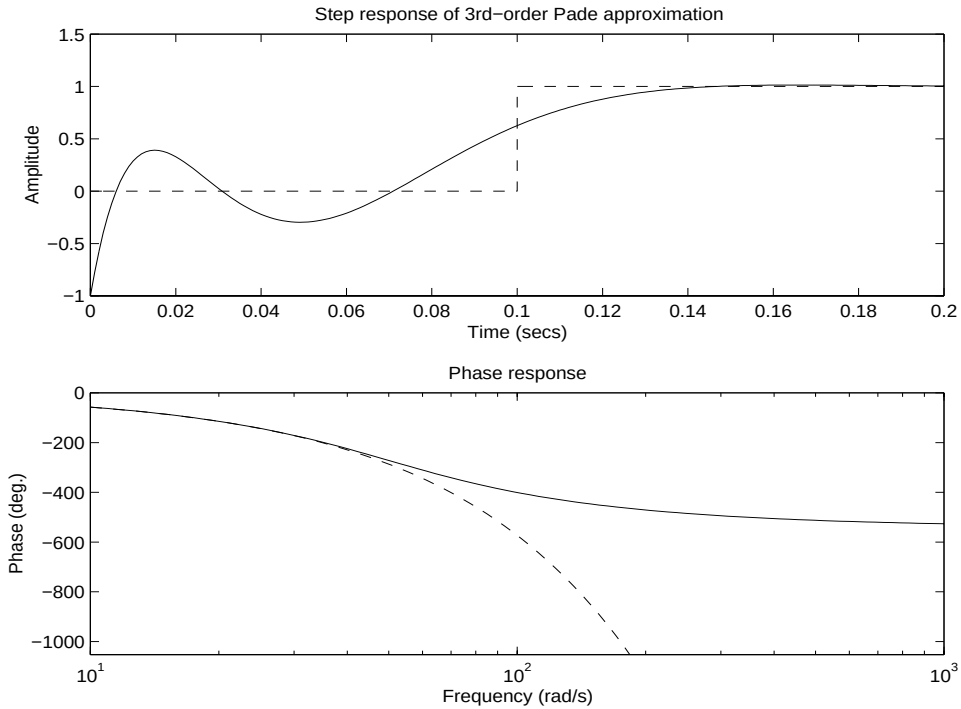
- `NI(j)` is the approximation order for the `j`-th input channel.
- `NO(i)` is the approximation order for the `i`-th output channel.
- `Nio(i,j)` is the approximation order for the I/O delay from input `j` to output `i`.

You can use scalar values to specify uniform approximation orders, and [] if there are no input, output, or I/O delays.

Example

Compute a third-order Padé approximation of a 0.1 second I/O delay and compare the time and frequency responses of the true delay and its approximation. To do this, type

```
pade(0.1,3)
```



Limitations

High-order Padé approximations produce transfer functions with clustered poles. Because such pole configurations tend to be very sensitive to perturbations, Padé approximations with order `N>10` should be avoided.

See Also

`c2d` Discretization of continuous system

`delay2z`	Changes transfer functions of discrete-time LTI models with delays to rational functions or absorbs FRD delays into the frequency response phase information

References

[1] Golub, G. H. and C. F. Van Loan, *Matrix Computations*, Johns Hopkins University Press, Baltimore, 1989, pp. 557–558.

parallel

Purpose

Parallel connection of two LTI models

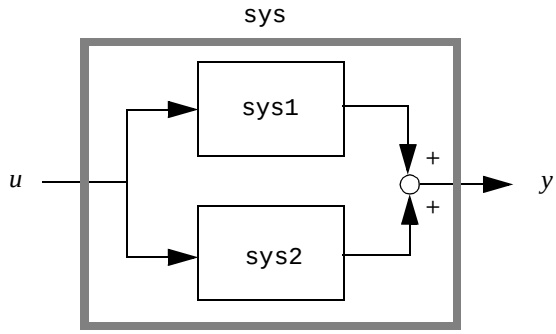
Syntax

```
sys = parallel(sys1,sys2)
sys = parallel(sys1,sys2,inp1,inp2,out1,out2)
```

Description

`parallel` connects two LTI models in parallel. This function accepts any type of LTI model. The two systems must be either both continuous or both discrete with identical sample time. Static gains are neutral and can be specified as regular matrices.

`sys = parallel(sys1,sys2)` forms the basic parallel connection shown below.

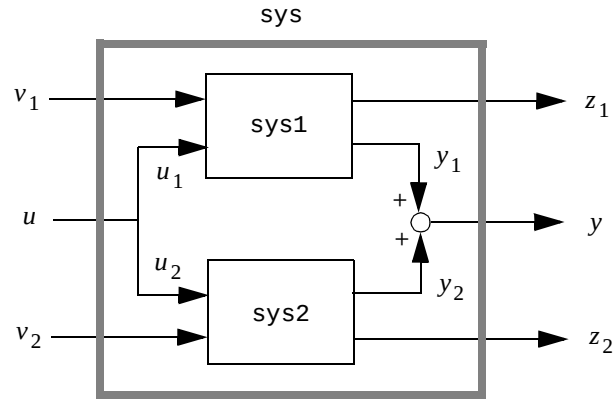


This command is equivalent to the direct addition

```
sys = sys1 + sys2
```

(See Addition and Subtraction for details on LTI system addition.)

`sys = parallel(sys1,sys2,inp1,inp2,out1,out2)` forms the more general parallel connection.



The index vectors `inp1` and `inp2` specify which inputs u_1 of `sys1` and which inputs u_2 of `sys2` are connected. Similarly, the index vectors `out1` and `out2` specify which outputs y_1 of `sys1` and which outputs y_2 of `sys2` are summed. The resulting model `sys` has $[v_1 ; u ; v_2]$ as inputs and $[z_1 ; y ; z_2]$ as outputs.

Example

See Kalman Filtering for an example.

See Also

`append`	Append LTI systems
`feedback`	Feedback connection
`series`	Series connection

place

Purpose

Pole placement design

Syntax

```
K = place(A,B,p)
[K,prec,message] = place(A,B,p)
```

Description

Given the single- or multi-input system

$$\dot{x} = Ax + Bu$$

and a vector `p` of desired self-conjugate closed-loop pole locations, `place` computes a gain matrix `K` such that the state feedback $u = -Kx$ places the closed-loop poles at the locations `p`. In other words, the eigenvalues of $A - BK$ match the entries of `p` (up to the ordering).

`K = place(A,B,p)` computes a feedback gain matrix `K` that achieves the desired closed-loop pole locations `p`, assuming all the inputs of the plant are control inputs. The length of `p` must match the row size of `A`. `place` works for multi-input systems and is based on the algorithm from [1]. This algorithm uses the extra degrees of freedom to find a solution that minimizes the sensitivity of the closed-loop poles to perturbations in A or B.

`[K,prec,message] = place(A,B,p)` also returns `prec`, an estimate of how closely the eigenvalues of $A - BK$ match the specified locations `p` (`prec` measures the number of accurate decimal digits in the actual closed-loop poles). If some nonzero closed-loop pole is more than 10% off from the desired location, `message` contains a warning message.

You can also use `place` for estimator gain selection by transposing the `A` matrix and substituting `C'` for `B`.

```
l = place(A',C',p).'
```

Example

Consider a state-space system `(a,b,c,d)` with two inputs, three outputs, and three states. You can compute the feedback gain matrix needed to place the closed-loop poles at `p = [1.1 23 5.0]` by

```
p = [1 1.23 5.0];
K = place(a,b,p)
```

Algorithm

`place` uses the algorithm of [1] which, for multi-input systems, optimizes the choice of eigenvectors for a robust solution. We recommend `place` rather than `acker` even for single-input systems.

In high-order problems, some choices of pole locations result in very large gains. The sensitivity problems attached with large gains suggest caution in the use of pole placement techniques. See [2] for results from numerical testing.

See Also

`acker`	Pole placement using Ackermann's formula
`lqr`	State-feedback LQ regulator design
`rlocus`	Root locus design

References

[1] Kautsky, J. and N.K. Nichols, "Robust Pole Assignment in Linear State Feedback," *Int. J. Control*, 41 (1985), pp. 1129–1155.

[2] Laub, A.J. and M. Wette, *Algorithms and Software for Pole Assignment and Observers*, UCRL-15646 Rev. 1, EE Dept., Univ. of Calif., Santa Barbara, CA, Sept. 1984.

pole

Purpose

Compute the poles of an LTI system

Syntax

```
p = pole(sys)
```

Description

`pole` computes the poles `p` of the SISO or MIMO LTI model `sys`.

Algorithm

For state-space models, the poles are the eigenvalues of the A matrix, or the generalized eigenvalues of $A - \lambda E$ in the descriptor case.

For SISO transfer functions or zero-pole-gain models, the poles are simply the denominator roots (see `roots`).

For MIMO transfer functions (or zero-pole-gain models), the poles are computed as the union of the poles for each SISO entry. If some columns or rows have a common denominator, the roots of this denominator are counted only once.

Limitations

Multiple poles are numerically sensitive and cannot be computed to high accuracy. A pole λ with multiplicity m typically gives rise to a cluster of computed poles distributed on a circle with center λ and radius of order

$$\rho \approx \varepsilon^{1/m}$$

where ε is the relative machine precision (`eps`).

See Also

`damp`	Damping and natural frequency of system poles
`esort`, `dsort`	Sort system poles
`pzmap`	Pole-zero map
`zero`	Compute (transmission) zeros

Purpose

Compute the pole-zero map of an LTI model

Syntax

```
pzmap(sys)
pzmap(sys1,sys2,...,sysN)
[p,z] = pzmap(sys)
```

Description

`pzmap(sys)` plots the pole-zero map of the continuous- or discrete-time LTI model `sys`. For SISO systems, `pzmap` plots the transfer function poles and zeros. For MIMO systems, it plots the system poles and transmission zeros. The poles are plotted as `x`'s and the zeros are plotted as `o`'s.

`pzmap(sys1,sys2,...,sysN)` plots the pole-zero map of several LTI models on a single figure. The LTI models can have different numbers of inputs and outputs and can be a mix of continuous and discrete systems.

When invoked without left-hand arguments,

```
[p,z] = pzmap(sys)
```

returns the system poles and (transmission) zeros in the column vectors `p` and `z`. No plot is drawn on the screen.

You can use the functions `sgrid` or `zgrid` to plot lines of constant damping ratio and natural frequency in the s- or z-plane.

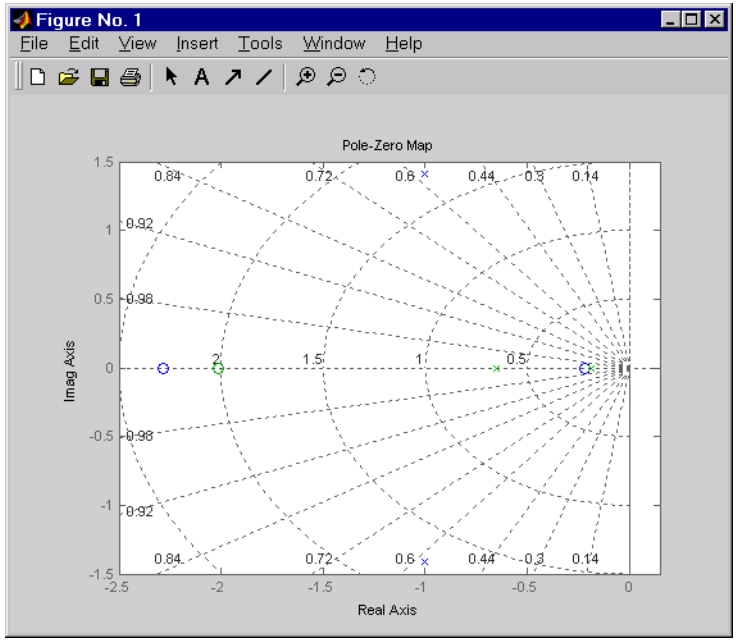
Example

Plot the poles and zeros of the continuous-time system.

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

```
H = tf([2 5 1],[1 2 3]); sgrid
```

```
pzmap(H)
```



Algorithm

`pzmap` uses a combination of `pole` and `zero`.

See Also

`damp`	Damping and natural frequency of system poles
`esort`, `dsort`	Sort system poles
`pole`	Compute system poles
`rlocus`	Root locus
`sgrid`, `zgrid`	Plot lines of constant damping and natural frequency
`zero`	Compute system (transmission) zeros

Purpose

Form regulator given state-feedback and estimator gains

Syntax

```
rsys = reg(sys,K,L)
rsys = reg(sys,K,L,sensors,known,controls)
```

Description

`rsys = reg(sys,K,L)` forms a dynamic regulator or compensator `rsys` given a state-space model `sys` of the plant, a state-feedback gain matrix `K`, and an estimator gain matrix `L`. The gains `K` and `L` are typically designed using pole placement or LQG techniques. The function `reg` handles both continuous- and discrete-time cases.

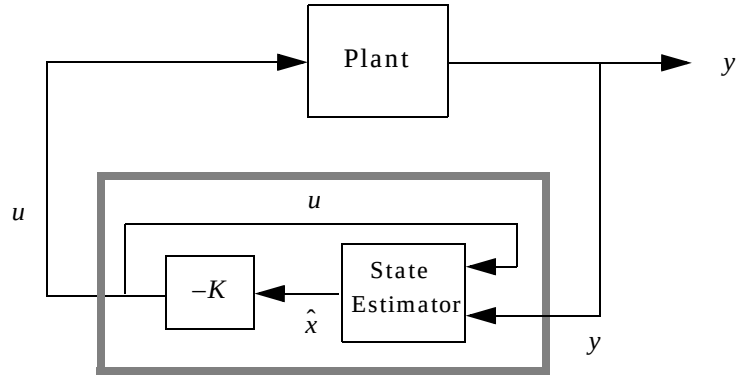
This syntax assumes that all inputs of `sys` are controls, and all outputs are measured. The regulator `rsys` is obtained by connecting the state-feedback law $u = -Kx$ and the state estimator with gain matrix `L` (see `estim`). For a plant with equations

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

this yields the regulator

$$\dot{\hat{x}} = \left[A - LC - (B - LD)K\right]\hat{x} + Ly$$
$$u = -K\hat{x}$$

This regulator should be connected to the plant using *positive* feedback.

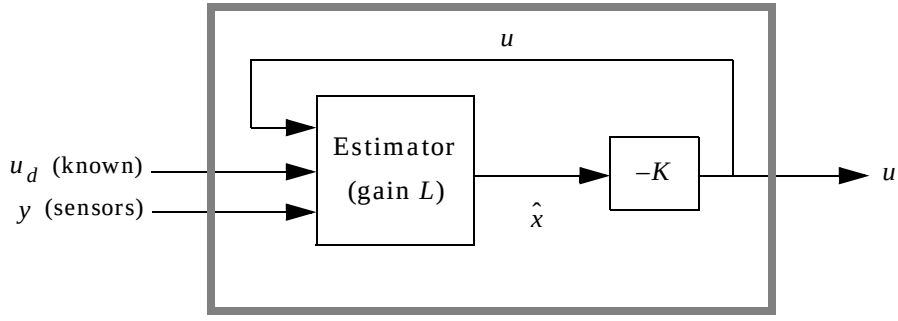


Regulator

`rsys = reg(sys,K,L,sensors,known,controls)` handles more general regulation problems where:

- The plant inputs consist of controls u, known inputs u_d, and stochastic inputs w.
- Only a subset y of the plant outputs is measured.

The index vectors `sensors`, `known`, and `controls` specify y, u_d, and u as subsets of the outputs and inputs of `sys`. The resulting regulator uses $[u_d ; y]$ as inputs to generate the commands u (see figure below).



Regulator `rsys`

Example

Given a continuous-time state-space model

```
sys = ss(A,B,C,D)
```

with seven outputs and four inputs, suppose you have designed:

- A state-feedback controller gain `K` using inputs 1, 2, and 4 of the plant as control inputs
- A state estimator with gain `L` using outputs 4, 7, and 1 of the plant as sensors, and input 3 of the plant as an additional known input

You can then connect the controller and estimator and form the complete regulation system by

```
controls = [1,2,4];
sensors = [4,7,1];
known = [3];
regulator = reg(sys,K,L,sensors,known,controls)
```

See Also

`estim`	Form state estimator given estimator gain
`kalman`	Kalman estimator design
`lqgreg`	Form LQG regulator
`lqr`, `dlqr`	State-feedback LQ regulator
`place`	Pole placement

reshape

Purpose

Change the shape of an LTI array

Syntax

```
sys = reshape(sys,s1,s2,...,sk)
sys = reshape(sys,[s1 s2 ... sk])
```

Description

`sys = reshape(sys,s1,s2,...,sk)` (or, equivalently, `sys = reshape(sys,[s1 s2 ... sk])`) reshapes the LTI array `sys` into an `s1`-by-`s2`-by...-`sk` array of LTI models. Equivalently, `sys = reshape(sys,[s1 s2 ... sk])` reshapes the LTI array `sys` into an `s1`-by-`s2`-by...-`sk` array of LTI models. With either syntax, there must be `s1*s2*...*sk` models in `sys` to begin with.

Example

```
sys = rss(4,1,1,2,3);
size(sys)

2x3 array of state-space models
Each model has 1 output, 1 input, and 4 states.

sys1 = reshape(sys,6);
size(sys1)

6x1 array of state-space models
Each model has 1 output, 1 input, and 4 states.
```

See Also

`ndims`	Provide the number of dimensions of an LTI array
`size`	Provide the lengths of each dimension of an LTI array

Purpose

Evans root locus

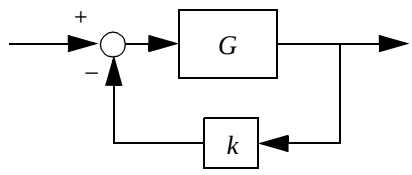
Syntax

```
rlocus(sys)
rlocus(sys,k)
rlocus(sys1,sys2,...)

[r,k] = rlocus(sys)
r = rlocus(sys,k)
```

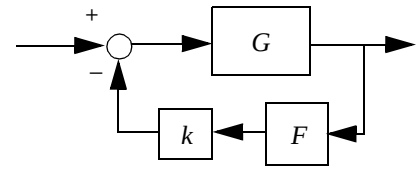
Description

`rlocus` computes the Evans root locus of a SISO open-loop model. The root locus gives the closed-loop pole trajectories as a function of the feedback gain k (assuming negative feedback). Root loci are used to study the effects of varying feedback gains on closed-loop pole locations. In turn, these locations provide indirect information on the time and frequency responses.

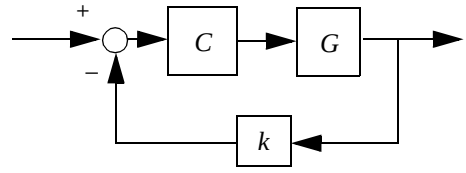
`rlocus(sys)` calculates and plots the root locus of the open-loop SISO model sys. This function can be applied to any of the following *negative* feedback loops by setting sys appropriately.



`sys = G`



`sys = F * G`



`sys = G * C`

If sys has transfer function

$$h(s) = \frac{n(s)}{d(s)}$$

the closed-loop poles are the roots of

$$d(s) + k\ n(s) = 0$$

`rlocus` adaptively selects a set of positive gains k to produce a smooth plot. Alternatively,

```
rlocus(sys,k)
```

uses the user-specified vector `k` of gains to plot the root locus.

`rlocus(sys1,sys2,...)` draws the root loci of multiple LTI models `sys1`, `sys2,...` on a single plot. You can specify a color, line style, and marker for each model, as in

```
rlocus(sys1,'r',sys2,'y:',sys3,'gx').
```

When invoked with output arguments,

```
[r,k] = rlocus(sys)
r = rlocus(sys,k)
```

return the vector `k` of selected gains and the complex root locations `r` for these gains. The matrix `r` has `length(k)` columns and its `j`th column lists the closed-loop roots for the gain `k(j)`.

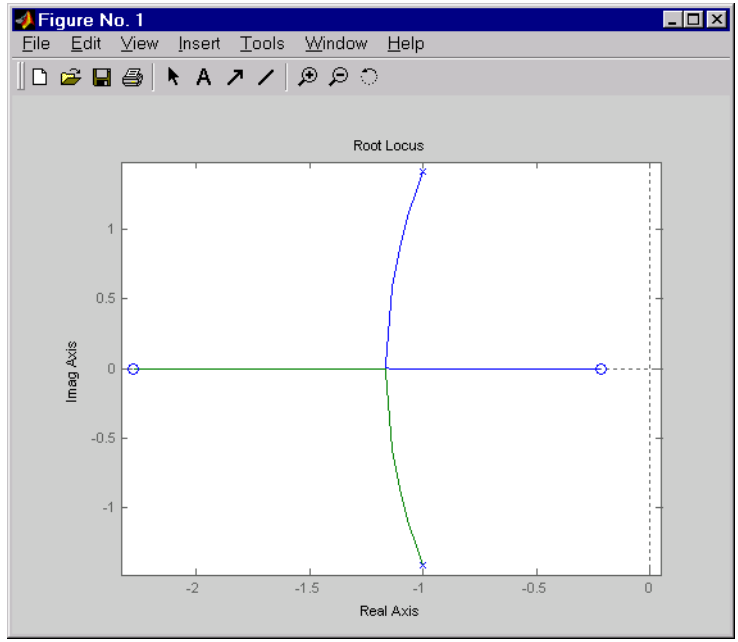
Example

Find and plot the root-locus of the following system.

$$h(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

```
h = tf([2 5 1],[1 2 3]);
```

```
rlocus(h)
```



You can use the right-click menu for rlocus to add grid lines, zoom in or out, and invoke the Property Editor to customize the plot. Also, click anywhere on the curve to activate a data marker that displays the gain value, pole, damping, overshoot, and frequency at the selected point.

See Also

`pole`	System poles
`pzmap`	Pole-zero map

Purpose

Generate stable random continuous test models

Syntax

```
sys = rss(n)
sys = rss(n,p)
sys = rss(n,p,m)
sys = rss(n,p,m,s1,...,sn)
```

Description

`rss(n)` produces a stable random `n`-th order model with one input and one output and returns the model in the state-space object `sys`.

`rss(n,p)` produces a random `n`th order stable model with one input and `p` outputs, and `rss(n,m,p)` produces a random `n`-th order stable model with `m` inputs and `p` outputs. The output `sys` is always a state-space model.

`rss(n,p,m,s1,...,sn)`produces an `s1`-by-...-by-`sn` array of random `n`-th order stable state-space models with `m` inputs and `p` outputs.

Use `tf`, `frd`, or `zpk` to convert the state-space object `sys` to transfer function, frequency response, or zero-pole-gain form.

Example

Obtain a stable random continuous LTI model with three states, two inputs, and two outputs by typing

```
sys = rss(3,2,2)

a =
                x1          x2          x3
     x1    -0.54175     0.09729     0.08304
     x2     0.09729    -0.89491     0.58707
     x3     0.08304     0.58707    -1.95271

b =
                u1          u2
     x1    -0.88844    -2.41459
     x2           0    -0.69435
     x3    -0.07162    -1.39139

c =
                x1          x2          x3
     y1     0.32965     0.14718           0
     y2     0.59854    -0.10144     0.02805
```

```
d =
                        u1           u2
           y1     -0.87631     -0.32758
           y2            0            0

Continuous-time system.
```

See Also

`drss`	Generate stable random discrete test models
`frd`	Convert LTI systems to frequency response form
`tf`	Convert LTI systems to transfer function form
`zpk`	Convert LTI systems to zero-pole-gain form

Purpose

Series connection of two LTI models

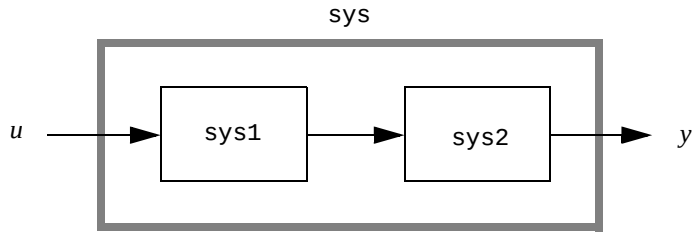
Syntax

```
sys = series(sys1,sys2)
sys = series(sys1,sys2,outputs1,inputs2)
```

Description

`series` connects two LTI models in series. This function accepts any type of LTI model. The two systems must be either both continuous or both discrete with identical sample time. Static gains are neutral and can be specified as regular matrices.

`sys = series(sys1,sys2)` forms the basic series connection shown below.

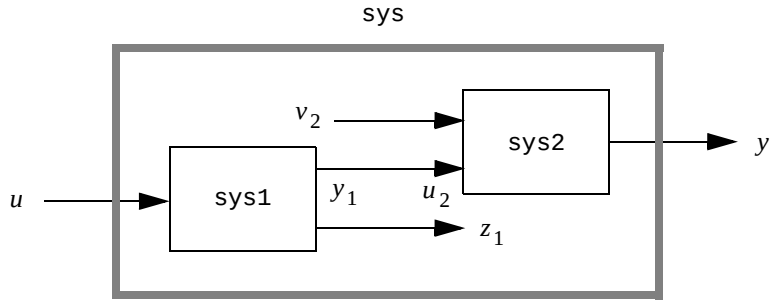


This command is equivalent to the direct multiplication

```
sys = sys2 * sys1
```

See Multiplication for details on multiplication of LTI models.

`sys = series(sys1,sys2,outputs1,inputs2)` forms the more general series connection.



The index vectors `outputs1` and `inputs2` indicate which outputs y_1 of `sys1` and which inputs u_2 of `sys2` should be connected. The resulting model `sys` has u as input and y as output.

Example

Consider a state-space system `sys1` with five inputs and four outputs and another system `sys2` with two inputs and three outputs. Connect the two systems in series by connecting outputs 2 and 4 of `sys1` with inputs 1 and 2 of `sys2`.

```
outputs1 = [2 4];
inputs2 = [1 2];
sys = series(sys1,sys2,outputs2,inputs1)
```

See Also

`append`	Append LTI systems
`feedback`	Feedback connection
`parallel`	Parallel connection

Purpose

Set or modify LTI model properties

Syntax

```
set(sys,'Property',Value)
set(sys,'Property1',Value1,'Property2',Value2,...)

set(sys,'Property')
set(sys)
```

Description

`set` is used to set or modify the properties of an LTI model (see LTI Properties for background on LTI properties). Like its Handle Graphics counterpart, `set` uses property name/property value pairs to update property values.

`set(sys,'Property',Value)` assigns the value `Value` to the property of the LTI model `sys` specified by the string `'Property'`. This string can be the full property name (for example, `'UserData'`) or any unambiguous case-insensitive abbreviation (for example, `'user'`). The specified property must be compatible with the model type. For example, if `sys` is a transfer function, `Variable` is a valid property but `StateName` is not (see Model-Specific Properties for details).

`set(sys,'Property1',Value1,'Property2',Value2,...)` sets multiple property values with a single statement. Each property name/property value pair updates one particular property.

`set(sys,'Property')` displays admissible values for the property specified by `'Property'`. See “Property Values” below for an overview of legitimate LTI property values.

`set(sys)` displays all assignable properties of `sys` and their admissible values.

Example

Consider the SISO state-space model created by

```
sys = ss(1,2,3,4);
```

You can add an input delay of 0.1 second, label the input as `torque`, reset the D matrix to zero, and store its DC gain in the `'Userdata'` property by

```
set(sys,'inputd',0.1,'inputn','torque','d',0,'user',dcgain(sys))
```

Note that `set` does not require any output argument. Check the result with `get` by typing

```
get(sys)

       a = 1
        b = 2
        c = 3
        d = 0
        e = []
        Nx = 1
        StateName = {''}
        Ts = 0
        InputDelay = 0.1
        OutputDelay = 0
        ioDelay = 0
        InputName = {'torque'}
        OutputName = {''}
        InputGroup = {0x2 cell}
        OutputGroup = {0x2 cell}
        Notes = {}
        UserData = -6
```

Property Values

The following table lists the admissible values for each LTI property. N_u and N_y denotes the number of inputs and outputs of the underlying LTI model. For K-dimensional LTI arrays, let $S_1, S_2, \ldots, S_K$ denote the array dimensions.

Table 16-17: LTI Properties

Property Name	Admissible Property Values
`Ts`	• `0` (zero) for continuous-time systems • Sample time in seconds for discrete-time systems • `-1` or `[]` for discrete systems with unspecified sample time **Note:** Resetting the sample time property does not alter the model data. Use `c2d`, `d2c`, or `d2d` for discrete/continuous and discrete/discrete conversions.
`ioDelay`	Input/Output delays specified with • Nonnegative real numbers for continuous-time models (seconds) • Integers for discrete-time models (number of sample periods) • Scalar when all I/O pairs have the same delay • N_y-by-N_u matrix to specify independent delay times for each I/O pair • Array of size N_y-by-N_u-by-S_1-by-...-by-S_n to specify different I/O delays for each model in an LTI array.
`InputDelay`	Input delays specified with • Nonnegative real numbers for continuous-time models (seconds) • Integers for discrete-time models (number of sample periods) • Scalar when $N_u = 1$ or system has uniform input delay • Vector of length N_u to specify independent delay times for each input channel • Array of size N_y-by-N_u-by-S_1-by-...-by-S_n to specify different input delays for each model in an LTI array.

Table 16-17: LTI Properties (Continued)

Property Name	Admissible Property Values
`OutputDelay`	Output delays specified with • Nonnegative real numbers for continuous-time models (seconds) • Integers for discrete-time models (number of sample periods) • Scalar when $N_y = 1$ or system has uniform output delay • Vector of length N_y to specify independent delay times for each output channel • Array of size N_y-by-N_u-by-S_1-by-...-by-S_n to specify different output delays for each model in an LTI array.
`Notes`	String, array of strings, or cell array of strings
`UserData`	Arbitrary MATLAB variable
`InputName`	• String for single-input systems, for example, `'thrust'` • Cell vector of strings for multi-input systems (with as many cells as inputs), for example, `{'u';'w'}` for a two-input system • Padded array of strings with as many rows as inputs, for example, `['rudder ' ; 'aileron']`
`OutputName`	Same as `InputName` (with "input" replaced by "output")
`InputGroup`	Cell array. See Input Groups and Output Groups.
`OutputGroup`	Same as `InputGroup`

Table 16-18: State-Space Model Properties

Property Name	Admissible Property Values
`StateName`	Same as `InputName` (with `Input` replaced by `State`)

Table 16-18: State-Space Model Properties (Continued)

Property Name	Admissible Property Values
a, b, c, d, e	Real-valued state-space matrices (multidimensional arrays, in the case of LTI arrays) with compatible dimensions for the number of states, inputs, and outputs. See The Size of LTI Array Data for SS Models.
Nx	• Scalar integer representing the number of states for single LTI models or LTI arrays with the same number of states in each model • S_1-by-…-by-S_K-dimensional array of integers when all of the models of an LTI array do not have the same number of states

Table 16-19: TF Model Properties

Property Name	Admissible Property Values
num, den	• Real-valued row vectors for the coefficients of the numerator or denominator polynomials in the SISO case. List the coefficients in *descending* powers of the variable s or z by default, and in *ascending* powers of $q = z^{-1}$ when the `Variable` property is set to `'q'` or `'z^-1'` (see note below). • N_y-by-N_u cell arrays of real-valued row vectors in the MIMO case, for example, `{[1 2];[1 0 3]}` for a two-output/one-input transfer function • N_y-by-N_u-by-S_1-by-…-by-S_K-dimensional real-valued cell arrays for MIMO LTI arrays
Variable	• String `'s'` (default) or `'p'` for continuous-time systems • String `'z'` (default), `'q'`, or `'z^-1'` for discrete-time systems

Table 16-20: ZPK Model Properties

Property Name	Admissible Property Values
z, p	• Vectors of zeros and poles (either real-valued or complex conjugate pairs of them) in SISO case • N_y-by-N_u cell arrays of vectors (entries are real-valued or in complex conjugate pairs) in MIMO case, for example, `z = {[],[-1 0]}` for a model with two inputs and one output • N_y-by-N_u-by-S_1-by-…-by-S_K-dimensional cell arrays for MIMO LTI arrays
Variable	• String `'s'` (default) or `'p'` for continuous-time systems • String `'z'` (default), `'q'`, or `'z^-1'` for discrete-time systems

Table 16-21: FRD Model Properties

Property Name	Admissible Property Values
Frequency	Real-valued vector of length N_f-by-1, where N_f is the number of frequencies
Response	• N_y-by-N_u-by-N_f-dimensional array of complex data for single LTI models • N_y-by-N_u-by-N_f-by-S_1-by-…-by-S_K-dimensional array for LTI arrays
Units	String `'rad/s'` (default), or `'Hz'`

Remark

For discrete-time transfer functions, the convention used to represent the numerator and denominator depends on the choice of variable (see the `tf` entry for details). Like `tf`, the syntax for `set` changes to remain consistent with the choice of variable. For example, if the `Variable` property is set to `'z'` (the default),

```
set(h,'num',[1 2],'den',[1 3 4])
```

produces the transfer function

$$h(z) = \frac{z+2}{z^2+3z+4}$$

However, if you change the `Variable` to `'z^-1'` (or `'q'`) by

```
set(h,'Variable','z^-1'),
```

the same command

```
set(h,'num',[1 2],'den',[1 3 4])
```

now interprets the row vectors `[1 2]` and `[1 3 4]` as the polynomials $1+2z^{-1}$ and $1+3z^{-1}+4z^{-2}$ and produces:

$$\bar{h}(z^{-1}) = \frac{1+2z^{-1}}{1+3z^{-1}+4z^{-2}} = zh(z)$$

Note Because the resulting transfer functions are different, make sure to use the convention consistent with your choice of variable.

See Also

`get`	Access/query LTI model properties
`frd`	Specify a frequency response data model
`ss`	Specify a state-space model
`tf`	Specify a transfer function
`zpk`	Specify a zero-pole-gain model

Purpose

Generate an *s*-plane grid of constant damping factors and natural frequencies

Syntax

```
sgrid
sgrid(z,wn)
```

Description

`sgrid` generates, for pole-zero and root locus plots, a grid of constant damping factors from zero to one in steps of 0.1 and natural frequencies from zero to 10 rad/sec in steps of one rad/sec, and plots the grid over the current axis. If the current axis contains a continuous *s*-plane root locus diagram or pole-zero map, `sgrid` draws the grid over the plot.

`sgrid(z,wn)` plots a grid of constant damping factor and natural frequency lines for the damping factors and natural frequencies in the vectors `z` and `wn`, respectively. If the current axis contains a continuous *s*-plane root locus diagram or pole-zero map, `sgrid(z,wn)` draws the grid over the plot.

Alternatively, you can select **Grid** from the right-click menu to generate the same s-plane grid.

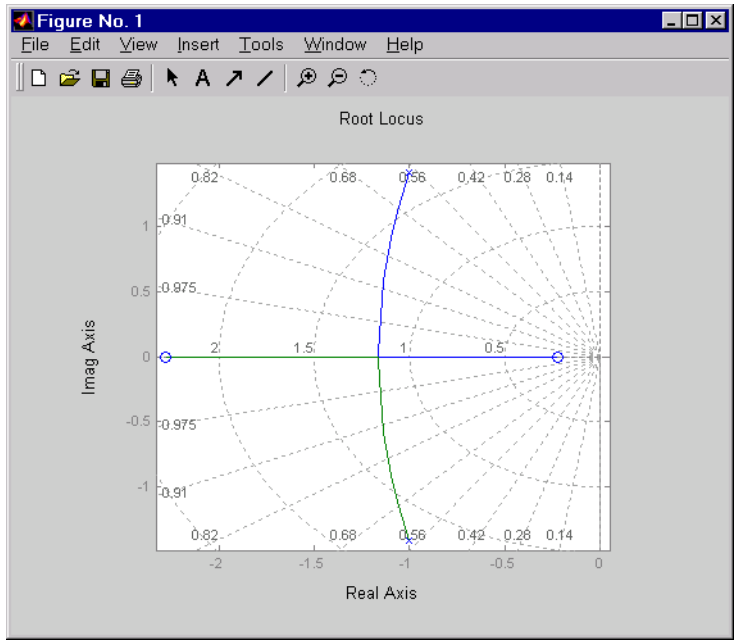
Example

Plot *s*-plane grid lines on the root locus for the following system.

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

You can do this by typing

```
H = tf([2 5 1],[1 2 3])

Transfer function:
2 s^2 + 5 s + 1
---------------
 s^2 + 2 s + 3

rlocus(H)
sgrid
```



See Also

`pzmap`	Plot pole-zero map
`rlocus`	Plot root locus
`zgrid`	Generate *z*-plane grid lines

Purpose

Singular values of the frequency response of LTI models

Syntax

```
sigma(sys)
sigma(sys,w)
sigma(sys,w,type)

sigma(sys1,sys2,...,sysN)
sigma(sys1,sys2,...,sysN,w)
sigma(sys1,sys2,...,sysN,w,type)
sigma(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[sv,w] = sigma(sys)
sv = sigma(sys,w)
```

Description

`sigma` calculates the singular values of the frequency response of an LTI model. For an FRD model, `sys`, `sigma` computes the singular values of `sys.Response` at the frequencies, `sys.frequency`. For continuous-time TF, SS, or ZPK models with transfer function $H(s)$, `sigma` computes the singular values of $H(j\omega)$ as a function of the frequency ω. For discrete-time TF, SS, or ZPK models with transfer function $H(z)$ and sample time T_s, `sigma` computes the singular values of

$$H(e^{j\omega T_s})$$

for frequencies ω between 0 and the Nyquist frequency $\omega_N = \pi / T_s$.

The singular values of the frequency response extend the Bode magnitude response for MIMO systems and are useful in robustness analysis. The singular value response of a SISO system is identical to its Bode magnitude response. When invoked without output arguments, `sigma` produces a singular value plot on the screen.

`sigma(sys)` plots the singular values of the frequency response of an arbitrary LTI model `sys`. This model can be continuous or discrete, and SISO or MIMO. The frequency points are chosen automatically based on the system poles and zeros, or from `sys.frequency` if `sys` is an FRD.

`sigma(sys,w)` explicitly specifies the frequency range or frequency points to be used for the plot. To focus on a particular frequency interval `[wmin,wmax]`, set

`w = {wmin,wmax}`. To use particular frequency points, set `w` to the corresponding vector of frequencies. Use `logspace` to generate logarithmically spaced frequency vectors. The frequencies must be specified in rad/sec.

`sigma(sys,[],type)` or `sigma(sys,w,type)` plots the following modified singular value responses:

`type = 1`	Singular values of the frequency response H^{-1}, where H is the frequency response of `sys`.
`type = 2`	Singular values of the frequency response $I + H$.
`type = 3`	Singular values of the frequency response $I + H^{-1}$.

These options are available only for square systems, that is, with the same number of inputs and outputs.

To superimpose the singular value plots of several LTI models on a single figure, use

```
sigma(sys1,sys2,...,sysN)
sigma(sys1,sys2,...,sysN,[],type)  % modified SV plot
sigma(sys1,sys2,...,sysN,w)        % specify frequency range/grid
```

The models `sys1,sys2,...,sysN` need not have the same number of inputs and outputs. Each model can be either continuous- or discrete-time. You can also specify a distinctive color, linestyle, and/or marker for each system plot with the syntax

```
sigma(sys1,'PlotStyle1',...,sysN,'PlotStyleN')
```

See `bode` for an example.

When invoked with output arguments,

```
[sv,w] = sigma(sys)
sv = sigma(sys,w)
```

return the singular values `sv` of the frequency response at the frequencies `w`. For a system with `Nu` input and `Ny` outputs, the array `sv` has `min(Nu,Ny)` rows and as many columns as frequency points (length of `w`). The singular values at the frequency `w(k)` are given by `sv(:,k)`.

Example

Plot the singular value responses of

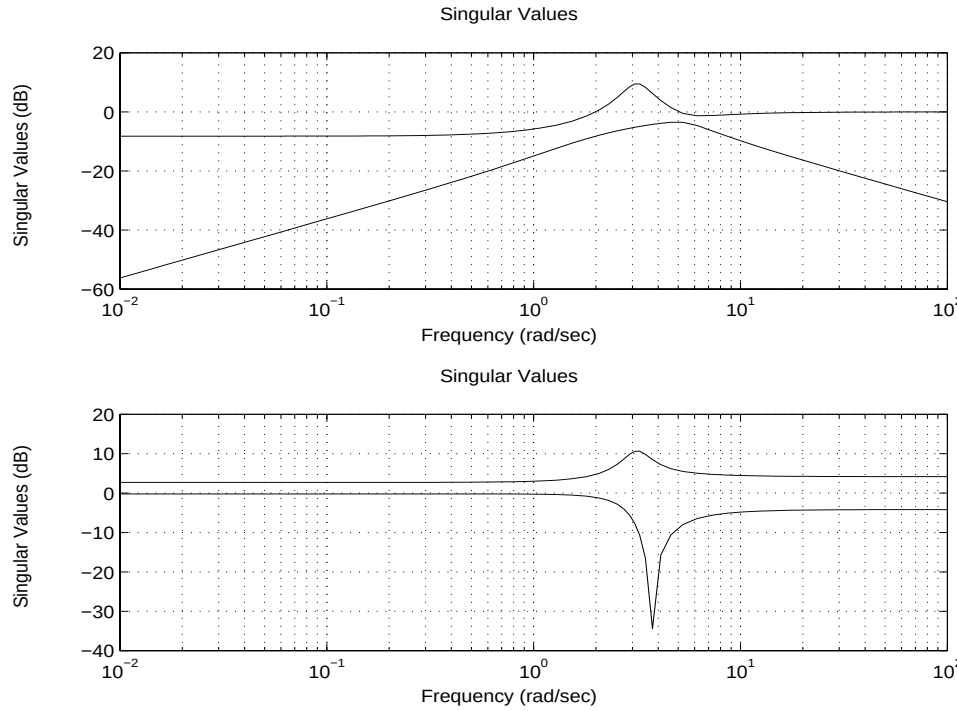
$$H(s) = \begin{bmatrix} 0 & \dfrac{3s}{s^2 + s + 10} \\ \dfrac{s+1}{s+5} & \dfrac{2}{s+6} \end{bmatrix}$$

and $I + H(s)$.

You can do this by typing

```
H = [0 tf([3 0],[1 1 10]) ; tf([1 1],[1 5]) tf(2,[1 6])]

subplot(211)
sigma(H)
subplot(212)
```

```
sigma(H,[],2)
```



Algorithm

`sigma` uses the `svd` function in MATLAB to compute the singular values of a complex matrix.

See Also

`bode`	Bode plot
`evalfr`	Response at single complex frequency
`freqresp`	Frequency response computation
`ltiview`	LTI system viewer
`nichols`	Nichols plot
`nyquist`	Nyquist plot

Purpose

Initialize the SISO Design Tool

Syntax

```
sisotool
sisotool(plant)
sisotool(plant,comp)
sisotool(views)
sisotool(views,plant,comp)
sisotool(views,plant,comp,options)
```

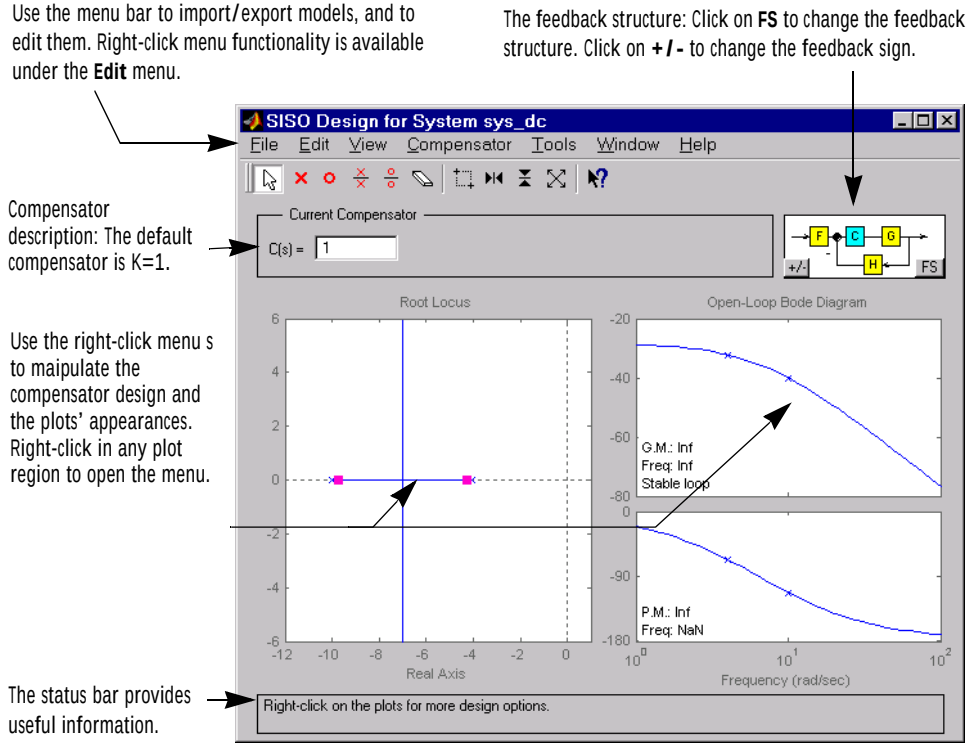
Description

When invoked without input arguments, `sisotool` opens a SISO Design GUI for interactive compensator design. This GUI allows you to design a single-input/single-output (SISO) compensator using root locus and Bode diagram techniques.

By default, the SISO Design Tool:

- Opens root locus and Bode diagrams.
- Places the compensator, **C**, in the forward path in series with the plant, **P**.
- Assumes the prefilter, **F**, and the sensor, **H**, are unity gains. If you want to include **F** and **H** in the design model, open the **Import Model** window and select the models you want to use for each. Once you specify **F**, **P**, and **H**, they are *fixed* in the feedback structure.

This picture shows the SISO Design Tool.



`sisotool(`*plant*`)` opens the SISO Design Tool with *plant* imported. If your *plant* is any SISO LTI object (created with `ss`, `tf`, or `zpk`) that exists in the MATLAB workspace, `sisotool(plant)` initializes a SISO Design Tool with the plant model **P** set to *plant* and initializes the plant model **P** to *plant* (any SISO LTI object).

`sisotool(`*plant*`,`*comp*`)` initializes both the plant model **P** to *plant* and the compenensator **C** to *comp*. Both *plant* and *comp* must be SISO LTI models.

`sisotool(views)` or `sisotool(views,`*plant*`,`*comp*`)` specifies the initial configuration of the SISO Design Tool. The argument `views` can be any of the following strings (or combination thereof):

- `'rlocus'` — Root Locus plot
- `'bode'` — Bode diagrams of the open-loop response

For example

```
sisotool('bode')
```

opens a SISO Design Tool with only the Bode Diagrams on.

`sisotool(`*`plant`*`,`*`comp`*`,options)` allows you to override the default compensator location and feedback sign by using an extra input argument `options` with the following fields:

- `options.Location = 'forward'` — Compensator in the forward loop
- `options.Location = 'feedback'` — Compensator in the feedback loop
- `options.Sign = -1` — Negative feedback
- `options.Sign = 1` — Positive feedback

You can design compensators for one of the following two feedback loop configurations.

Compensator in the Forward Path

Compensator in the Feedback Path

Figure 16-1: The SISO Design Tool Supports Two Feedback Structures.

For more details on the SISO Design Tool, see “Designing Compensators” in **Getting Started with the Control System Toolbox**.

See Also

`bode`	Select gain from the root locus plot
`ltiview`	Open an LTI Viewer
`rlocus`	Plot root locus

Purpose

Provide the output/input/array dimensions of LTI models, the model order of TF, SS, and ZPK models, and the number of frequencies of FRD models

Syntax

```
size(sys)
d = size(sys)
Ny = size(sys,1)
Nu = size(sys,2)
Sk = size(sys,2+k)
Ns = size(sys,'order')
Nf = size(sys,'frequency')
```

Description

When invoked without output arguments, `size(sys)` returns a vector of the number of outputs and inputs for a single LTI model. The lengths of the array dimensions are also included in the response to `size` when `sys` is an LTI array. `size` is the overloaded version of the MATLAB function `size` for LTI objects.

`d = size(sys)` returns:

- The row vector `d = [Ny Nu]` for a single LTI model `sys` with `Ny` outputs and `Nu` inputs
- The row vector `d = [Ny Nu S1 S2 ... Sp]` for an `S1`-by-`S2`-by-...-by-`Sp` array of LTI models with `Ny` outputs and `Nu` inputs

`Ny = size(sys,1)` returns the number of outputs of `sys`.

`Nu = size(sys,2)` returns the number of inputs of `sys`.

`Sk = size(sys,2+k)` returns the length of the `k`-th array dimension when `sys` is an LTI array.

Ns = size(sys,'order') returns the model order of a TF, SS, or ZPK model. This is the same as the number of states for state-space models. When `sys` is an LTI array, ns is the maximum order of all of the models in the LTI array.

`Nf = size(sys,'frequency')` returns the number of frequencies when `sys` is an FRD. This is the same as the length of `sys.frequency`.

Example

Consider the random LTI array of state-space models

```
sys = rss(5,3,2,3);
```

Its dimensions are obtained by typing

```
size(sys)

3x1 array of state-space models
Each model has 3 outputs, 2 inputs, and 5 states.
```

See Also

`isempty`	Test if LTI model is empty
`issiso`	Test if LTI model is SISO
`ndims`	Number of dimensions of an LTI array

sminreal

Purpose

Perform model reduction based on structure

Syntax

```
msys = sminreal(sys)
```

Description

`msys = sminreal(sys)` eliminates the states of the state-space model `sys` that don't affect the input/output response. All of the states of the resulting state-space model `msys` are also states of `sys` and the input/output response of `msys` is equivalent to that of `sys`.

`sminreal` eliminates only structurally non minimal states, i.e., states that can be discarded by looking only at hard zero entries in the *A*, *B*, and *C* matrices. Such structurally nonminimal states arise, for example, when linearizing a Simulink model that includes some unconnected state-space or transfer function blocks.

Remark

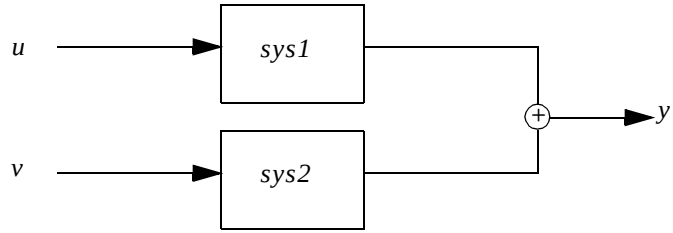
The model resulting from `sminreal(sys)` is not necessarily minimal, and may have a higher order than one resulting from `minreal(sys)`. However, `sminreal(sys)` retains the state structure of `sys`, while, in general, `minreal(sys)` does not.

Example

Suppose you concatenate two SS models, `sys1` and `sys2`.

```
sys = [sys1,sys2];
```

This operation is depicted in the diagram below.



If you extract the subsystem `sys1` from `sys`, with

```
sys(1,1)
```

all of the states of sys, including those of sys2 are retained. To eliminate the unobservable states from sys2, while retaining the states of sys1, type

```
sminreal(sys(1,1))
```

See Also

minreal	Model reduction by removing unobservable/ uncontrollable states or cancelling pole/zero pairs

Purpose

Specify state-space models or convert an LTI model to state space

Syntax

```
sys = ss(a,b,c,d)
sys = ss(a,b,c,d,Ts)
sys = ss(d)
sys = ss(a,b,c,d,ltisys)

sys = ss(a,b,c,d,'Property1',Value1,...,'PropertyN',ValueN)
sys = ss(a,b,c,d,Ts,'Property1',Value1,...,'PropertyN',ValueN)

sys_ss = ss(sys)
sys_ss = ss(sys,'minimal')
```

Description

ss is used to create real-valued state-space models (SS objects) or to convert transfer function or zero-pole-gain models to state space.

Creation of State-Space Models

`sys = ss(a,b,c,d)` creates the continuous-time state-space model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

For a model with `Nx` states, `Ny` outputs, and `Nu` inputs:

- `a` is an `Nx`-by-`Nx` real-valued matrix.
- `b` is an `Nx`-by-`Nu` real-valued matrix.
- `c` is an `Ny`-by-`Nx` real-valued matrix.
- `d` is an `Ny`-by-`Nu` real-valued matrix.

The output `sys` is an SS model that stores the model data (see “State-Space Models” on page 2-14). If $D = 0$, you can simply set `d` to the scalar `0` (zero), regardless of the dimension.

`sys = ss(a,b,c,d,Ts)` creates the discrete-time model

$$x[n+1] = Ax[n] + Bu[n]$$
$$y[n] = Cx[n] + Du[n]$$

with sample time `Ts` (in seconds). Set `Ts = -1` or `Ts = []` to leave the sample time unspecified.

`sys = ss(d)` specifies a static gain matrix D and is equivalent to

```
sys = ss([],[],[],d)
```

`sys = ss(a,b,c,d,ltisys)` creates a state-space model with generic LTI properties inherited from the LTI model `ltisys` (including the sample time). See "Generic Properties" on page 2-26 for an overview of generic LTI properties.

See "Building LTI Arrays" on page 4-12 for information on how to build arrays of state-space models.

Any of the previous syntaxes can be followed by property name/property value pairs.

```
'PropertyName',PropertyValue
```

Each pair specifies a particular LTI property of the model, for example, the input names or some notes on the model history. See the `set` entry and the example below for details. Note that

```
sys = ss(a,b,c,d,'Property1',Value1,...,'PropertyN',ValueN)
```

is equivalent to the sequence of commands.

```
sys = ss(a,b,c,d)
set(sys,'Property1',Value1,...,'PropertyN',ValueN)
```

Conversion to State Space

`sys_ss = ss(sys)` converts an arbitrary TF or ZPK model `sys` to state space. The output `sys_ss` is an equivalent state-space model (SS object). This operation is known as *state-space realization*.

`sys_ss = ss(sys,'minimal')` produces a state-space realization with no uncontrollable or unobservable states. This is equivalent to `sys_ss = minreal(ss(sys))`.

Examples

Example 1

The command

```
sys = ss(A,B,C,D,0.05,'statename',{'position' 'velocity'},...
                      'inputname','force',...
                      'notes','Created 10/15/96')
```

creates a discrete-time model with matrices A, B, C, D and sample time 0.05 second. This model has two states labeled `position` and `velocity`, and one input labeled `force` (the dimensions of A, B, C, D should be consistent with these numbers of states and inputs). Finally, a note is attached with the date of creation of the model.

Example 2

Compute a state-space realization of the transfer function

$$H(s) = \begin{bmatrix} \dfrac{s+1}{s^3+3s^2+3s+2} \\ \dfrac{s^2+3}{s^2+s+1} \end{bmatrix}$$

by typing

```
H = [tf([1 1],[1 3 3 2]) ; tf([1 0 3],[1 1 1])];
sys = ss(H);
size(sys)

State-space model with 2 outputs, 1 input, and 5 states.
```

Note that the number of states is equal to the cumulative order of the SISO entries of $H(s)$.

To obtain a minimal realization of $H(s)$, type

```
sys = ss(H,'min');
size(sys)

State-space model with 2 outputs, 1 input, and 3 states.
```

The resulting state-space model order has order three, the minimum number of states needed to represent $H(s)$. This can be seen directly by factoring $H(s)$ as the product of a first order system with a second order one.

$$H(s) = \begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{s+1}{s^2+s+1} \\ \frac{s^2+3}{s^2+s+1} \end{bmatrix}$$

See Also

`dss`	Specify descriptor state-space models.
`frd`	Specify FRD models or convert to an FRD.
`get`	Get properties of LTI models.
`set`	Set properties of LTI models.
`ssdata`	Retrieve the A, B, C, D matrices of state-space model.
`tf`	Specify transfer functions or convert to TF.
`zpk`	Specify zero-pole-gain models or convert to ZPK.

ss2ss

Purpose

State coordinate transformation for state-space models

Syntax

```
sysT = ss2ss(sys,T)
```

Description

Given a state-space model sys with equations

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

(or their discrete-time counterpart), ss2ss performs the similarity transformation $\bar{x} = Tx$ on the state vector x and produces the equivalent state-space model sysT with equations.

$$\dot{\bar{x}} = TAT^{-1}\bar{x} + TBu$$
$$y = CT^{-1}\bar{x} + Du$$

sysT = ss2ss(sys,T) returns the transformed state-space model sysT given sys and the state coordinate transformation T. The model sys must be in state-space form and the matrix T must be invertible. ss2ss is applicable to both continuous- and discrete-time models.

Example

Perform a similarity transform to improve the conditioning of the A matrix.

```
T = balance(sys.a)
sysb = ss2ss(sys,inv(T))
```

See ssbal for a more direct approach.

See Also

balreal	Grammian-based I/O balancing
canon	Canonical state-space realizations
ssbal	Balancing of state-space models using diagonal similarity transformations

Purpose

Balance state-space models using a diagonal similarity transformation

Syntax

```
[sysb,T] = ssbal(sys)
[sysb,T] = ssbal(sys,condT)
```

Description

Given a state-space model sys with matrices (A, B, C, D),

```
[sysb,T] = ssbal(sys)
```

computes a diagonal similarity transformation T and a scalar α such that

$$\begin{bmatrix} TAT^{-1} & TB/\alpha \\ \alpha CT^{-1} & 0 \end{bmatrix}$$

has approximately equal row and column norms. `ssbal` returns the balanced model sysb with matrices

$$(TAT^{-1}, TB/\alpha, \alpha CT^{-1}, D)$$

and the state transformation $\bar{x} = Tx$ where $\bar{x}$ is the new state.

`[sysb,T] = ssbal(sys,condT)` specifies an upper bound `condT` on the condition number of T. Since balancing with ill-conditioned T can inadvertently magnify rounding errors, `condT` gives control over the worst-case roundoff amplification factor. The default value is `condT=Inf`.

`ssbal` returns an error if the state-space model sys has varying state dimensions.

Example

Consider the continuous-time state-space model with the following data.

$$A = \begin{bmatrix} 1 & 10^4 & 10^2 \\ 0 & 10^2 & 10^5 \\ 10 & 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0.1 & 10 & 100 \end{bmatrix}$$

```
a = [1 1e4 1e2;0 1e2 1e5;10 1 0];
b = [1;1;1];
c = [0.1 10 1e2];
sys = ss(a,b,c,0)
```

Balance this model with `ssbal` by typing

```
ssbal(sys)
```

```
a =
                    x1          x2          x3
           x1        1        2500     0.39063
           x2        0         100      1562.5
           x3     2560          64           0


b =
                    u1
           x1     0.125
           x2       0.5
           x3        32


c =
                    x1          x2          x3
           y1       0.8          20       3.125


d =
                    u1
           y1        0

Continuous-time system.
```

Direct inspection shows that the range of numerical values has been compressed by a factor 100 and that the B and C matrices now have nearly equal norms.

Algorithm

`ssbal` uses the MATLAB function `balance` to compute T and α.

See Also

`balreal`	Grammian-based I/O balancing
`ss2ss`	State coordinate transformation

Purpose

Quick access to state-space model data

Syntax

```
[a,b,c,d] = ssdata(sys)
[a,b,c,d,Ts] = ssdata(sys)
```

Description

`[a,b,c,d] = ssdata(sys)` extracts the matrix (or multidimensional array) data (A, B, C, D) from the state-space model (LTI array) `sys`. If `sys` is a transfer function or zero-pole-gain model (LTI array), it is first converted to state space. See Table 11-16, "State-Space Model Properties," on page 11-195 for more information on the format of state-space model data.

`[a,b,c,d,Ts] = ssdata(sys)` also returns the sample time `Ts`.

You can access the remaining LTI properties of `sys` with `get` or by direct referencing, for example,

```
sys.statename
```

See Also

`dssdata`	Quick access to descriptor state-space data
`get`	Get properties of LTI models
`set`	Set model properties
`ss`	Specify state-space models
`tfdata`	Quick access to transfer function data
`zpkdata`	Quick access to zero-pole-gain data

stack

Purpose

Build an LTI array by stacking LTI models or LTI arrays along array dimensions of an LTI array

Syntax

```
sys = stack(arraydim,sys1,sys2,...)
```

Description

`sys = stack(arraydim,sys1,sys2,...)` produces an array of LTI models `sys` by stacking (concatenating) the LTI models (or LTI arrays) `sys1,sys2,...` along the array dimension `arraydim`. All models must have the same number of inputs and outputs (the same I/O dimensions). The I/O dimensions are not counted in the array dimensions. See "Dimensions, Size, and Shape of an LTI Array" on page 4-7, and "Building LTI Arrays Using the stack Function" on page 4-15 for more information.

Example

If `sys1` and `sys2` are two LTI models with the same I/O dimensions:

- `stack(1,sys1,sys2)` produces a 2-by-1 LTI array.
- `stack(2,sys1,sys2)` produces a 1-by-2 LTI array.
- `stack(3,sys1,sys2)` produces a 1-by-1-by-2 LTI array.

Purpose

Step response of LTI systems

Syntax

```
step(sys)
step(sys,t)

step(sys1,sys2,...,sysN)
step(sys1,sys2,...,sysN,t)
step(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[y,t,x] = step(sys)
```

Description

`step` calculates the unit step response of a linear system. Zero initial state is assumed in the state-space case. When invoked with no output arguments, this function plots the step response on the screen.

`step(sys)` plots the step response of an arbitrary LTI model `sys`. This model can be continuous or discrete, and SISO or MIMO. The step response of multi-input systems is the collection of step responses for each input channel. The duration of simulation is determined automatically based on the system poles and zeros.

`step(sys,t)` sets the simulation horizon explicitly. You can specify either a final time `t = Tfinal` (in seconds), or a vector of evenly spaced time samples of the form

```
t = 0:dt:Tfinal
```

For discrete systems, the spacing `dt` should match the sample period. For continuous systems, `dt` becomes the sample time of the discretized simulation model (see "Algorithm"), so make sure to choose `dt` small enough to capture transient phenomena.

To plot the step responses of several LTI models `sys1`,..., `sysN` on a single figure, use

```
step(sys1,sys2,...,sysN)
step(sys1,sys2,...,sysN,t)
```

All systems must have the same number of inputs and outputs but may otherwise be a mix of continuous- and discrete-time systems. This syntax is useful to compare the step responses of multiple systems.

You can also specify a distinctive color, linestyle, and/or marker for each system. For example,

```
step(sys1,'y:',sys2,'g--')
```

plots the step response of `sys1` with a dotted yellow line and the step response of `sys2` with a green dashed line.

When invoked with output arguments,

```
[y,t] = step(sys)
[y,t,x] = step(sys)      % for state-space models only
y = step(sys,t)
```

return the output response `y`, the time vector `t` used for simulation, and the state trajectories `x` (for state-space models only). No plot is drawn on the screen. For single-input systems, `y` has as many rows as time samples (length of `t`), and as many columns as outputs. In the multi-input case, the step responses of each input channel are stacked up along the third dimension of `y`. The dimensions of `y` are then

(length of `t`) × (number of outputs) × (number of inputs)

and `y(:,:,j)` gives the response to a unit step command injected in the `j`th input channel. Similarly, the dimensions of `x` are

(length of `t`) × (number of states) × (number of inputs)

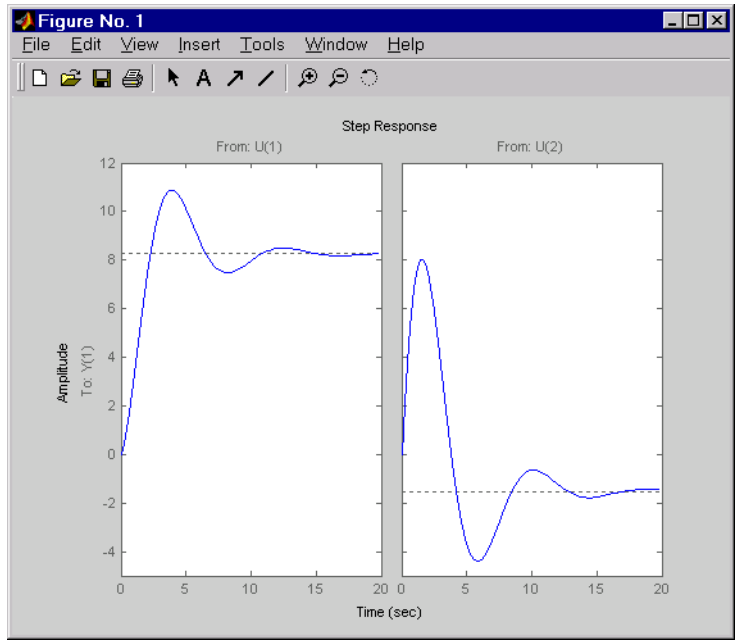
Example

Plot the step response of the following second-order state-space model.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-0.5572 & -0.7814\\ 0.7814 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$y = \begin{bmatrix}1.9691 & 6.4493\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

```
a = [-0.5572   -0.7814;0.7814  0];
b = [1 -1;0 2];
c = [1.9691  6.4493];
sys = ss(a,b,c,0);
step(sys)
```



The left plot shows the step response of the first input channel, and the right plot shows the step response of the second input channel.

Algorithm

Continuous-time models are converted to state space and discretized using zero-order hold on the inputs. The sampling period is chosen automatically based on the system dynamics, except when a time vector `t = 0:dt:Tf` is supplied (`dt` is then used as sampling period).

See Also

`impulse`	Impulse response
`initial`	Free response to initial condition
`lsim`	Simulate response to arbitrary inputs
`ltiview`	LTI system viewer

Purpose

Specify transfer functions or convert LTI model to transfer function form

Syntax

```
sys = tf(num,den)
sys = tf(num,den,Ts)
sys = tf(M)
sys = tf(num,den,ltisys)

sys = tf(num,den,'Property1',Value1,...,'PropertyN',ValueN)
sys = tf(num,den,Ts,'Property1',Value1,...,'PropertyN',ValueN)

sys = tf('s')
sys = tf('z')

tfsys = tf(sys)
tfsys = tf(sys,'inv')   % for state-space sys only
```

Description

`tf` is used to create real-valued transfer function models (TF objects) or to convert state-space or zero-pole-gain models to transfer function form.

Creation of Transfer Functions

`sys = tf(num,den)` creates a continuous-time transfer function with numerator(s) and denominator(s) specified by `num` and `den`. The output `sys` is a TF object storing the transfer function data (see "Transfer Function Models" on page 2-8).

In the SISO case, `num` and `den` are the real-valued row vectors of numerator and denominator coefficients ordered in *descending* powers of s. These two vectors need not have equal length and the transfer function need not be proper. For example, `h = tf([1 0],1)` specifies the pure derivative $h(s) = s$.

To create MIMO transfer functions, specify the numerator and denominator of each SISO entry. In this case:

- `num` and `den` are cell arrays of row vectors with as many rows as outputs and as many columns as inputs.
- The row vectors `num{i,j}` and `den{i,j}` specify the numerator and denominator of the transfer function from input `j` to output `i` (with the SISO convention).

If all SISO entries of a MIMO transfer function have the same denominator, you can set den to the row vector representation of this common denominator. See "Examples" for more details.

`sys = tf(num,den,Ts)` creates a discrete-time transfer function with sample time `Ts` (in seconds). Set `Ts = -1` or `Ts = []` to leave the sample time unspecified. The input arguments `num` and `den` are as in the continuous-time case and must list the numerator and denominator coefficients in *descending* powers of `z`.

`sys = tf(M)` creates a static gain `M` (scalar or matrix).

`sys = tf(num,den,ltisys)` creates a transfer function with generic LTI properties inherited from the LTI model `ltisys` (including the sample time). See "Generic Properties" on page 2-26 for an overview of generic LTI properties.

There are several ways to create LTI arrays of transfer functions. To create arrays of SISO or MIMO TF models, either specify the numerator and denominator of each SISO entry using multidimensional cell arrays, or use a `for` loop to successively assign each TF model in the array. See "Building LTI Arrays" on page 4-12 for more information.

Any of the previous syntaxes can be followed by property name/property value pairs

```
'Property',Value
```

Each pair specifies a particular LTI property of the model, for example, the input names or the transfer function variable. See `set` entry and the example below for details. Note that

```
sys = tf(num,den,'Property1',Value1,...,'PropertyN',ValueN)
```

is a shortcut for

```
sys = tf(num,den)
set(sys,'Property1',Value1,...,'PropertyN',ValueN)
```

Transfer Functions as Rational Expressions in s or z

You can also use real-valued rational expressions to create a TF model. To do so, first type either:

- `s = tf('s')` to specify a TF model using a rational function in the Laplace variable, `s`.
- `z = tf('z',Ts)` to specify a TF model with sample time `Ts` using a rational function in the discrete-time variable, `z`.

Once you specify either of these variables, you can specify TF models directly as rational expressions in the variable `s` or `z` by entering your transfer function as a rational expression in either `s` or `z`.

Conversion to Transfer Function

`tfsys = tf(sys)` converts an arbitrary SS or ZPK LTI model `sys` to transfer function form. The output `tfsys` (TF object) is the transfer function of `sys`. By default, `tf` uses `zero` to compute the numerators when converting a state-space model to transfer function form. Alternatively,

```
tfsys = tf(sys,'inv')
```

uses inversion formulas for state-space models to derive the numerators. This algorithm is faster but less accurate for high-order models with low gain at $s = 0$.

Examples

Example 1

Create the two-output/one-input transfer function

$$H(p) = \begin{bmatrix} \dfrac{p+1}{p^2+2p+2} \\ \dfrac{1}{p} \end{bmatrix}$$

with input `current` and outputs `torque` and `ang velocity`.

To do this, type

```
num = {[1 1] ; 1}
den = {[1 2 2] ; [1 0]}
H = tf(num,den,'inputn','current',...
                'outputn',{'torque' 'ang. velocity'},...
                'variable','p')
```

```
Transfer function from input "current" to output...
              p + 1
 torque:  -------------
          p^2 + 2 p + 2

                 1
 ang. velocity:  -
                 p
```

Note how setting the `'variable'` property to `'p'` causes the result to be displayed as a transfer function of the variable p.

Example 2

To use a rational expression to create a SISO TF model, type

```
s = tf('s');
H = s/(s^2 + 2*s +10);
```

This produces the same transfer function as

```
h = tf([1 0],[1 2 10]);
```

Example 3

Specify the discrete MIMO transfer function

$$H(z) = \begin{bmatrix} \frac{1}{z+0.3} & \frac{z}{z+0.3} \\ \frac{-z+2}{z+0.3} & \frac{3}{z+0.3} \end{bmatrix}$$

with common denominator $d(z) = z + 0.3$ and sample time of 0.2 seconds.

```
nums = {1 [1 0];[-1 2] 3}
Ts = 0.2
H = tf(nums,[1 0.3],Ts)     % Note: row vector for common den. d(z)
```

Example 4

Compute the transfer function of the state-space model with the following data.

$$A = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

To do this, type

```
sys = ss([-2 -1;1 -2],[1 1;2 -1],[1 0],[0 1])
tf(sys)
```

```
Transfer function from input 1 to output:
      s
-------------
s^2 + 4 s + 5

Transfer function from input 2 to output:
s^2 + 5 s + 8
-------------
s^2 + 4 s + 5
```

Example 5

You can use a `for` loop to specify a 10-by-1 array of SISO TF models.

```
s = tf('s')
H = tf(zeros(1,1,10));
   for k=1:10,
       H(:,:,k) = k/(s^2+s+k);
   end
```

The first statement pre-allocates the TF array and fills it with zero transfer functions.

Discrete-Time Conventions

The control and digital signal processing (DSP) communities tend to use different conventions to specify discrete transfer functions. Most control engineers use the z variable and order the numerator and denominator terms in descending powers of z, for example,

$$h(z) = \frac{z^2}{z^2 + 2z + 3}$$

The polynomials z^2 and $z^2 + 2z + 3$ are then specified by the row vectors `[1 0 0]` and `[1 2 3]`, respectively. By contrast, DSP engineers prefer to write this transfer function as

$$h(z^{-1}) = \frac{1}{1 + 2z^{-1} + 3z^{-2}}$$

and specify its numerator as `1` (instead of `[1 0 0]`) and its denominator as `[1 2 3]`.

`tf` switches convention based on your choice of variable (value of the `'Variable'` property).

Variable	Convention
`'z'` (default)	Use the row vector `[ak ... a1 a0]` to specify the polynomial $a_k z^k + ... + a_1 z + a_0$ (coefficients ordered in *descending* powers of z).
`'z^-1'`, `'q'`	Use the row vector `[b0 b1 ... bk]` to specify the polynomial $b_0 + b_1 z^{-1} + ... + b_k z^{-k}$ (coefficients in *ascending* powers of z^{-1} or q).

For example,

```
g = tf([1 1],[1 2 3],0.1)
```

specifies the discrete transfer function

$$g(z) = \frac{z + 1}{z^2 + 2z + 3}$$

because z is the default variable. In contrast,

```
h = tf([1 1],[1 2 3],0.1,'variable','z^-1')
```

uses the DSP convention and creates

$$h(z^{-1}) = \frac{1 + z^{-1}}{1 + 2z^{-1} + 3z^{-2}} = zg(z)$$

See also `filt` for direct specification of discrete transfer functions using the DSP convention.

Note that `tf` stores data so that the numerator and denominator lengths are made equal. Specifically, `tf` stores the values

```
num = [0 1 1]; den = [1 2 3]
```

for `g` (the numerator is padded with zeros on the left) and the values

```
num = [1 1 0]; den = [1 2 3]
```

for `h` (the numerator is padded with zeros on the right).

Algorithm

`tf` uses the MATLAB function `poly` to convert zero-pole-gain models, and the functions `zero` and `pole` to convert state-space models.

See Also

`filt`	Specify discrete transfer functions in DSP format
`frd`	Specify a frequency response data model
`get`	Get properties of LTI models
`set`	Set properties of LTI models
`ss`	Specify state-space models or convert to state space
`tfdata`	Retrieve transfer function data
`zpk`	Specify zero-pole-gain models or convert to ZPK

Purpose

Quick access to transfer function data

Syntax

```
[num,den] = tfdata(sys)
[num,den] = tfdata(sys,'v')
[num,den,Ts] = tfdata(sys)
```

Description

`[num,den] = tfdata(sys)` returns the numerator(s) and denominator(s) of the transfer function for the TF, SS or ZPK model (or LTI array of TF, SS or ZPK models) `sys`. For single LTI models, the outputs `num` and `den` of `tfdata` are cell arrays with the following characteristics:

- `num` and `den` have as many rows as outputs and as many columns as inputs.
- The `(i,j)` entries `num{i,j}` and `den{i,j}` are row vectors specifying the numerator and denominator coefficients of the transfer function from input `j` to output `i`. These coefficients are ordered in *descending* powers of s or z.

For arrays `sys` of LTI models, `num` and `den` are multidimensional cell arrays with the same sizes as `sys`.

If `sys` is a state-space or zero-pole-gain model, it is first converted to transfer function form using `tf`. See Table 11-15, “LTI Properties,” on page 11-194 for more information on the format of transfer function model data.

For SISO transfer functions, the syntax

```
[num,den] = tfdata(sys,'v')
```

forces `tfdata` to return the numerator and denominator directly as row vectors rather than as cell arrays (see example below).

`[num,den,Ts] = tfdata(sys)` also returns the sample time `Ts`.

You can access the remaining LTI properties of `sys` with `get` or by direct referencing, for example,

```
sys.Ts
sys.variable
```

Example

Given the SISO transfer function

```
h = tf([1 1],[1 2 5])
```

you can extract the numerator and denominator coefficients by typing

```
[num,den] = tfdata(h,'v')

num =
     0     1     1

den =
     1     2     5
```

This syntax returns two row vectors.

If you turn `h` into a MIMO transfer function by typing

```
H = [h ; tf(1,[1 1])]
```

the command

```
[num,den] = tfdata(H)
```

now returns two cell arrays with the numerator/denominator data for each SISO entry. Use `celldisp` to visualize this data. Type

```
celldisp(num)
```

and MATLAB returns the numerator vectors of the entries of `H`.

```
num{1} =
     0     1     1

num{2} =
     0     1
```

Similarly, for the denominators, type

```
celldisp(den)

den{1} =
     1     2     5

den{2} =
     1     1
```

See Also

`get`	Get properties of LTI models
`ssdata`	Quick access to state-space data

`tf`	Specify transfer functions
`zpkdata`	Quick access to zero-pole-gain data

totaldelay

Purpose

Return the total combined I/O delays for an LTI model

Syntax

```
td = totaldelay(sys)
```

Description

`td = totaldelay(sys)` returns the total combined I/O delays for an LTI model `sys`. The matrix `td` combines contributions from the `InputDelay`, `OutputDelay`, and `ioDelay` properties, (see `set` on page 11-192 or type `ltiprops` for details on these properties).

Delays are expressed in seconds for continuous-time models, and as integer multiples of the sample period for discrete-time models. To obtain the delay times in seconds, multiply `td` by the sample time `sys.Ts`.

Example

```
sys = tf(1,[1 0]);  % TF of 1/s
sys.inputd = 2;                    % 2 sec input delay
sys.outputd = 1.5;                 % 1.5 sec output delay
td = totaldelay(sys)

td =
    3.5000
```

The resulting I/O map is

$$e^{-2s} \times \frac{1}{s} e^{-1.5s} = e^{-3.5s} \frac{1}{s}$$

This is equivalent to assigning an I/O delay of 3.5 seconds to the original model `sys`.

See Also

`delay2z`	Change transfer functions of discrete-time LTI models with delays to rational functions or absorbs FRD delays into the frequency response phase information
`hasdelay`	True for LTI models with delays

Purpose

Transmission zeros of LTI models

Syntax

```
z = zero(sys)
[z,gain] = zero(sys)
```

Description

`zero` computes the zeros of SISO systems and the transmission zeros of MIMO systems. For a MIMO system with matrices (A, B, C, D), the transmission zeros are the complex values λ for which the normal rank of

$$\begin{bmatrix} A - \lambda I & B \\ C & D \end{bmatrix}$$

drops.

`z = zero(sys)` returns the (transmission) zeros of the LTI model `sys` as a column vector.

`[z,gain] = zero(sys)` also returns the gain (in the zero-pole-gain sense) if `sys` is a SISO system.

Algorithm

The transmission zeros are computed using the algorithm in [1].

See Also

`pole`	Compute the poles of an LTI model
`pzmap`	Compute the pole-zero map

References

[1] Emami-Naeini, A. and P. Van Dooren, "Computation of Zeros of Linear Multivariable Systems," *Automatica*, 18 (1982), pp. 415–430.

zgrid

Purpose

Generate a *z*-plane grid of constant damping factors and natural frequencies

Syntax

```
zgrid
zgrid(z,wn)
```

Description

`zgrid` generates, for root locus and pole-zero maps, a grid of constant damping factors from zero to one in steps of 0.1 and natural frequencies from zero to π in steps of $\pi/10$, and plots the grid over the current axis. If the current axis contains a discrete *z*-plane root locus diagram or pole-zero map, `zgrid` draws the grid over the plot without altering the current axis limits.

`zgrid(z,wn)` plots a grid of constant damping factor and natural frequency lines for the damping factors and normalized natural frequencies in the vectors `z` and `wn`, respectively. If the current axis contains a discrete *z*-plane root locus diagram or pole-zero map, `zgrid(z,wn)` draws the grid over the plot. The frequency lines for unnormalized (true) frequencies can be plotted using

```
zgrid(z,wn/Ts)
```

where `Ts` is the sample time.

`zgrid([],[])` draws the unit circle.

Alternatively, you can select **Grid** from the right-click menu to generate the same z-plane grid.

Example

Plot *z*-plane grid lines on the root locus for the system

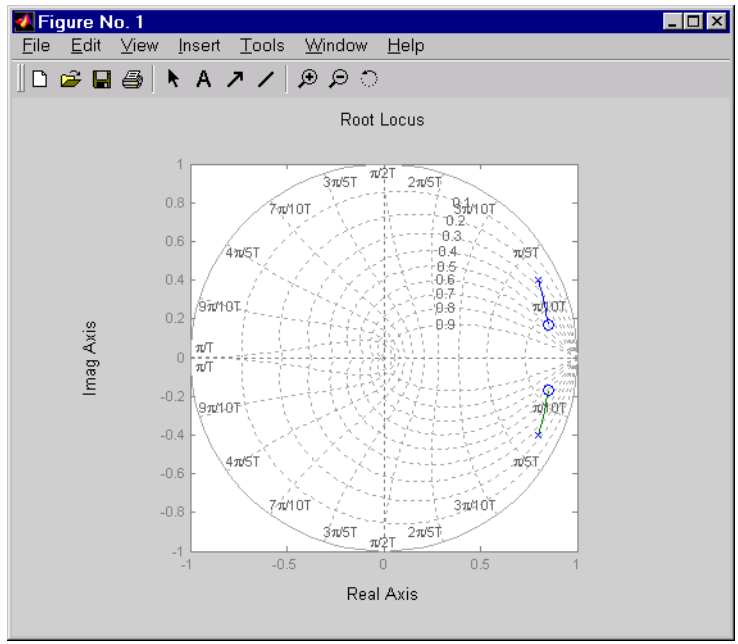
$$H(z) = \frac{2z^2 - 3.4z + 1.5}{z^2 - 1.6z + 0.8}$$

by typing

```
H = tf([2 -3.4 1.5],[1 -1.6 0.8],-1)

Transfer function:
2 z^2 - 3.4 z + 1.5
-------------------
 z^2 - 1.6 z + 0.8

Sampling time: unspecified
```

To see the z-plane grid on the root locus plot, type

```
rlocus(H)
zgrid
axis('square')
```



See Also

`pzmap`	Plot pole-zero map of LTI systems
`rlocus`	Plot root locus
`sgrid`	Generate *s*-plane grid lines

zpk

Purpose

Specify zero-pole-gain models or convert LTI model to zero-pole-gain form

Syntax

```
sys = zpk(z,p,k)
sys = zpk(z,p,k,Ts)
sys = zpk(M)
sys = zpk(z,p,k,ltisys)

sys = zpk(z,p,k,'Property1',Value1,...,'PropertyN',ValueN)
sys = zpk(z,p,k,Ts,'Property1',Value1,...,'PropertyN',ValueN)

sys = zpk('s')
sys = zpk('z')

zsys = zpk(sys)
zsys = zpk(sys,'inv')   % for state-space sys only
```

Description

zpk is used to create zero-pole-gain models (ZPK objects) or to convert TF or SS models to zero-pole-gain form.

Creation of Zero-Pole-Gain Models

`sys = zpk(z,p,k)` creates a continuous-time zero-pole-gain model with zeros `z`, poles `p`, and gain(s) `k`. The output `sys` is a ZPK object storing the model data (see "LTI Objects" on page 2-3).

In the SISO case, `z` and `p` are the vectors of real or complex conjugate zeros and poles, and `k` is the real-valued scalar gain.

$$h(s) = k\,\frac{(s-z(1))(s-z(2))\ldots(s-z(m))}{(s-p(1))(s-p(2))\ldots(s-p(n))}$$

Set `z` or `p` to `[]` for systems without zeros or poles. These two vectors need not have equal length and the model need not be proper (that is, have an excess of poles).

You can also use rational expressions to create a ZPK model. To do so, use either:

- `s = zpk('s')` to specify a ZPK model from a rational transfer function of the Laplace variable, `s`.

- `z = zpk('z',Ts)` to specify a ZPK model with sample time `Ts` from a rational transfer function of the discrete-time variable, `z`.

Once you specify either of these variables, you can specify ZPK models directly as real-valued rational expressions in the variable `s` or `z`.

To create a MIMO zero-pole-gain model, specify the zeros, poles, and gain of each SISO entry of this model. In this case:

- `z` and `p` are cell arrays of vectors with as many rows as outputs and as many columns as inputs, and `k` is a matrix with as many rows as outputs and as many columns as inputs.
- The vectors `z{i,j}` and `p{i,j}` specify the zeros and poles of the transfer function from input `j` to output `i`.
- `k(i,j)` specifies the (scalar) gain of the transfer function from input `j` to output `i`.

See below for a MIMO example.

`sys = zpk(z,p,k,Ts)` creates a discrete-time zero-pole-gain model with sample time `Ts` (in seconds). Set `Ts = -1` or `Ts = []` to leave the sample time unspecified. The input arguments `z`, `p`, `k` are as in the continuous-time case.

`sys = zpk(M)` specifies a static gain `M`.

`sys = zpk(z,p,k,ltisys)` creates a zero-pole-gain model with generic LTI properties inherited from the LTI model `ltisys` (including the sample time). See “Generic Properties” on page 2-26 for an overview of generic LTI properties.

To create an array of ZPK models, use a `for` loop, or use multidimensional cell arrays for `z` and `p`, and a multidimensional array for `k`.

Any of the previous syntaxes can be followed by property name/property value pairs.

```
'PropertyName',PropertyValue
```

Each pair specifies a particular LTI property of the model, for example, the input names or the input delay time. See `set` entry and the example below for details. Note that

```
sys = zpk(z,p,k,'Property1',Value1,...,'PropertyN',ValueN)
```

is a shortcut for the following sequence of commands.

```
sys = zpk(z,p,k)
set(sys,'Property1',Value1,...,'PropertyN',ValueN)
```

Zero-Pole-Gain Models as Rational Expressions in s or z

You can also use rational expressions to create a ZPK model. To do so, first type either:

- `s = zpk('s')` to specify a ZPK model using a rational function in the Laplace variable, `s`.
- `z = zpk('z',Ts)` to specify a ZPK model with sample time `Ts` using a rational function in the discrete-time variable, `z`.

Once you specify either of these variables, you can specify ZPK models directly as rational expressions in the variable `s` or `z` by entering your transfer function as a rational expression in either `s` or `z`.

Conversion to Zero-Pole-Gain Form

`zsys = zpk(sys)` converts an arbitrary LTI model `sys` to zero-pole-gain form. The output `zsys` is a ZPK object. By default, `zpk` uses `zero` to compute the zeros when converting from state-space to zero-pole-gain. Alternatively,

```
zsys = zpk(sys,'inv')
```

uses inversion formulas for state-space models to compute the zeros. This algorithm is faster but less accurate for high-order models with low gain at $s = 0$.

Variable Selection

As for transfer functions, you can specify which variable to use in the display of zero-pole-gain models. Available choices include s (default) and p for continuous-time models, and z (default), z^{-1}, or $q = z^{-1}$ for discrete-time models. Reassign the `'Variable'` property to override the defaults. Changing the variable affects only the display of zero-pole-gain models.

Example

Example 1

Specify the following zero-pole-gain model.

$$H(z) = \begin{bmatrix} \dfrac{1}{z-0.3} \\ \dfrac{2(z+0.5)}{(z-0.1+j)(z-0.1-j)} \end{bmatrix}$$

To do this, type

```
z = {[] ; -0.5}
p = {0.3 ; [0.1+i 0.1-i]}
k = [1 ; 2]
H = zpk(z,p,k,-1)    % unspecified sample time
```

Example 2

Convert the transfer function

```
h = tf([-10 20 0],[1 7 20 28 19 5])
```

```
Transfer function:
              -10 s^2 + 20 s
---------------------------------------
s^5 + 7 s^4 + 20 s^3 + 28 s^2 + 19 s + 5
```

to zero-pole-gain form by typing

```
zpk(h)
```

```
Zero/pole/gain:
     -10 s (s-2)
----------------------
(s+1)^3 (s^2 + 4s + 5)
```

Example 3

Create a discrete-time ZPK model from a rational expression in the variable z, by typing

```
z = zpk('z',0.1);
H = (z+.1)*(z+.2)/(z^2+.6*z+.09)
```

```
Zero/pole/gain:
(z+0.1) (z+0.2)
---------------
```

```
     (z+0.3)^2

  Sampling time: 0.1
```

Algorithm

zpk uses the MATLAB function roots to convert transfer functions and the functions zero and pole to convert state-space models.

See Also

frd	Convert to frequency response data models
get	Get properties of LTI models
set	Set properties of LTI models
ss	Convert to state-space models
tf	Convert to TF transfer function models
zpkdata	Retrieve zero-pole-gain data

Purpose

Quick access to zero-pole-gain data

Syntax

```
[z,p,k] = zpkdata(sys)
[z,p,k] = zpkdata(sys,'v')
[z,p,k,Ts,Td] = zpkdata(sys)
```

Description

`[z,p,k] = zpkdata(sys)` returns the zeros `z`, poles `p`, and gain(s) `k` of the zero-pole-gain model `sys`. The outputs `z` and `p` are cell arrays with the following characteristics:

- `z` and `p` have as many rows as outputs and as many columns as inputs.
- The `(i,j)` entries `z{i,j}` and `p{i,j}` are the (column) vectors of zeros and poles of the transfer function from input `j` to output `i`.

The output `k` is a matrix with as many rows as outputs and as many columns as inputs such that `k(i,j)` is the gain of the transfer function from input `j` to output `i`. If `sys` is a transfer function or state-space model, it is first converted to zero-pole-gain form using `zpk`. See Table 11-15, "LTI Properties," on page 11-194 for more information on the format of state-space model data.

For SISO zero-pole-gain models, the syntax

```
[z,p,k] = zpkdata(sys,'v')
```

forces `zpkdata` to return the zeros and poles directly as column vectors rather than as cell arrays (see example below).

`[z,p,k,Ts,Td] = zpkdata(sys)` also returns the sample time `Ts` and the input delay data `Td`. For continuous-time models, `Td` is a row vector with one entry per input channel (`Td(j)` indicates by how many seconds the `j`th input is delayed). For discrete-time models, `Td` is the empty matrix `[]` (see `d2d` for delays in discrete systems).

You can access the remaining LTI properties of `sys` with `get` or by direct referencing, for example,

```
sys.Ts
sys.inputname
```

Example

Given a zero-pole-gain model with two outputs and one input

```
H = zpk({[0];[-0.5]},{[0.3];[0.1+i 0.1-i]},[1;2],-1)
```

```
Zero/pole/gain from input to output...
         1
 #1:  -------
      (z-0.3)

           2 (z+0.5)
 #2:  -------------------
      (z^2 - 0.2z + 1.01)

Sampling time: unspecified
```

you can extract the zero/pole/gain data embedded in `H` with

```
[z,p,k] = zpkdata(H)

z =
    [      0]
    [-0.5000]
p =
    [    0.3000]
    [2x1 double]
k =
     1
     2
```

To access the zeros and poles of the second output channel of `H`, get the content of the second cell in `z` and `p` by typing

```
z{2,1}

ans =
   -0.5000

p{2,1}

ans =
   0.1000+ 1.0000i
   0.1000- 1.0000i
```

See Also

`get`	Get properties of LTI models
`ssdata`	Quick access to state-space data
`tfdata`	Quick access to transfer function data

zpk	Specify zero-pole-gain models

Index

D

G

H

I

K

L

M

N

O

P

R

S

T

U

V

Z