The capacitance of a wire

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The inductance of a straight wire with radius r and length l is given by:

$$L = \frac{\mu_0}{2\pi} l \,\mathrm{Ln}\left(\frac{2l}{r} - \frac{3}{4}\right) \qquad [1]$$

The capacitance of a cylinder can be calculated by decomposing it in many rings, as described in [1].

The speed of propagation of an electric signal in a lossless transmission line is given by:

$$v = \sqrt{\frac{1}{L_u C_u}} \qquad [2]$$

where L_u and C_u are the inductance and the capacitance per unit of length of the line.

Considering a straight wire as a transmission line, this speed is limited by the speed of light, c, and so, for a given length l and radius r, the capacitance of a straight wire can be obtained as:

$$C \approx \frac{2\pi l}{\mu_0 c^2 \,\mathrm{Ln}\left(\frac{2l}{r} - \frac{3}{4}\right)} = \frac{2\pi\varepsilon_0 l}{\mathrm{Ln}\left(\frac{2l}{r} - \frac{3}{4}\right)} \approx \frac{5.56\times10^{-11} l}{\mathrm{Ln}\left(\frac{2l}{r} - \frac{3}{4}\right)} \qquad [3]$$

Verification:

l	*r*	*C* calculated (pF)	200 rings	20 rings
1	0.1	18.8	16.8	16.6
1	0.01	10.5	9.88	10.21
1	0.001	7.32	7.20	9.75
1	0.0001	5.61	6.97	9.75
2	0.001	13.4	14.1	19.5

If the wire is curved into a loop, forming a thin toroid with diameter l/π, the capacitance can be compared with a capacitance of a toroid. Considering the length measured along the center of the toroid:

l	*r*	*C* toroid (pF)
1	0.1	17.7
1	0.01	10.1
1	0.001	7.09
1	0.0001	5.48
2	0.001	13.04

References

[1] Antonio C. M. de Queiroz, “Capacitance calculations”. Available at: http://www.coe.ufrj.br/~acmq/tesla/capcalc.pdf