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Multiple Resonance Networks

Antonio Carlos M. de Queiroz

Abstract—This brief shows how "multiple resonance networks" of any order and with many possible structures can be systematically designed using standard lossless impedance synthesis techniques. These networks are composed of linear lumped or distributed capacitors, inductors, and transformers, with a switch separating one of the capacitors from the remaining circuit. They have the property of transferring completely the energy initially stored in the capacitor insulated by the switch, to another, much smaller, capacitor in the circuit, through a linear transient when the switch is closed. These circuits find applications in the production of very high voltages for pulsed power systems.

Index Terms-Linear network synthesis, power converters, resonance.

I. INTRODUCTION

"Multiple resonance networks" [1] is a name that generalizes the "double resonance" [2], [3], "triple resonance" [4]–[6], and the higher order networks discussed in this brief. These circuits are usually composed of a transformer and some extra capacitors and inductors and work by transferring the energy initially stored in a capacitor at one side of the transformer to another, much smaller, capacitor at the other side of the transformer, through a linear transient composed (in the ideal lossless case) of a sum of several cosinusoidal waveforms (Fig. 1).

The "double resonance" case is long known [2], [7] as the "Tesla coil" [3]. In this case, only two capacitors and one transformer are used, resulting in a fourth-order system with a transient formed by two oscillatory modes (Fig. 2). With the system properly designed, after some cycles all the initial energy in C_1 is transferred to C_2 , and the obtained voltage is given, by energy conservation, by

$$v_{\text{out}} \max = v_{\text{in}}(0) \sqrt{\frac{C_1}{C_p}} \tag{1}$$

(with p = 2). This same equation fixes the maximum output voltage for all the systems of this type.

More recently, triple resonance systems were developed [4]–[6] for instrumentation used in high-energy physics. An additional capacitor and an inductor were added to the output side (Fig. 3), with the aim of reducing the voltage stress over the transformer and of taking into consideration the output capacitance of the transformer. With only the extra inductor added, the system is still a double resonance system, long known as the "Tesla magnifier." With the extra capacitor the system is of sixth order and the transient has three oscillatory modes, but operation with complete energy transfer is equally possible.

In all the cases found in the literature, the design of these systems is based on the analysis of a fixed structure. The following sections show that the design can be made by synthesis, can be applied to a wide range of structures, and can be extended to systems of any order.

II. SYNTHESIS APPROACH

The transformer can be left out of the problem, because it can be inserted after the synthesis of a "ladder" structure composed of series

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Fig. 1. Multiple resonance network. An initial energy in C_1 is totally transferred to C_p through the transformer and two possible *LC* networks, during the transient after the closure of the switch.



Fig. 2. Typical double resonance network, and voltages in C_1 and C_2 after the closure of the switch. The voltage gain was designed as $10. C_1 = 1$ nF, $C_2 = 10$ pF, $L_a = 129.8 \mu$ H, $L_b = 12.98$ mH, and $k_{ab} = 0.2195. v_{C1}(0) = 10$ kV.

inductors and shunt capacitors, with a shunt inductor somewhere, as shown in Fig. 4(a). An ideal transformer with turns ratio 1 : n is inserted at the left side of where the shunt inductor appears and is then converted into a real transformer by using the equivalence shown in Fig. 4(b), where

$$L_{a} = \frac{L_{x}}{n^{2}} \quad L_{b} = L_{x} + L_{y}$$

$$k_{ab} = \sqrt{\frac{L_{x}}{L_{x} + L_{y}}} \quad Z'(s) = \frac{Z(s)}{n^{2}}.$$
(2)

The turns ratio can be chosen as convenient for the desired voltage gain. It multiplies the gain in (1) directly because the input capacitor is multiplied by n^2 . k_{ab} is the coupling coefficient of the resulting real transformer that can be quite small if the energy transfer occurs in many cycles.

If the shunt inductor is at the low-voltage end (as is the case for the fourth- and sixth-order cases in Figs. 2 and 3, with the transformer eliminated, the problem is reduced to the synthesis of the output impedance of the circuit by a succession of complete pole removals at infinity, or an impedance synthesis in Cauer's first form. The following discussion shows how to find the required impedance.

With the switch closed, the impedance seen across any of the capacitors has a denominator of order p, even, and a numerator of order p-1, with a zero at s = 0. The voltage response of one of these impedances to a current impulse applied in parallel with the corresponding capacitor is proportional to the response to a charged capacitor there. It appears as a sum of p pure cosinusoidal oscillations with positive



Fig. 3. Typical triple resonance network and voltages in C_1 , C_2 , and C_3 after the closure of the switch. The voltage gain was also designed as 10. $C_1 = 1$ nF, $C_2 = 126.3$ pF, $C_3 = 10$ pF, $L_a = 110 \mu$ H, $L_b = 780.7 \mu$ H, $L_3 = 10.13$ mH, $k_{\rm ab} = 0.28131$, and $v_{C1}(0) = 10$ kV.



Fig. 4. (a) General structure without transformer. (b) Equivalence that allows the insertion of a transformer where a shunt inductor appears.

multiplying factors. Sinusoidal components don't appear and the multiplying factors must be positive, due to the proportionality between the Laplace transform of the voltage waveform and the impedance at that point in Foster's first form. With the *p* oscillation frequencies considered as distinct integer multiples of a basic frequency ω_0 by factors $k_i, j = 1, \dots, p$, all the capacitor voltages have the forms

$$Z_{\text{in},i}(s) \propto V_i(s) = \sum_{j=1}^{p} \frac{A_{ijs}}{s^2 + k_j^2 \omega_0^2}$$
(3)

(Laplace transform) and

$$v_i(t) = \sum_{j=1}^p A_{ij} \cos\left(k_j \omega_0 t\right) \tag{4}$$

(time domain).

The currents in all the inductors are then proportional to the derivatives of the capacitor voltages (4), and so are all sums of p sinusoids at the same frequencies

$$i_i(t) = \sum_{j=1}^p B_{ij} \sin(k_j \omega_0 t).$$
 (5)

It is convenient to work from the output of the network and compute the output impedance. Considering C_p initially charged to v_p , the energy there is transferred to C_1 using the "return" part of the (perpetual) transient waveform. As all the k_j are different positive integers, all the currents reduce to 0 at $t = \pi/\omega_0$. If the output is excited by a unit impulse source, the proportionality in (3) becomes an identity. From (3) and the structure of the network [Fig. 4(a)], when $s \to \infty$:

$$\sum_{j=1}^{p} A_{pj} = \frac{1}{C_p}.$$
 (6)

At $t = \pi/\omega_0$, the backward energy transfer is complete, and $v_p = 0$. From (4) we have

$$v_p\left(\frac{\pi}{\omega_0}\right) = \sum_{j=1}^p A_{\rm pj}\cos(k_j\pi) = \sum_{j=1}^p A_{\rm pj}\left(-1\right)^{k_j} = 0.$$
(7)

At the same instant, $v_{p-1} = 0$, $v_{p-2} = 0$, ..., $v_2 = 0$. At any time after t = 0, considering the currents i_i in the inductors in Fig. 4(a) going to the right, we have

$$v_{i-1}(t) = v_i(t) + L_i \frac{di_i(t)}{dt}, i = p, p - 1, \dots, 3.$$
 (8)

Expanding these expressions as functions of $v_p(t)$, it can be shown that these voltages are all zero at $t = \pi/\omega_0$ if all the even derivatives of $v_p(t)$ up to order 2p - 4 are null at this instant. Combining this condition with (6) and (7), and eliminating powers of ω_0 and of -1that multiply the derivatives of $v_p(t)$, the following system of equations results:

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ (-1)^{k_1} & (-1)^{k_2} & \cdots & (-1)^{k_p} \\ k_1^2 (-1)^{k_1} & k_2^2 (-1)^{k_2} & \cdots & k_p^2 (-1)^{k_p} \\ \vdots & \vdots & \ddots & \vdots \\ k_1^{2p-4} (-1)^{k_1} & k_2^{2p-4} (-1)^{k_2} & \cdots & k_p^{2p-4} (-1)^{k_p} \end{bmatrix} \cdot \begin{bmatrix} A_{p1} \\ A_{p2} \\ \vdots \\ A_{pp} \end{bmatrix} = \begin{bmatrix} \frac{1}{C_p} \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \quad (9)$$

It is observed that positive solutions for all the A_{pj} are only obtained if the powers of -1 have alternate signs for increasing k_i . This adds a condition on the k_j , mentioned in [2], [4]–[6], and that extends for higher orders: Given a positive integer as k_i , the next value k_{i+1} is obtained by adding an odd positive integer to k_j . Valid sequences are then 1, 2, 3, ...; 2, 3, 4, ...; 1, 2, 5, ...; 1, 4, 5, ...; etc. This "rule" is stated here without a formal proof, but no exceptions could be found. Even differences between all the successive k_j or identical k_j result in a singular system. Sequences with mixed odd and even differences result also in solvable systems, but produce negative residues. The rule allows a further simplification of (9) [1] with the elimination of the powers of -1, and is assumed in the deduction of the formulas presented in the following sections. A particularly interesting alternative method for the calculation of the residues of the output impedance of the network, that does not require the solving of a system of equations, is discussed in [8].

With the A_{pj} computed, an *LC* network can be obtained by the expansion of the output impedance (3) in ladder form. Alternative forms for the expansion of the impedance are also possible, for example, extracting the shunt inductor at other points of the expansion, or extracting more than one shunt inductor. With this, a transformer can be inserted at other points, or more than one transformer can be inserted.

III. EXAMPLES

A. Fourth-Order Case

This is the classic double resonance circuit, but without a transformer. The system of equations in (9) with C_2 normalized to 1 reduces to two equations that give $A_{21} = A_{22} = 1/2$. The output impedance



Fig. 5. Structures for transformerless multiple resonance networks of: (a) fourth, (b) sixth, and (c) eighth orders.

of the network is then, normalizing ω_0 to 1 and naming $k_1 = k$ and $k_2 = l$:

$$Z_{\text{out}} = \frac{\frac{1}{2}s}{s^2 + k^2} + \frac{\frac{1}{2}s}{s^2 + l^2} = \frac{s^3 + \frac{1}{2}\left(k^2 + l^2\right)s}{s^4 + \left(k^2 + l^2\right)s^2 + k^2l^2}.$$
 (10)

This impedance, expanded in Cauer's first form, results in the structure in Fig. 5(a), with the values

$$C_{2} = 1; \qquad L_{2} = \frac{2}{k^{2} + l^{2}}$$

$$C_{1} = \left(\frac{k^{2} + l^{2}}{l^{2} - k^{2}}\right)^{2}; \qquad L_{1} = \frac{\left(l^{2} - k^{2}\right)^{2}}{2\left(k^{2} + l^{2}\right)k^{2}l^{2}} \qquad (11)$$

This circuit produces the voltage gain [from (1)]

$$\frac{v_{C\,2\,\max}}{v_{C\,1}(0)} = \sqrt{\frac{C_1}{C_2}} = \frac{k^2 + l^2}{l^2 - k^2}.$$
(12)

If a transformer is inserted through the use of the relations shown in (2), the relations for the elements in Fig. 2 reduce to

$$C_{1}L_{a} = C_{2}L_{b} = \frac{k^{2} + l^{2}}{2k^{2}l^{2}}$$

$$k_{12} = \frac{l^{2} - k^{2}}{k^{2} + l^{2}}.$$
(13)

The turns ratio n, actually just a number because the coils can have different geometries, affects only the voltage gain (12), multiplying it directly. The second equality in the first equation just sets the energy transfer time to π seconds.

B. Sixth-Order Case

For the triple resonance case, the system (9) has three equations. Symbolic expressions for the element values can be obtained as

$$A_{31} = \frac{l^2 - m^2}{2(k^2 - m^2)}; A_{32} = \frac{1}{2}; A_{33} = \frac{k^2 - l^2}{2(k^2 - m^2)}$$

$$C_3 = 1; L_3 = \frac{1}{l^2}; C_2 = \frac{2l^4}{(l^2 - m^2)(k^2 - l^2)}$$

$$L_2 = \frac{(l^2 - m^2)(k^2 - l^2)}{l^2(k^2(l^2 + m^2) - l^2(l^2 - m^2))}$$

$$C_1 = \left(\frac{k^2(l^2 + m^2) - l^2(l^2 - m^2)}{(k^2 - l^2)(l^2 - m^2)}\right)^2$$

$$L_1 = \frac{(k^2 - l^2)^2(l^2 - m^2)^2}{2k^2l^2m^2(k^2(l^2 + m^2) - l^2(l^2 - m^2))}$$
(14)

for the structure in Fig. 5(b), with $k_1 = k$, $k_2 = l$, and $k_3 = m$. The voltage gain is given by

$$\frac{v_{C3\max}}{v_{C1}(0)} = \sqrt{\frac{C_1}{C_3}} = \frac{k^2 \left(l^2 + m^2\right) - l^2 \left(l^2 - m^2\right)}{(k^2 - l^2) \left(l^2 - m^2\right)}.$$
 (15)

Values/Mode	1, 2, 3, 4	2, 3, 4, 5	3, 4, 5, 6	1, 2, 3, 6	1, 2, 5, 6	1, 4, 5, 6
C ₄	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
L_4	0.181818	0.086957	0.051282	0.222222	0.117647	0.047187
C ₃	1.920635	3.918519	6.584416	1.285714 ⁻	0.583838	5.841289
L ₃	0.078456	0.020310	0.007395	0.045752	0.172294	0.012073
C ₂	3.383492	12.626002	34.290043	1.651429	1.177530	6.949514
L ₂	0.051554	0.006828	0.001491	0.083353	0.043393	0.024434
C ₁	7.060408	50.342404	227.082720	5.533696	3.644628	3.134893
L,	0.051453	0.002312	0.000266	0.077492	0.126042	0.039203
Voltage gain	2.657143	7.095238	15.069264	2.352381	1.909091	1.770563

 TABLE I

 NORMALIZED ELEMENT VALUES FOR QUADRUPLE RESONANCE NETWORKS [FIG. 5(C)], AS FUNCTIONS OF THE FREQUENCY MULTIPLIERS k_1 , k_2 , k_3 , and k_4 . In all Cases the Total Energy Transfer Occurs in π Seconds



Fig. 6. Experimental voltage waveforms obtained for an eighth-order network [Fig. 5(c)] in mode 1, 2, 3, 4. $C_4 = 100$ nF, $L_4 = 461 \mu$ H, $C_3 = 192$ nF, $L_3 = 199 \mu$ H, $C_2 = 338$ nF, $L_2 = 131 \mu$ H, $C_1 = 706$ nF, and $L_1 = 130 \mu$ H. Measured voltage gain: 2.23. Energy transfer time: 50 μ s. The dark traces are v_{C1} and v_{C2} , and the light traces are v_{C3} and v_{C4} .

For the structure with a transformer (see Fig. 3), convenient design equations, adapted and normalized to $\omega_0 = 1$, are

$$\frac{L_b}{L_3} = \frac{\left(l^2 - m^2\right)\left(k^2 - l^2\right)}{2k^2m^2}, \ \frac{C_2}{C_3} = \frac{2l^4}{\left(l^2 - m^2\right)\left(k^2 - l^2\right)}$$
$$L_a C_1 = \left(L_b + L_3\right)C_3, \ k_{12} = \sqrt{\frac{L_b}{L_b + L_3}}.$$
 (16)

These equations show several curious dependencies among the components of the structure, similar to what happens in the fourth-order case [see (13)].

C. Eighth-Order Case

The extension to higher orders results in higher voltage gain for the same basic frequencies of operation, and maybe smaller voltage differences across the series inductors (with the exception of the last, that always has to sustain the full output voltage). No attempts of design or of applications for networks with orders greater than 6 could be found

in the literature. Symbolical expressions continue to be relatively easy to derive for the element values, but become rather impractical for the "quadruple resonance" case and above. Table I lists numerical normalized ($C_p = 1, \omega_0 = 1$) element values for the structure in Fig. 5(c) for some of the possibly more practical combinations of frequency multipliers.

An experimental circuit was constructed, operating in mode 1, 2, 3, 4. The values in Table I were denormalized for resonances at 10, 20, 30, and 40 kHz, with $C_4 = 100$ nF. The element values were adjusted for maximum error of 1%, and the resistances of the inductors, wound on ferrite pot-cores, were kept around 1 Ohm. In this example, it was not necessary to compensate for parasitic capacitances in the inductors and all the capacitances were lumped. A mechanical switch was used to start the energy transfer. Fig. 6 shows the resulting voltage waveforms obtained. The measured voltage gain was just 84% of the ideal, mainly due to the losses, but the waveforms are very similar to the ideal ones.

IV. CONCLUSION

A systematic procedure for the design of *LC* voltage multipliers that are a generalization of the "double resonance" and "triple resonance" networks was presented. The procedure first obtains a special *LC* impedance that is then expanded in ladder form. A transformer is not necessary, but can be easily included. Only lossless circuits were considered, but the applications of these circuits generally require low losses, and they are designed to behave as lossless circuits. The presence of small losses does not significantly affect the waveforms in practical circuits of this type, adding essentially only a decay with time in the waveforms.

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Advanced Feedback Control of the Chaotic Duffing Equation

Zhong-Ping Jiang

Abstract—This brief deals with the celebrated chaotic Duffing equation with external control force. It is shown that Lyapunov direct method in conjunction with recent developments in nonlinear control yields a promising way of engineering chaotic dynamics. Among the three types of feedback controllers introduced in the paper, we particularly emphasize the value of linear feedback strategy in controlling chaos. For the forced Duffing equation, it is shown that linear feedback control laws are inherently robust to (even large) sensor errors.

Index Terms—Adaptive nonlinear control, Duffing equation, exponential convergence, global stability, linear feedback.

I. INTRODUCTION

Chaos control has been an active research field in recent years. Various control methodologies have been developed by many researchers from a point of view of dynamic system theory and traditional feedback control. Among these creative control algorithms are the celebrated OGY method of small time-dependent pertubations of an available system parameter [16] and Lyapunov control methods [3], [5], [7], [14] (see the books [1], [2], [10], [15] for a rather complete list of references in this quickly expanding area). Possibilities of applying chaotic system theory to secure communication have also been considered and justified by experimental work (see, e.g., [2], [4], [8], [17], [22] and references therein).

The purpose of this brief is to make novel contributions to Lyapunov control of chaotic continuous-time dynamic systems. Because of the tremendous complexity of chaotic dynamics, we will restrict ourselves to Duffing's equation which has been investigated as a benchmark chaotic system in several articles [3], [6], [7], [13], [14]. It is hoped that our methodologies developed for this pecular chaotic system will be applicable to other types of chaotic dynamic systems such as Chua's circuits and Lorenz chaotic attractor [2], [10].

In this paper, we consider a general form of Duffing's equation with external control input u

$$\ddot{x} + p\dot{x} + p_1x + p_2x^3 = u + q\cos(\omega t).$$
(1)

The control u is added in order to order or guide the chaotic dynamics to meet our specific requirements. We are interested in driving the state

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x to an appropriately defined reference signal x_d . This issue is widely known as the tracking problem in the control community. The first contribution of this paper is that new solutions to the tracking problem are obtained with the help of advanced nonlinear control theory. Previous work of others [3], [6], [7], [13], [14] have presented interesting results on the tracking of a more restrictive form of the forced Duffing equation (1). For example, the seemingly first solution developed by Chen and Dong [3] is applicable to Duffing's equation only when p > 0and $p_2 = 1$. In addition, their result solves the local tracking problem, i.e., only those trajectories of (1) starting from a small neighborhood of the desired reference orbit can be asymptotically controlled to the desired trajectory. The assumption that p > 0 was relaxed by Nijmeijer and Berghuis [14] following classical Lyapunov direct method. Global tracking results were obtained in [13], [14]. In this paper, we do not require that p > 0 and $p_2 = 1$. When all system parameters are known, we present two different feedback controllers to solve the global tracking problem with arbitrary rate of exponential convergence. Such a property of stability is introduced in Section II. The first such controller is a linear time-varying state-feedback control law. The second one is derived from the first one in conjunction with a nonlinear observer without assuming that \dot{x} is measurable. When the system parameters p, p_1 , p_2 and q are unknown, we develop a nonlinear adaptive controller to achieve the global tracking task. It is simply shown that the popular method of adaptive backstepping [12] proves useful for chaos control, in particular for the forced Duffing equation in the general form (1).

The second contribution of this paper is to argue the importance of linear feedback strategy in the context of controlling chaos. Linear feedback control laws are simpler to implement in practice than nonlinear controllers and therefore more acceptable by practicing engineers. More importantly, linear controllers are often less sensitive to sensor errors and are inherently robust against measurement errors. It is well-known in the nonlinear control community that a globally stabilizing nonlinear controller may not be robust in front of (even small) sensor errors—see [9] and references therein for detailed discussions. Intuitively, the effect of measurement errors can be amplified through the nonlinearity of the control law in question, therefore leading to instability. A redesign of nonlinear control is needed to guarantee the robustness property. In case when the full-state information is available, we construct a linear feedback controller to guarantee the property of global exponential convergence with arbitrary rate for the tracking error. In particular, we show that this linear feedback control law enjoys the inherent robustness to (even large) measurement errors (see Proposition 1 below).

The rest of this paper is organized as follows. In Section II, we recall some definitions from the literature of stability theory and introduce a new notion of stability. The statement of the control problem is also given in Section II. Section III presents two tracking algorithms for the controlled Duffing equation (1). Linear state-feedback and nonlinear observer-based output-feedback controllers are obtained. It should be mentioned that our observer structure is quite different from the mechanism of observing chaos proposed in [20]. In Section IV, we show how to remove the assumption of requiring the precise knowledge of system parameters and propose nonlinear adaptive controllers. Simulation results are offered in Section V to support our theoretic findings. Some concluding remarks are contained in Section VI.

II. DEFINITIONS AND PROBLEM STATEMENT

First of all, recall some definitions that we will frequently use throughout this paper. A new concept of stability is introduced. The goal of this paper is to show that we can achieve this type of stability

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