Wideband Asymmetrical Bandpass LC-Ladder Matching Networks for Low-Noise Amplifiers

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***Abstract*— The use of asymmetrical bandpass LC filters as input matching networks for low noise amplifiers is investigated. The use of these filters instead of symmetrical bandpass filters for wideband impedance matching allows simpler structures for the matching of unequal resistances, some voltage gain, and stronger attenuation at high-frequency.**

I. INTRODUCTION

The application of LC ladder filters as input matching networks for wideband low noise amplifiers (LNA) received some attention in the last years [1]-[4]. In the usual circuits, bandpass filters are formed by the combination of a few inductances and capacitances with a common-source or common-emitter amplifier using inductive degeneration [5]. The input impedance of an amplifier as the one in fig. 1, with the transistor having gate-source capacitance C_{gs} and transconductance G_m results as:

$$Z_{in}(s) = sL + \frac{1}{sC_{gs}} + \frac{G_m}{C_{gs}} L \qquad (1)$$

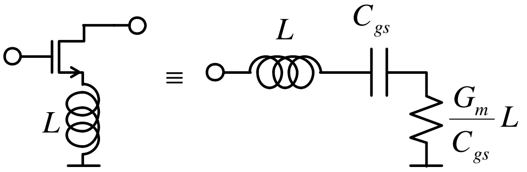

Figure 1. Input equivalent circuit of a common-source amplifier with inductive degeneration.

This is the impedance of an RLC series network. This kind of circuit can be directly employed to adjust the input impedance of the amplifier to be resistive in a narrow band of frequencies, with an extra series inductor used to adjust the frequency, but the RLC elements can also be made to be part of a bandpass filter, what results in wideband resistive input impedance for the amplifier, and low noise due to the (ideal) absence of resistive elements. A common choice of filter is a bandpass Chebyshev filter. In [1] a 4th-order filter is used (fig. 2), in [2] a 6th-order filter, in [3] a differential 4th-order filter, and in [4] a 4th-order filter with a transformer. In all these cases, the transistor output goes to an inductive impedance, to compensate for the integration caused by the input current generating V_{gs} by passing trough the capacitance C_{gs}.
The filters are usually obtained by applying a lowpass to bandpass transformation to a Chebyshev lowpass filter [6]. This results in fixed ratios between the loading resistances at both sides of the filter, that are 1:1 for a 6th-order bandpass filter and depends on the passband ripple for a 4th-order bandpass filter. The input transistor must be designed according to this limitation, or resources as the use of circuit transformations [1] or a transformer [4] employed to allow more freedom in the choice of transistor dimensions and input resistance of the amplifier. These resources may be somewhat inconvenient, because transformations require the use of more components in the matching filter, and integrated transformers require very careful modeling.

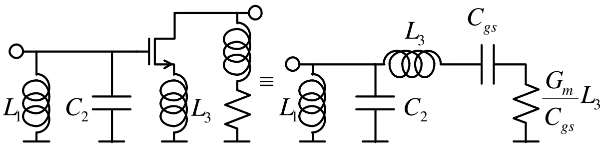

Figure 2. Wideband impedance matching with a symmetrical 4th-order bandpass filter.

It is observed that polynomial bandpass doubly terminated filters having asymmetrical frequency response, with different numbers of transmission zeros at zero and infinity, practically always require different termination resistances. In this work, a class of these filters, having maximally flat passbands, that is particularly easy to design, is proposed as alternative to the realization of the input matching network for wideband LNAs. The ratio of the terminating resistances depends on a single parameter, that can be adjusted so the network realizes a wide range of resistance transformations with the same structure. The resulting bandwidths are easily wide enough for application in ultrawideband radio.

II. MAXIMALLY FLAT IMPEDANCE MATCHING NETWORKS

To introduce the idea, consider first a low-pass filter having the structure shown in fig. 3. The filter can be designed to exhibit a maximally flat passband around the normalized frequency of 1 rad/s, if it is designed from the characteristic function:

$$K(s) = \frac{F(s)}{P(s)} = \frac{\pm(s^2+1)^2}{p} \qquad (2)$$

The transducer function $H(s) = E(s)/P(s)$ corresponding to this characteristic function is obtained by solving Feldtkeller's equation in the usual way [6]:

$$E(s)E(-s) = P(s)P(-s) + F(s)F(-s) \qquad (3)$$

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From $F(s)$ and $E(s)$, the input impedances or admittances of the LC network between the terminations can be found, and the structure can be obtained [6]. For the lowpass case, it is easy to obtain the parameter p in (2) in function of the termination resistances. Observing that the impedance seen by the input termination R_1 is:

$$Z_1(0) = R_1 \frac{H(0)+K(0)}{H(0)-K(0)} = \frac{\sqrt{1+\varepsilon^2}+\varepsilon}{\sqrt{1+\varepsilon^2}-\varepsilon} = R_2 \qquad (4)$$

The constant that p is readily obtained as:

$$p = \frac{1}{\varepsilon} = \sqrt{\left(\frac{R_2+R_1}{R_2-R_1}\right)^2 - 1} \qquad (5)$$

As an example of what can be obtained, consider the case of matching R_1=1Ω with R_2=2Ω. The required functions are obtained as:

$F(s) = -\left(s^4 + 2s^2 + 1\right)$

$P(s) = \sqrt{8} = 2.828427$

$E(s) = s^4 + 2.44672s^3 + 4.99321s^2 + 5.09502s + 3$

The element values are obtained as:
R_1 = 1 Ω; R_2 = 2Ω;
L_1 = 0.817422 H; C_2 = 0.865045 F;
L_3 = 1.73009 H; C_4 = 0.408711 F
The input impedance and the frequency response are shown in fig. 4.

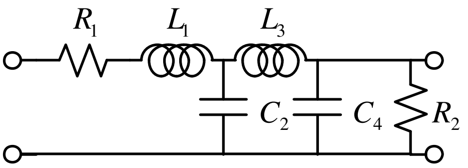

Figure 3. Low-pass impedance matching network.

The input impedance Z_1 is "maximally resistive" around 1 rad/s, and the voltage gain is maximally flat. Note that the single parameter p controls the ratio between the terminations, and also the bandwidth (smaller for larger ratios). More sections can be added by just adding more pairs of roots at $\pm j$ in $F(s)$, what extends the flatness and the frequency range where the matching is good enough. Eq. (5) remains valid for higher-order networks.

III. 4TH-ORDER IRREGULAR BANDPASS MATCHING NETWORK

The previous circuit can't be directly employed for the input matching of an LNA. A convenient structure can be obtained by adding a pole at 0 in $K(s)$, by using $P(s) = ps$. The resulting structure is shown in fig. 5. The section connected to R_2 can be realized by the input impedance of an amplifier, as in fig. 1. A complication is that it's not possible anymore to obtain a simple expression relating the terminations with the parameter p, although they continue to determinate it uniquely. A way around the problem is the observation that the real part of the impedance seen by R_2, Z_2, is equal to R_1 at low frequency. If $Z_2(s)/R_2$ is a ratio of polynomials as in (6) [6], then the ratio of the terminations can be calculated as (7). E(s) must be calculated, and part of $Z_2(s)/R_2$, but the expansion of the network from both sides, that would be required for the determination of the ratio by the usual method [6], is not necessary.

$$\frac{Z_2(s)}{R_2} = \frac{E(s)-F_e(s)+F_o(s)}{E(s)+F_e(s)-F_o(s)} = \frac{\cdots + a_1 s + a_0}{\cdots + b_2 s^2 + b_1 s} \qquad (6)$$

$$\lim_{\omega\to 0} \mathrm{Re}\left(\frac{Z_2(j\omega)}{R_2}\right) = \frac{R_1}{R_2} = \frac{a_1 b_1 - a_0 b_2}{b_1^2} \qquad (7)$$

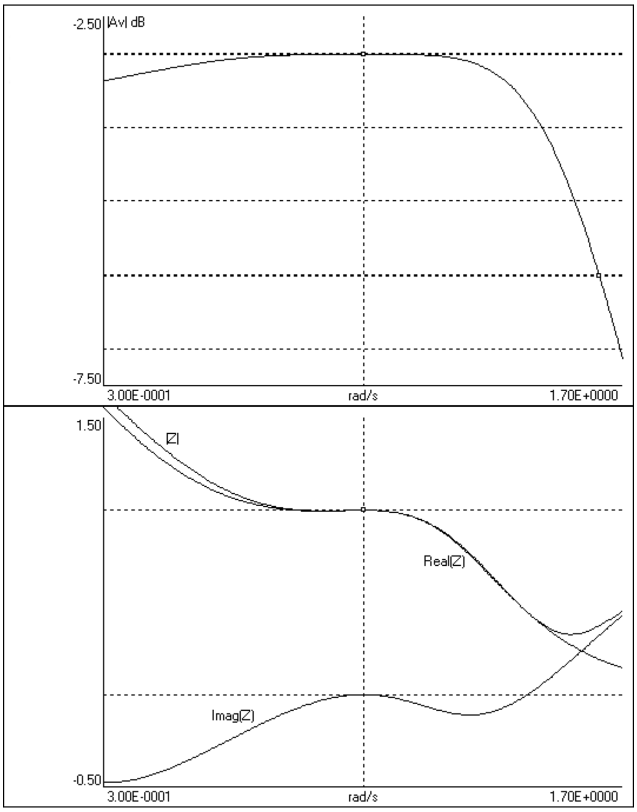

Figure 4. Voltage gain and input impedance (Z_1, after R_1) for the filter in fig. 3.

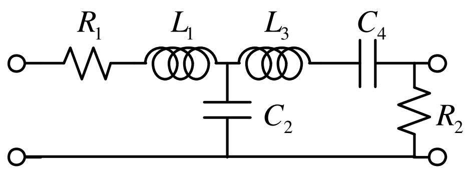

Figure 5. Irregular bandpass impedance matching network.

After a few tries with a computer program using (7), the value of p that results in R_2/R_1 = 2 is found, and the required functions are:

$F(s) = -\left(s^4 + 2s^2 + 1\right)$

$P(s) = 1.820359s$

$E(s) = s^4 + 1.82036s^3 + 3.65685s^2 + 2.57438s + 1$

The synthesis results in:
$R_2 = 2\ \Omega$; $R_1 = 1\ \Omega$;
$L_1 = 1.09868$ H; $C_2 = 0.643594$ F;
$L_3 = 2.19737$ H; $C_4 = 0.643594$ F

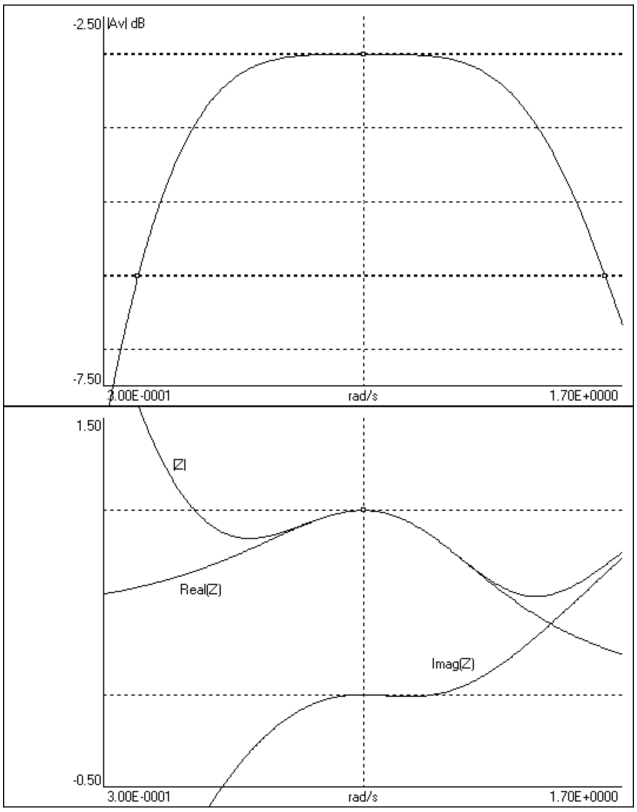

Figure 6. Voltage gain and input impedance and for the filter in fig. 5.

Fig. 6 shows the obtained input impedance seen by R_1 and the voltage magnitude gain. The ratio of the upper to lower passband border frequencies, considering 3 dB of attenuation, is 4.2, that is enough for application in a ultrawideband radio system, operating between 3 and 10 GHz, with a 3.3 ratio only.

IV. 5TH-ORDER IRREGULAR BAND PASS MATCHING NETWORK

This matching network has still an inconvenience that is the inductor at the input. A practical integrated inductor has always a significant parasitic capacitance to the substrate at both terminals, and if the input comes from out of the chip a significant pad capacitance is present at the input end. Adding a capacitance at the input transforms the network in the one in fig. 7. It is a 5th-order network, and so $F(s)$ must have a real root. The exact value is not critical, and so as example a root at $s = 2$ is chosen. The ratio of the terminations continues to be given by (7). After some tries, the following functions are obtained, for $R_2/R_1 = 2$:

$$F(s) = \left(s^2+1\right)^2(s-2) = s^5 - 2s^4 + 2s^3 - 4s^2 + s - 2$$

$$P(s) = 3.192459s$$

$$E(s) = s^5 + 3.61292s^4 + 6.52660s^3 + 8.80536s^2 + 5.51482s + 2$$

The synthesis results in:
$R_2 = 2\ \Omega$; $R_1 = 1\ \Omega$;
$C_1 = 0.356321$ F; $L_2 = 1.41605$ H;
$C_3 = 0.708032$ F; $L_4 = 2.47997$ H;
$C_5 = 0.564352$ F

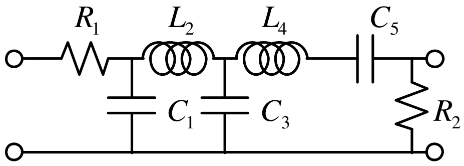

Figure 7. 5th-order irregular bandpass impedance matching network.

The results are similar to the previous circuit, but the ratio of the cutoff frequencies results now as 3.62. The filter is a bit more selective, but still suitable. The free choice of the real root of $F(s)$ can be used to adjust the bandwidth and the element values. Of course, p changes with it too.

V. 6TH-ORDER IRREGULAR BAND PASS MATCHING NETWORK

The two last matching networks exhibit just one transmission zero at 0. This may be inconvenient if high-intensity signals are present below the filter passband, because the attenuation provided by the filter may be insufficient. Moreover, the drain current of a transistor connected as in figs. 1 and 2 shows a 20 dB/decade roll-off in frequency, because of the integration in the capacitance C_{gs}. The net effect is that the single transmission zero at 0 is cancelled, with the voltage gain becoming constant below a frequency determined by the RL load network. A solution is to add another transmission zero at 0 to the matching network, by making $P(s) = ps^2$. If a 4th-order network is used, the result is a regular Butterworth bandpass filter with the structure of fig. 2, where the ratio of the terminations is fixed at 1. To allow different ratios, a 6th-order network can be used, that compensates the added complexity with wider passband. Two possibilities are shown in fig. 8, one suitable for $R_1>R_2$ and another for $R_2>R_1$.

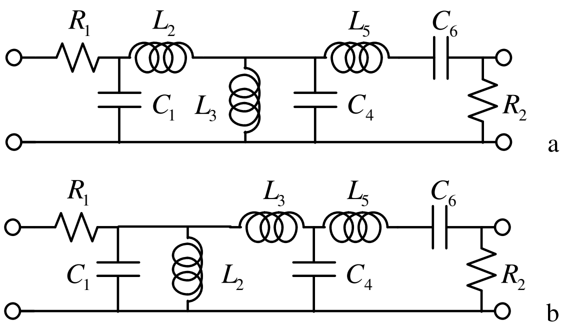

Figure 8. 6th-order irregular bandpass impedance matching networks, for $R_1>R_2$ (a) and for $R_1<R_2$ (b).

The ratio of the terminations for these filters is more difficult to find, but it still possible to apply the idea (7) in admittance basis after removing the series LC tank from Z_2. Considering the first structure, the results are:

$$\lim_{\omega\to 0}\mathrm{Re}\left(\frac{Z_2(j\omega)}{R_2}-\frac{1}{j\omega C_6}-j\omega L_5\right)^{-1}=\frac{R_2}{R_1}$$

$$\frac{Z_2(s)}{R_2}=\frac{E(s)-F_e(s)+F_o(s)}{E(s)+F_e(s)-F_o(s)}=\frac{a_6s^6+a_5s^5+\cdots+a_0}{b_5s^5+b_4s^4+\cdots+b_1s} \qquad (8)$$

$$C_6=b_1/a_0;\; L_5=a_6/b_5$$

$$\frac{R_2}{R_1}=-\frac{b_5^2b_1\left(a_3b_1^2-a_2b_2b_1+a_0\left(b_3b_2-b_4b_1\right)\right)}{\left(a_6b_1^2-b_5\left(a_2b_1-a_0b_3\right)\right)^2}$$

For the second structure the new resistance ratio can be obtained from the ratio in the first structure. The tank L_5C_6 is removed from Z_2 as in the first case, but from the resulting admittance C_4 is removed too. A Norton transformation is then applied to convert the first structure into the second. The ratio of inductances L_2/L_3 is necessary, but can be obtained from the ratio $L_3/(L_2//L_3)$, easy to obtain by looking at the impedance at $s = 0$ and $s = \infty$. After all the algebra, the result is shown below.

$$\frac{R_2'}{R_1'}=\frac{R_2}{R_1}\left(\frac{L_2}{L_3}+1\right)^2=\frac{R_2}{R_1}\left(\frac{1}{\frac{L_3}{L_2//L_3}-1}+1\right)^2 \qquad (9)$$

$$\frac{L_3}{L_2//L_3}=\frac{\left(b_5\left(a_2b_1-a_0b_3\right)-a_6b_1^2\right)\left(a_6\left(b_5b_1-b_3^2\right)+b_5\left(a_4b_3-a_2b_5\right)\right)}{\left(a_6b_3b_1-b_5\left(a_4b_1-a_0b_5\right)\right)^2}$$

For $R_1 = 1$ and $R_2 = 2$, the required functions are:

$$F(s)=\left(s^2+1\right)^3=s^6+3s^4+3s^2+1$$

$$P(s)=7.941998s^2$$

$$E(s)=s^6+3.75676s^5+10.0566s^4+15.2523s^3$$
$$+14.2317s^2+4.73955s+1$$

The synthesis for the structure in fig. 8b gives the elements listed below, and fig. 9 shows the obtained results. The ratio of passband border frequencies reaches 6.7:

$R_2 = 2\ \Omega$; $R_1 = 1\ \Omega$;
$C_1 = 0.532374$ F; $L_2 = 2.36978$ H;
$L_3 = 1.12096$ H; $C_4 = 0.560481$ F;
$L_5 = 1.06475$ H; $C_6 = 1.18489$ F

VI. Conclusions

The use of irregular passband filters is an interesting option for the realization of input wideband matching networks for low-noise amplifiers, when matching between different terminations is required. The described filters have maximally flat passbands, and the resulting bandwidth is a function of the order and of the termination resistances, except for the odd-order cases where a real reflection zero adds another degree of freedom. Equations were derived to help in finding the required characteristic function from the required terminations. It is surely also possible to design equal ripple passband versions using numerical methods (as an adaptation of the algorithm described in [7]), what allows a compromise between bandwidth and passband ripple. The equations for the ratio of the terminations remain valid in this case.

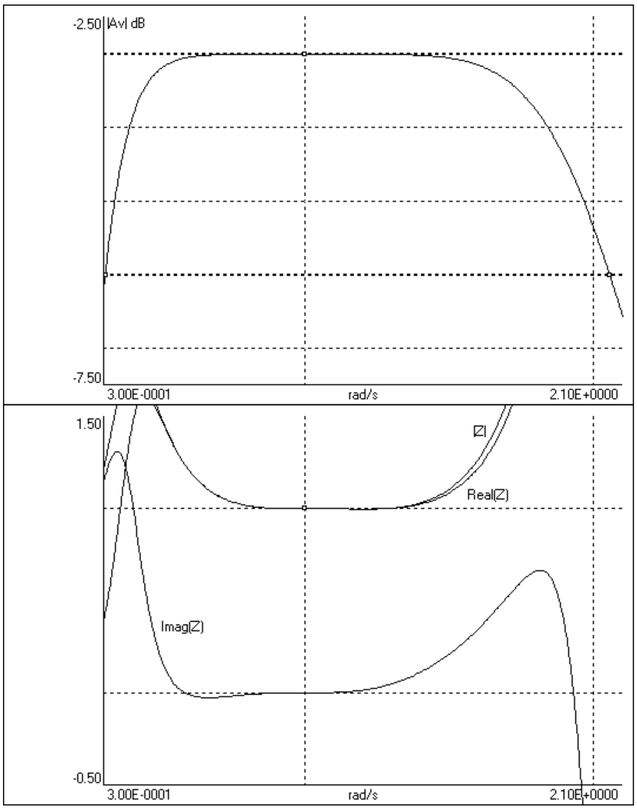

Figure 9. Voltage gain and input impedance for the network in fig. 8b.

VII. References

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