Component simulation switched-current filters using backward Euler transformation

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Abstract—**The paper describes a structure for switched-current filters using the component simulation technique that, through the backward Euler transformation, results in a filter realizing the LDI transformation with a minimum number of elements. The structure is also particularly convenient for sensitivity reduction through component swapping**

I. Introduction

The "component simulation" technique for the construction of switched-current (SI) filters was introduced in [1]-[3]. It consists in the simulation of a continuous-time *Gm-C* filter by switched-current blocks that are identifiable after the application of a suitable *s*-to-*z* transformation to the nodal equations that describe the network. The *Gm-C* structure (fig. 1b), that can be the simulation of a passive network (fig. 1a), is first decomposed into transconductors and transcapacitors (fig. 1c). These elements can then be implemented as shown in fig. 2, for three common types of *s*-to-*z* transformations. The final filter is obtained after simplifications in the structure to remove redundant switches [2]. All the transconductors would be normally implemented with single biased MOS transistors, with their input capacitances holding voltages when the switches are open. Something as cascodes would be added to improve the precision of the resulting network, or more complex transconductors can be used. The structure requires just two nonoverlapping clock signals for correct operation, and works with "modulated" signals, where the polarity of all the signals is alternated from phase to phase.

In this paper, the *s*-to-*z* transformation to be considered is the backward Euler transformation:

$$s = \frac{1 - z^{-1}}{T} \tag{1}$$

where T is the switching period (one half of the switching frequency period, in the case of these circuits). As shown in fig. 2, the continuous-time transconductor is implemented by a single unswitched transconductor, and the structure of the transcapacitor is simple too. A precise filter, however,

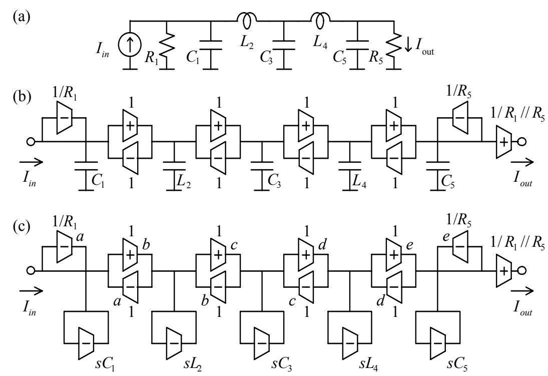

Figure 1. Transformation of a passive RLC network into a transconductor-transcapacitor network. a) Passive network. b) *Gm-C* equivalent. c) Final *Gm-Cm* network.

can't be directly built with the backward Euler transformation, because it produces severe distortion in the frequency response, mapping the $j\omega$ axis in a circle inside the unit circle in the *z* domain (fig. 3). The consequence is that poles have their quality factors significantly decreased, and imaginary zeros are lost.

II. Prewarping of Poles and Zeros by a Combination of the LDI and Backward Euler Transformations

Precise filters can be obtained with the "lossless discrete integration" (LDI) transformation:

$$s = \frac{1 - z^{-1}}{T z^{-1/2}} \tag{2}$$

This transformation maps the section of the $j\omega$ axis between $\omega = \pm 2/T$ over the unit circle with just a simple frequency warping effect (fig. 3). The LDI transformation can't be directly implemented with a switched-current circuit, but the required poles and zeros in the *z* domain can be obtained by solving (2) for *z* by (3), where *s* is a pole or zero of the continuous-time transfer function, previously

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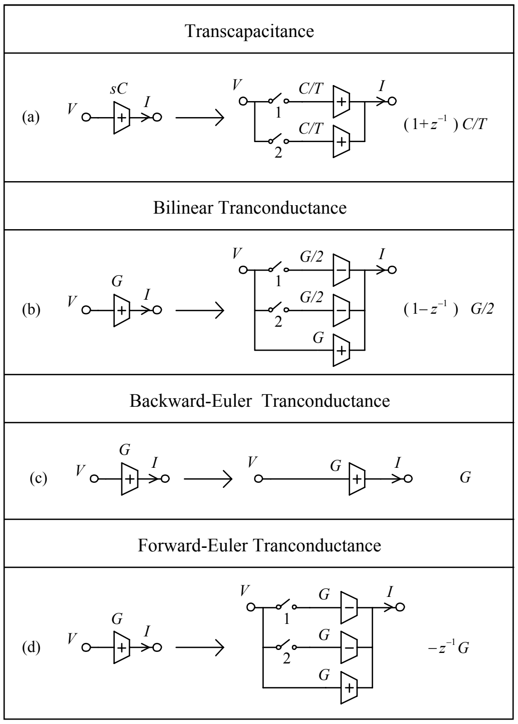

Figure 2. Equivalents for operation with modulated signals.

prewarped to compensate for the warping effect ((4) below).

$$z=\left(\frac{Ts+\sqrt{T^2s^2+4}}{2}\right)^2 \tag{3}$$

The poles and zeros obtained through this transformation realize a given filter approximation correctly, without the introduction of additional zeros. The bilinear transformation, if used, would produce a correct filter too, but would introduce additional zeros at z = –1. Note that the poles and zeros obtained in this way are the unique solution that produces a correct filter.

A filter built through the backward Euler transformation, that realizes certain given poles and zeros in the *z* domain can be obtained by designing it from a continuous-time prototype that has these poles and zeros transformed back to the *s* domain through the relation (1). If the *z* domain poles and zeros are first obtained trough the LDI transformation (3), a correct filter is obtained. The design procedure is:

a) The filter specifications are prewarped, with the frequencies transformed according to the relation:

$$\omega_s=\frac{2}{T}\sin\frac{\omega_z T}{2} \tag{4}$$

In the simplest cases, as in a low-pass filter, the normalized poles and zeros are simply multiplied by this ω_s.

b) The poles and zeros are then transformed first to the *z* domain using (3) and then back to the *s* domain using (1).

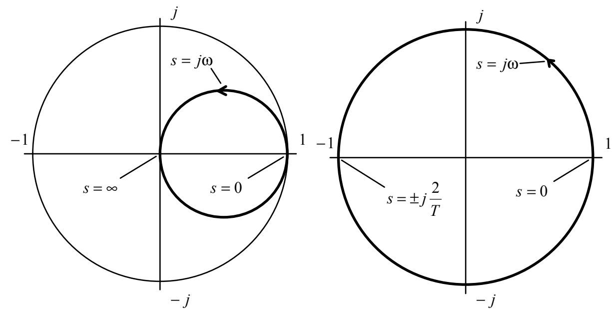

Figure 3. Mapping of the *j*ω axis in the *z* plane by the backward Euler (left) and LDI (right) transformations.

Or, combining (1) and (3), directly:

$$s'=\frac{s}{z^{1/2}}=\frac{2s}{Ts+\sqrt{T^2s^2+4}} \tag{5}$$

A *Gm-C* filter can then be designed, that realizes the transformed poles and zeros. The SI component simulation of its structure using the backward Euler transformation results in the desired filter.

There are restrictions on the applicability of this technique. One of them is that transmission zeros in the *j*ω axis are mapped into zeros in the right side of the complex plane. These may be impractical to realize by a passive structure that can be easily transformed into a *Gm-C* equivalent. This limitation restricts the technique to the simulation of passive polynomial filters, but doesn't restrict other types of active realizations. Another restriction is that when the switching frequency is not much higher than the frequencies where the filter operates, the predistortion results in poles in the right side of the complex plane, or in an unstable continuous-time prototype. Although the final discrete-time filter results stable, a "passive" prototype for an unstable filter would have negative elements, possibly with very irregular distribution of values, and very high sensitivities to their variation. The discrete-time implementation would suffer the same problems. This limitation restricts the technique to cases where the switching frequency is much higher than the filter operating frequencies. A third restriction is that it's not possible to exploit the low sensitivity properties of LC doubly terminated filters as prototypes. The predistortion of any usual filter approximation results in a filter where the passband has increasing gain along it, allowing maximum power transfer and low sensitivities to be obtained only close to the end of the passband. But the sensitivities at this single frequency are not preserved exactly in the discrete time implementation, because the backward Euler transformation does not map the imaginary axis over the unit circle. This limitation may turn the use of LC doubly terminated prototypes less attractive, although they continue to be a good option, when compared to singly terminated passive prototypes or direct active realizations that do not simulate passive structures.

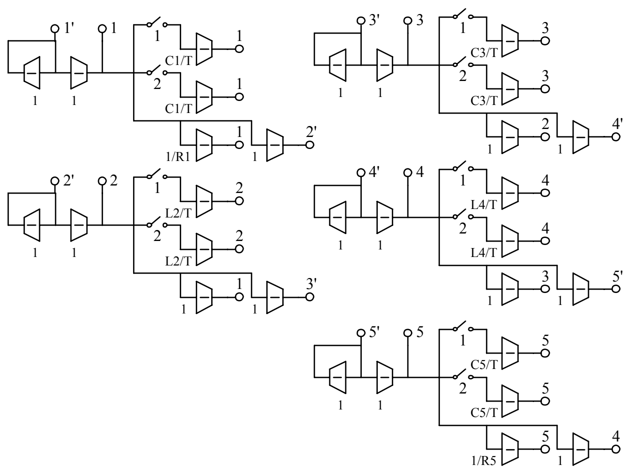

Figure 4. Basic SI structure using the backward Euler transformation, with modulated signals.

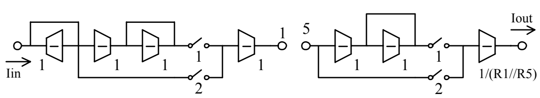

Figure 5. Input modulator and output demodulator.

III. Design Example

As example, the design a 5th-order low-pass Chebyshev filter with 1 dB maximum passband ripple, operating with a switching frequency 20π times greater than the passband border will be detailed: The poles for a continuous-time Chebyshev filter with passband edge at 1 rad/s are:

$s_{1,2} = -0.0894583622 \pm 0.9901071120j$
$s_{3,4} = -0.2342050328 \pm 0.6119198477j$
$s_5 = -0.2894933412$

The normalized switching frequency is 10 Hz, and so $T = 0.1$. The prewarping factor (4) maps 1 rad/s into $\omega_s = 0.999583385$ rad/s (small correction, because the switching frequency is high). This factor multiplies the poles, and the desired z domain poles are obtained trough (3):

$z_{1,2} = 0.9862331832 \pm 0.0979661904j$
$z_{3,4} = 0.9750238663 \pm 0.0597185061j$
$z_5 = 0.9714783808$

Returning these poles to the s domain trough (1) in the reverse direction, the predistorted s domain poles are obtained:

$s'_{1,2} = -0.0405182374 \pm 0.9973618189j$
$s'_{3,4} = -0.2178285702 \pm 0.6258241246j$
$s'_5 = -0.2935898503$

The resulting transfer function, with the maximum gain normalized to 1 is:

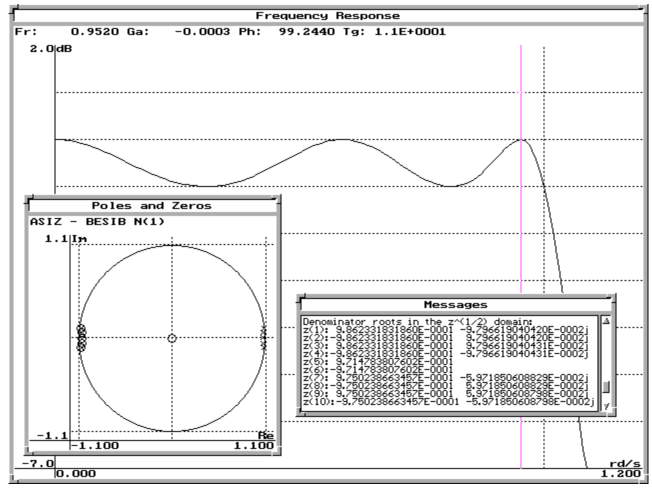

Figure 6. Simulation of the filter in fig. 6.

$$T(s) = \frac{b_0}{s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$a_0 = 0.1284491385$ $\quad a_1 = 0.5753996727$
$a_2 = 0.9014667888$ $\quad a_3 = 1.6224775664$
$a_4 = 0.8102834657$ $\quad b_0 = 0.0584800888$

A passive LC doubly terminated ladder filter was designed in the usual way to realize this filter, with a single frequency of maximum power transfer, at the frequency where the gain is maximum, close to the end of the passband. There are several possible designs, as for this filter the characteristic function is not unique. In this case the reflection zeros that were not imaginary were chosen to be at the left side of the complex plane. The resulting element values, for a load resistance of 1 Ohm in the structure shown if fig. 1a, are:

$C_1 = 0.8069311526$ $\quad L_2 = 1.8823708612$
$C_3 = 2.3710770091$ $\quad L_4 = 1.6887319084$
$C_5 = 1.3542807579$ $\quad R_1 = 17.239729357$
$R_5 = 1.0$

The normalized component simulation SI filter has the structure shown in fig. 4. Each node (1...5) has a current mirror attached, that serves as input (1'…5') for positive transconductances feeding the node. Note that the mirror at node 1 is not used in this case, and could be omitted. The structure takes as input a modulated current at node 1, and has as output the, also modulated, currents at the transconductors with input at node 5, or the voltage at node 5, that can be used to drive other transconductors. Convenient input modulator and output demodulator are shown in fig. 5. Fig. 6 shows a simulation of this structure in the ASIZ program [5]. The computed poles and zeros are exactly the poles obtained from the LDI transformation of the s domain poles, as expected (the ASIZ program computes poles and zeros in the $z^{1/2}$ domain, but this filter

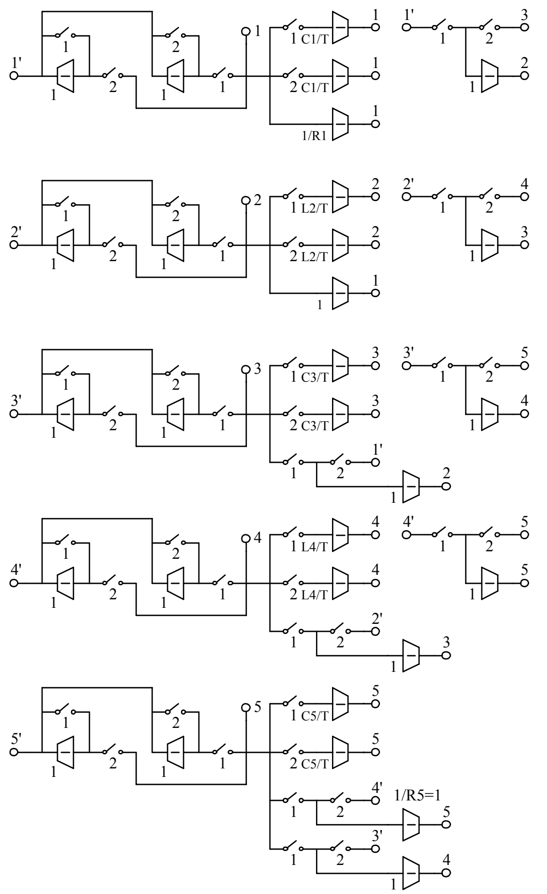

Figure 7. SI structure with component swapping to reduce errors.

doubles the switching frequency of 5 Hz, and so the calculated poles appear as in the design).

IV. Swapping elements to reduce sensitivities

Component swapping [4] is a technique that reduces errors in the filter transfer function due to component variations, by swapping elements with opposite, or approximately opposite, sensitivities of the filter transfer function to their values, at each phase. The operation results in the sensitivities of the swapped elements being replaced by their average values. An examination of the sensitivities of the circuit in fig. 4 shows that the input mirrors at the nodes have always exactly opposite sensitivities, and so are natural candidates for swapping. It's also easy to swap the pairs of transconductors forming gyrators and the output termination, indicated as b-b..e-e in fig. 1c, that have outputs at the same nodes and identical values (the transconductors a-a are not swapped because their values are different). These elements don't have opposite sensitivities, but just reducing their pairs of sensitivities to their average values reduces significantly the expected errors (statistical deviations). Note that as in this realization the continuous-time transconductors are realized by unswitched transcondutors only, they can be entirely

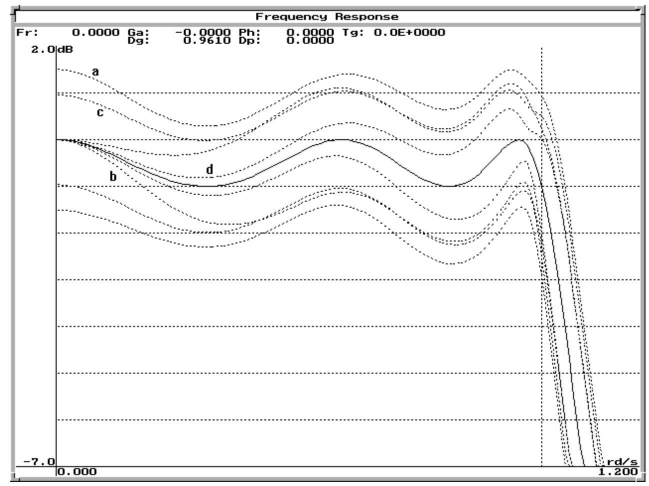

Figure 8. Passband errors expected for 5% tolerances in the element values. a) Fig. 4. b) Fig. 4 with discounted sensitivities. c) Fig. 7. d) Fig. 7 with discounted sensitivities.

swapped. In implementations using the bilinear transformation or the forward Euler transformation (bilinear realization with Euler integrators), all or some of the transconductors have switched parts that can't be swapped [4]. The SI structure is also modified by replacing the current inverters at the input of the nodes by voltage inverters that drive the positive transconductors with inputs connected to the nodes. In this way the transconductors that make the inverters can be swapped without current switching [4]. The resulting structure is shown in fig. 7. Fig. 8 shows a comparison of the passband errors expected, counting all the elements with 5% random variabilities, and discounting the sensitivities of their values at DC.

V. Conclusions

It was shown how to obtain a component simulation switched-current filter using only the simplest building blocks, through the use of a combined LDI - backward Euler transformation. The technique has limitations on its applicability, but in many cases can be used in the generation of practical filter structures.

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