The 47th IEEE International Midwest
Symposium on Circuits and Systems

Band-Pass Multiple Resonance Networks

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***Abstract*—The idea of "multiple resonance networks" is reviewed and extended to networks with the structure of band-pass LC ladders. This realization combines the properties of the previously described low-pass and high-pass realizations. It allows the generation, for example, of triple resonance networks with a transformer at one side and capacitive coupling at the other, overcoming the limitations in voltage gain of high-pass realizations. A simple transformation allows the generation of networks with single input and balanced output.**

I. Introduction

Multiple resonance networks are a class of LC networks that can completely transfer all the energy initially stored at a set of capacitors or inductors of the network to another set of elements. One of the simplest cases is the well-known Tesla transformer, one of the first nontrivial linear circuits to be analyzed. Ref. [1] contains a review of some early works. These circuits have found applications in many areas, when the fast lossless conversion of low voltage to high voltage is desired. Examples range from early radio transmitters [1] to modern pulsed power systems. In [2], a 6th-order triple resonance network was described for this kind of application, showing that there are justifications for the study of more complex versions of these networks, that besides practical applications have also interesting properties in the point of view of linear circuit theory. This author developed methods to extend the design of multiple resonance networks to any order, using a synthesis approach instead of an analysis approach, what lead to quite simple design procedures for several different structures[1]. Refs. [3][5] introduced the low-pass versions (figs. 1a and 1b), that include the Tesla transformer and the triple resonance network mentioned in [2] as the simplest cases of fig. 1b, transferring energy between the capacitors C_1 and C_p. Ref. [4] presented a very simple design method for these circuits that avoids the solving of systems of equations. [6] extended the idea to networks where the energy is transferred from or to inductors, what allows optimized design of induction coils and generalizations of them (energy transfer between L_1 and C_p in figs. 1a and 1b).

Ref. [7] presented high-pass versions, that have the structure of a high-pass ladder network (fig. 1c). The transformerless low-pass networks (fig. 1a) allow just a single design possibility. The high-pass versions, however, allow variations in the mechanism of energy transfer, depending on which elements hold the energy at the start and at the end of the energy transfer cycle. Four cases were identified for the energy transfer between capacitors and two for the energy transfer between inductor and capacitors:

The "symmetrical design" transfers energy between C_1 and C_2 (fig. 1c) charged to same voltage to C_p and C_{pa} only, that also get charged to the same voltage. The same network also works if the starting energy is applied to the "high-frequency input capacitance" of the network, through a current impulse across C_1, transferring the energy to the "high-frequency output capacitance" of the network, where it can be completely extracted by a current impulse caused by short-circuiting the output.

The "asymmetrical design" transfers energy between C_1 and C_2 and the high-frequency output capacitance of the network. The network can also be designed in a reverse form, that transfers energy between the high-frequency input capacitance and C_p and C_{pa} only.

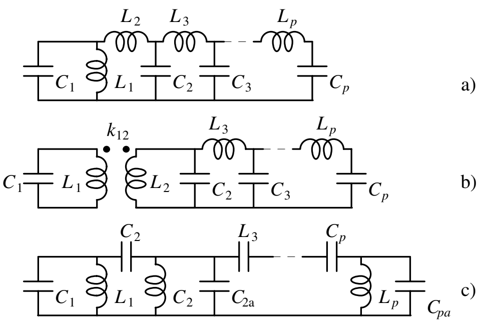

Fig. 1. Previously described multiple resonance networks: a) Low-pass network. b) Low-pass network with a transformer. c) High-pass network.

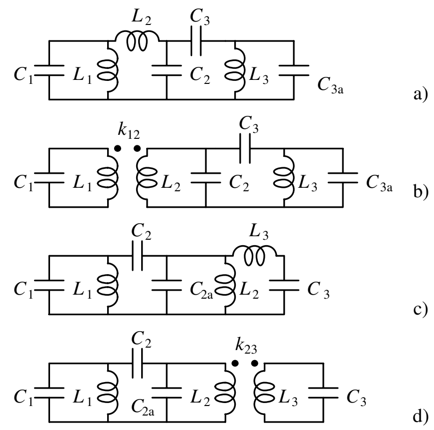

Fig. 2. Band-pass 6th-order multiple resonance networks.

A variation of the symmetrical design transfers energy between L_1 and C_p and C_{pa} only, and a variation of the asymmetrical design transfers energy between L_1 and the high-frequency output capacitance. Variations of the same simple design method described in [4] can be applied to all of these cases.

The following sections show that the design procedures described in the previous papers can be combined for the generation of other structures. Low-pass and high-pass sections can be combined in any order for orders 6 and above. The introduction of a low-pass section in a high-pass network allows the inclusion of a transformer, overcoming the

[1] The referred papers, and computer programs for the design of multiple resonance networks, can be found at: http://www.coe.ufrj.br/~acmq.

association between the operating mode and the voltage gain that limits the application of pure high-pass networks. As examples of these structures, fig. 2 shows the possibilities for 6^{th}-order networks, without and with a transformer. In all cases, energy transfer between two sets of capacitors or an inductor at both sides is possible. In this paper, only the energy transfer from a set of capacitors, or from L_1, to another set of capacitors at the output side will be considered. Designs with energy transfer to the output inductor, or inductors, can be obtained by simple dualization.

II. Design procedure for band-pass multiple resonance networks

The simple design method described in [4] departs from the observation that the output voltage of one of these networks can be predicted, from the natural frequencies of the network, and from the number of transmission zeros at zero that the structure of the network places between an impulsive source that has the same effect or the initial conditions (in figs. 1 and 2, a current source in parallel with C_1 or a voltage source in series with L_1) and the output. Supposing that the circuit resonates at frequencies $\omega_j = k_j\omega_0$, $j = 1 \ldots p$, where p is the number of conjugate pairs of natural frequencies, in all the cases the output voltage $V_{out}(s)$ must have the form:

$$V_{out}(s) = \frac{\beta s^m}{(s^2 + k_1{}^2\omega_0{}^2)(s^2 + k_2{}^2\omega_0{}^2)\cdots(s^2 + k_p{}^2\omega_0{}^2)} \qquad (1)$$

For complete energy transfer after a finite time, the multipliers k_j must be successive integers with odd difference for the case of energy transfer between capacitors [5] (or in the dual case between inductors), or must be all odd with double odd differences for energy transfer between inductors and capacitors [6]. The power of s, m, is odd for energy transfer between elements of the same kind, and even for energy transfer between inductors and capacitors. The expansion of (1) in partial fractions results in a sum of pure cosinusoids (2a) in the first case, and in a sum of pure sinusoids (2b) in the second case. With the condition imposed on the multipliers k_j, the waveform components add destructively (or are null in the second case) at the start of the energy transfer from C_1 or L_1, at $t = 0$, and add constructively when the energy transfer is complete, at $t = \pi/\omega_0$ in the first case, and at $t = \pi/(2\omega_0)$ in the second case.

$$V_{out}(s) = \sum_{j=1}^{p} \frac{A_j{}'s}{s^2 + k_j{}^2\omega_0{}^2} \qquad (2a)$$

$$V_{out}(s) = \sum_{j=1}^{p} \frac{B_j{}'k_j\omega_0}{s^2 + k_j{}^2\omega_0{}^2} \qquad (2b)$$

In a pure high-pass network with energy transfer between capacitors [7], the power of s in the numerator of (1) is $m = 2p-3$ for the asymmetrical design and $m = 2p-1$ for the symmetrical design. For the energy transfer from L_1, considerations similar to the ones in [7] lead to $m = 2p-2$ for the asymmetrical design and $m = 2p$ for the symmetrical design (in this case, a constant term appears in the partial fraction expansion of (1), but it is an artifact of the special calculation for the symmetrical design and is ignored).

The next consideration [4] is that the same waveforms can be obtained, shifted in time by the total energy transfer time, if the input is considered as being an impulsive current source in parallel with the output. With this excitation, the output voltage is proportional to the output impedance of the network. This consideration works for the symmetrical design in energy transfer between capacitors too, because one of the possibilities is energy transfer to the high-frequency output capacitance [7]. As this is the impedance of an LC network, it has the form (2a), but the residues are all positive. All that has to be done to obtain this impedance is then to calculate the expansions (2a) or (2b) for an arbitrary constant β in (1), as 1, and then use the absolute values of the obtained residues A'_j or B'_j as residues for the expansion of the impedance in Foster's first form, multiplying it by a convenient constant λ:

$$Z_{out}(s) = \lambda \sum_{j=1}^{p} \frac{A_j s}{s^2 + k_j{}^2\omega_0{}^2}, \quad A_j = |A'_j| \text{ or } A_j = |B'_j| \qquad (3)$$

The constant λ can be simply the inverse of the sum of the A_j. This results in a normalized impedance with a high-frequency capacitance of 1 F. The normalized network is then obtained by the expansion of $Z_{out}(s)$ in a ladder with the required structure. A transformer with arbitrary turns ratio can be inserted where an "L" of inductors appears [1].

The inclusion of a low-pass section has the effect of decreasing by 2 the power of s in (1), by the addition of two transmission zeros at infinity. From this point the design proceeds as before, with the transformerless network being obtained by a proper expansion of the output impedance. There are always two possible designs:

The symmetrical design, transferring energy from the input capacitors, or the input inductor, to the output capacitors (C_3 and C_{3a} in fig. 2a, or C_3, C_{2a}, and C_2 in fig. 2c). The symmetrical design also transfers energy correctly from the high-frequency input capacitance (C_1 in fig. 2a, C_1+C_2//C_{2a} in fig. 2c) to the high-frequency output capacitance.

The asymmetrical design, transferring energy from the input capacitors (C_1, C_2, and C_3 in fig. 2a or C_1 and C_2 in fig. 2c, charged to the same voltage) or from the input inductor (L_1) to the high-frequency output capacitance (C_{3a}+C_3//C_2 in fig. 2a or C_3 in fig. 2c). The network can also be designed or operated in the reverse direction.

The inclusion of a transformer keeps the same capacitors charged at both ends of the energy transfer cycle, and so the asymmetrical design doesn't make sense in fig. 2b for energy transfer between capacitors, since this network is obtained from fig. 2a, where C_2 and C_3 would be charged along with C_1, that ends at the other side of the included transformer. The same problem occurs with the inverted asymmetrical design in the case of fig. 2d.

It is always possible to expand the networks in the reverse order if convenient. For example, it may be required that the network in Fig. 2a shall transfer energy from C_1 (that is the high-frequency input capacitance) to C_3 and C_{3a} only (not to the high-frequency output capacitance C_{3a}+C_3//C_2). This can be obtained by expanding the structure in Fig. 2a using the asymmetrical design, starting from the input side. C_2 would be then a selectable fraction of the capacitance seen at infinite frequency after the extraction of C_1, L_1, and L_2. The other elements are all determined. The transformation to the structure with transformer, fig. 2b, is then possible, by the replacement of the "L" L_1-L_2, with correct operation.

III. Examples

Consider the design of a network with the structure in fig. 2a, with the specifications: Mode = 5:6:7, L_3 = 30 mH, C_1 = 1 nF, symmetrical design with energy transfer between capacitors. The normalized resonance frequencies are 5, 6, and 7 rad/s, and the power of s in (1) is $2p-1-2 = 3$. The expansion in partial fractions is then:

$$\frac{s^3}{(s^2+25)(s^2+36)(s^2+49)} = \frac{\frac{-25}{264}s}{(s^2+25)} + \frac{\frac{36}{143}s}{(s^2+36)} + \frac{\frac{-49}{312}s}{(s^2+49)} \quad (4)$$

Taking the residues in absolute value and scaling them so they add to 1, the normalized output impedance is obtained as:

$$Z_{out}(s) = \frac{0.1880787037s}{(s^2+25)} + \frac{0.5s}{(s^2+36)} + \frac{0.3119212963s}{(s^2+49)} \quad (5)$$

This impedance is expanded with the structure in fig. 2a, starting from the extraction of, say, one half of the high-frequency output capacitance, as C_{3a}. The values are then denormalized for C_1 and L_3 as specified. See Table I.

TABLE I
ELEMENT VALUES FOR ENERGY TRANSFER BETWEEN CAPACITORS.

Element:	normalized	final
C_{3a}	0.5 F	5.463487 pF
L_3	0.027777777778 H	30 mH
C_3	0.558367346939 F	6.101265 pF
C_2	4.783216783217 F	52.266082 pF
L_2	0.005501692829 H	5941.828255 µH
L_1	0.000303527035 H	327.809198 µH
C_1	91.516651180987 F	1 nF

This circuit transfers energy in 10.79 µs, and resonates at 231.65 kHz, 277.98 kHz, and 324.31 kHz. The final capacitances C_3 and C_{3a} are small enough to be distributed capacitances, allowing a realization as the example in [7]. Fig. 3 shows the voltage waveforms when the input capacitors are charged to 10 kV, as if they were slowly charged by a voltage source with nonzero series resistance in series with L_1, and then the source had its output short-circuited. The output voltage reaches 95.7 kV. Observe that the voltages over C_1, C_2, and C_3 start from the same value, and that all the voltages are null, except the output voltage over C_3 and C_{3a}, at the end of the energy transfer. The currents are also null, because the derivatives of all the voltages are null at that instant.

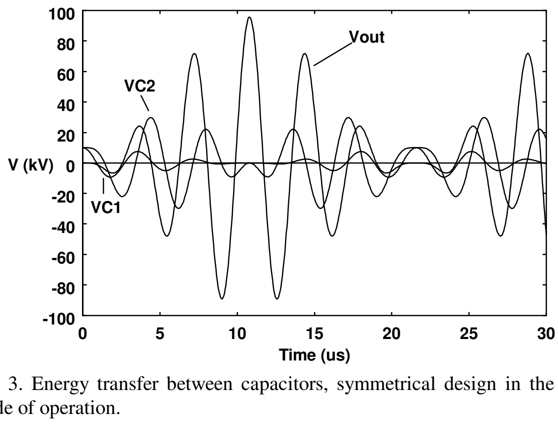

Fig. 3. Energy transfer between capacitors, symmetrical design in the first mode of operation.

Fig. 4 shows the alternative operating mode for this same circuit. Only C_1 is initially charged (as if it were slowly charged by a negative voltage source with nonzero series resistance inserted at its connection with the ground and then the source had its output short-circuited). The output voltage is almost the same, but there is some voltage over C_2 when the energy transfer is complete. A short-circuit at the output at this time would extract all the energy, discharging completely all the capacitors. The asymmetrical design would result in an initial state as in fig. 3 and a final state as in fig. 4. A reverted asymmetrical design would result in the reverse.

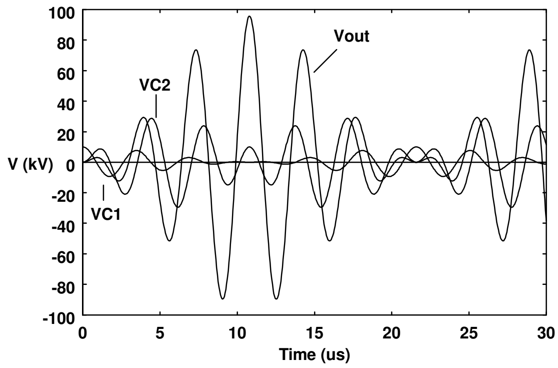

Fig. 4. Energy transfer between capacitors, symmetrical design in the second mode of operation.

Consider now the same structure (fig. 2a) but with initial energy in L_1. A mode that results in voltage waveforms similar to the ones in figs. 3 and 4 is mode 7:9:11. The normalized resonance frequencies are 7, 9, and 11 rad/s, and the power of s in (1) is $2p-2 = 4$. The expansion in partial fractions, now with sinusoids instead of cosinusoids, is then:

$$\frac{s^4}{(s^2+49)(s^2+81)(s^2+121)} = \frac{\frac{2401}{16128}\times 7}{(s^2+49)} + \frac{-\frac{6561}{11520}\times 9}{(s^2+81)} + \frac{\frac{14641}{31680}\times 11}{(s^2+121)} \quad (6)$$

Now, taking the amplitudes of the sinusoids in absolute value and scaling them so they add to 1, the normalized output impedance, corresponding to an output voltage in sum of cosinusoids, is obtained as:

$$Z_{out}(s) = \frac{0.1261029412s}{(s^2+49)} + \frac{0.4824264706s}{(s^2+81)} + \frac{0.3914705882s}{(s^2+121)} \quad (7)$$

The expansion of this impedance in the structure of fig. 2a, again extracting first one half of the high-frequency capacitance, results in the values listed in Table II. As the normalized C_1 resulted smaller than in the previous case, The final C_1 has to be smaller, in order to keep the final capacitances similar to what was obtained before, suitable for distributed realization. C_1=350 pF was used. Complete energy transfer from L_1 to C_{3a} and C_3 occurs in 8.27 µs. The circuit resonates at 211.60 kHz, 272.05 kHz, and 332.51 kHz. Figs. 5 and 6 shows the resulting voltage and current waveforms for a starting current of 10 A in L_1 (currents down and to the right in fig. 2a). The output voltage reaches –92.6 kV. Note the inversion of the energy transfer cycle after the energy returns to L_1. The asymmetrical design of this circuit would result in some voltage in C_2 at the end of the energy transfer cycle, as in fig. 4, and again all the energy could be extracted by a short-circuit at the output.

TABLE II
ELEMENTS FOR INDUCTOR TO CAPACITORS ENERGY TRANSFER.

Element:	normalized	final
C_{3a}	0.5 F	5.435582 pF
L_3	0.011764705882 H	30 mH
C_3	0.603896103896 F	6.565053 pF
C_2	2.906250000000 F	31.594318 pF
L_2	0.003882558320 H	9900.523717 µH
L_1	0.000403382683 H	1028.625841 µH
C_1	32.195266544118 F	350 pF

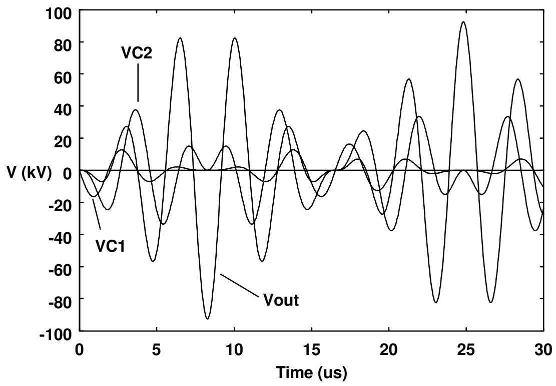

Fig. 5. Voltage waveforms in inductor to capacitor energy transfer, symmetrical mode.

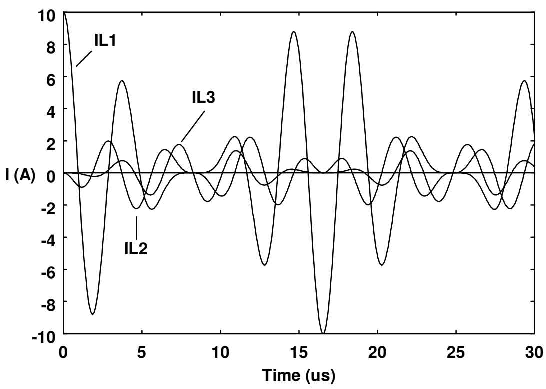

Fig. 6. Current waveforms in inductor to capacitor energy transfer, symmetrical mode.

A transformer could be inserted in this circuit replacing L_1 and L_2, resulting in the structure of fig. 2b. This would reduce the voltage over C_1, at the expense of an increase in the required input current. The circuit could then be scaled in impedance and frequency to compensate for the current increase, increasing the inductances and keeping the capacitances. Eventually a structure similar to conventional induction coils, with high output inductances and low-frequency operation would obtained.

IV. STRUCTURES WITH BALANCED OUTPUT

An interesting possibility with these circuits is to generate a balanced output. The last three elements in figs. 2a or 2b , C_3, L_3, and C_{3a}, can be duplicated and the circuit arranged as in fig. 7. With the circuit operating in the modes shown in figs. 3 or 5, at the end of the energy transfer the voltage over the tank L_3-C_{3a} is identical to the voltage over C_3. If the copy of these elements is assembled reversed, two opposite copies of the output voltage are obtained (observe the signs). If C_{3a} is chosen so $C_{3a} = C_3$, two similar structures with distributed capacitances, as the one shown in [7], can be used, and the total output voltage is doubled. This can also be done with the pure high-pass realizations, but the band-pass realization allows the use of a transformer to set the voltage gain independently of the operating mode.

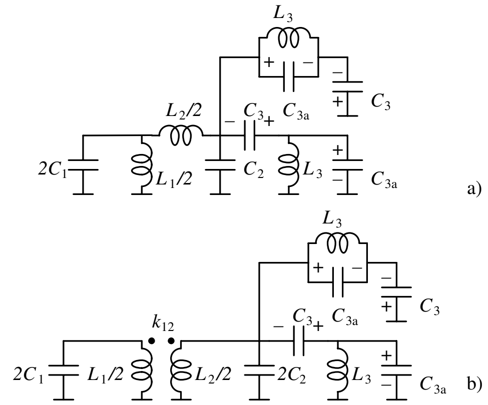

Fig. 7. Band-pass 6[th]-order multiple resonance networks with balanced output. a) transformerless. b) with transformer.

V. CONCLUSIONS

The concept of multiple resonance networks was extended to polynomial bandpass ladder structures, that mix low-pass and high-pass sections. The simple design procedure previously developed was shown to be effective in these cases too. In all cases where high-pass sections are present, four possible designs are possible for the same structure in the case of energy transfer between capacitors, and two designs are possible with energy transfer from inductors to capacitors. A curious form of network with balanced output was also proposed.

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