Triple resonance pulse transformer circuit

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A modified, resonant pulse transformer circuit, based on a generalization of the well-known double-resonance pulse transformer circuit, is described. This modified circuit allows complete energy transfer in the presence of non-negligible internal capacitance of realistic pulse transformers, and substantially reduces peak transformer voltage, compared to the double-resonance circuit. Conditions under which the internal capacitance significantly affects energy transfer, and sensitivity of overall efficiency to circuit component values are discussed.

I. INTRODUCTION

Circuits for pulse-charging high-voltage pulse forming lines (PFL) are generally based either on a Marx generator or on a pulse transformer. Pulse-transformer-driven circuits are typically designed to operate as a Tesla, or double-resonance circuit.[1–3] When the circuit is tuned to resonance, energy is efficiently transformed from an energy storage capacitor to the PFL. However, the simple equivalent circuit model for a Tesla transformer neglects the internal capacitance of the transformer. The presence of the additional capacitance in the circuit can significantly affect circuit operation. A modified circuit design that takes into account the internal capacitance was patented in 1989.[4] The effect of transformer capacitance on the circuit is described in Sec. II, and the modified circuit design is described in Sec. III.

II. DOUBLE RESONANCE CIRCUIT

The Tesla transformer circuit is a special case of the circuit shown in Fig. 1. Here C_1 is the energy storage capacitor, C_2 is the internal capacitance of the transformer, C_3 is the PFL capacitance, L_1, L_2, and L_3 are the inductances of the transformer primary, transformer secondary, and an external tuning/isolation inductor, respectively, and M is the mutual inductance of the transformer. The circuit (assuming C_2 of Fig. 1 is zero) becomes resonant when it meets the conditions $L_1C_1 = (L_2 + L_3)C_3$ and $k_m = M/\sqrt{L_1(L_2 + L_3)} = 0.6$. A complete coupling of the energy, neglecting damping, takes place from C_1 to C_3 when the primary switch is closed.

Internal capacitance of the transformer significantly affects circuit operation under some conditions. If the internal capacitance is comparable to the capacitance of the PFL, the double resonance design becomes inefficient because a large fraction of the available energy remains stored in the transformer, and is unavailable to the output. The reduction in output efficiency was observed on the MEDEA-II device.[3] Internal capacitance may be estimated for an air-core spiral transformer[5] as the capacitance between the coaxial inner core and outer ring cage of the transformer. Distributed turn-to-turn capacitance of the transformer windings is not considered in this model.

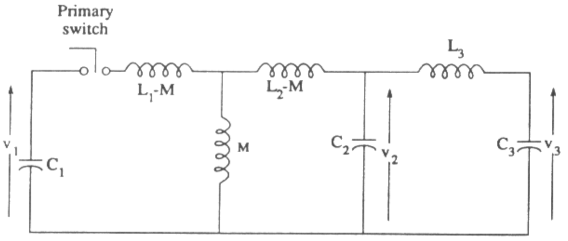

FIG. 1. Equivalent circuit of a transformer-coupled pulse-line charging circuit, neglecting resistive elements.

Figure 2 illustrates the effect of a finite internal capacitance on circuit efficiency. As C_2 increases, the energy transfer efficiency of the double-resonance transformer circuit is reduced from the theoretical 100% in the absence of C_2. For small external inductance L_3, the drop is monotonic; however, as L_3 increases, peaks appear in the efficiency curves. This paper describes the conditions under which these peaks approach 100%, again allowing complete energy transfer.

III. TRIPLE RESONANCE CIRCUIT

The coupled circuit equation describing the voltages across the three capacitors of the equivalent circuit shown in Fig. 1 is

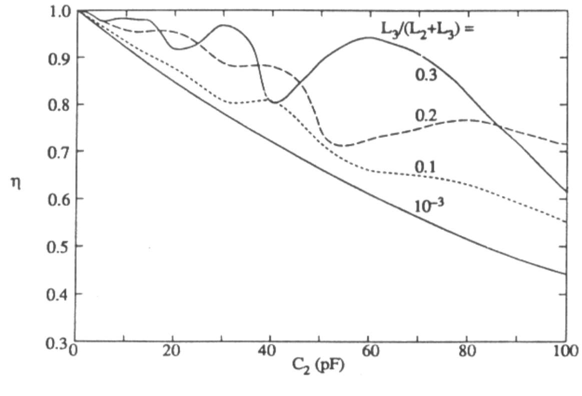

FIG. 2. Effect of internal capacitance and tuning inductor on double-resonance transformer. $C_1 = 0.2\ \mu$F, $C_3 = 125$ pF, $L_1 = 1.0\ \mu$H, $L_2 + L_3 = 1600\ \mu$H, $M = 0.6\sqrt{L_1(L_2 + L_3)}$.

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TABLE I. Triple resonance circuit parameters.

	1:2:3	1:2:5	$k{:}l{:}m$
$\omega_0^2 L_1 C_1$	$\frac{11}{24}$	$\frac{113}{200}$	$\frac{2m^2k^2+(m^2-l^2)(l^2)(l^2-k^2)}{2k^2l^2m^2}$
$\omega_0^2 L_2 C_2$	$\frac{4}{9}$	$\frac{4}{25}$	$\frac{l^2}{k^2m^2}$
$\omega_0^2 L_3 C_3$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{l^2}$
k_m^2	$\frac{5}{11}$	$\frac{63}{113}$	$\frac{(m^2-l^2)(l^2-k^2)}{2m^2k^2+(m^2-l^2)(l^2-k^2)}$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = - \begin{bmatrix} L_1C_1 & MC_2 & MC_3 \\ MC_1 & L_2C_2 & L_2C_3 \\ MC_1 & L_2C_2 & (L_2+L_3)C_3 \end{bmatrix} \begin{bmatrix} \ddot{v}_1 \\ \ddot{v}_2 \\ \ddot{v}_3 \end{bmatrix}.$$

The circuit has a purely oscillatory transient response with three natural frequencies ω_1, ω_2, and ω_3. If the capacitor C_1 is initially charged to voltage v_{10}, the voltages may be described as $v_i = \Sigma\, a_{ij} \cos \omega_j t$ where $i, j = 1,2,3$ and where $t = 0$ corresponds to the time at which the primary switch closes. A resonant condition may be found at which 100% of the energy initially stored in the capacitor C_1 is transferred to the capacitor C_3. The circuit becomes resonant if the voltages v_1 and v_2, and their time derivatives $\dot{v}_1$ and $\dot{v}_2$, are zero when the PFL voltage v_3 reaches its peak. This can only occur when the three normal mode frequencies $\omega_1 = k\omega_0$, $\omega_2 = l\omega_0$, and $\omega_3 = m\omega_0$ are in the ratio of whole numbers k, l, and m where $l = k + 1, k + 3, k + 5, \ldots$ and $m = l + 1, l + 3, l + 5, \ldots$. The two simplest resonant modes, which are also the modes of greatest practical interest, are those in which the three normal mode frequencies of the coupled circuit are in the ratios 1:2:3 and 1:2:5. Considering the coupled circuit equation, and requiring 100% energy transfer, a solution[4] for the coefficients a_{ij} and thereby, the conditions on circuit components may be found (Table I).

Figure 3 shows the voltage wave forms for two resonant conditions, calculated for typical conditions: an overall voltage step-up of 40 and a fundamental resonant frequency of 250 kHz. The entire energy in the circuit may be extracted from C_3 when v_3 reaches its peak at $t = 2\ \mu$s. The energy remaining in the transformer primary and secondary circuits is zero at this time. A parameter of particular interest is the ratio of peak transformer voltage to peak PFL voltage. For the simplest case of a 1:2:3 resonance, this ratio is 0.36; for the case of a 1:2:5 resonance, the ratio is 0.60. The voltage appearing across the pulse transformer is significantly less than the output voltage; instead, the full output voltage appears across the external inductor. This transfer of peak voltage stress is significant because, unlike the transformer, the external inductor does not suffer from the conflicting requirements to maintain close magnetic coupling and good electrical insulation. Therefore, it is easier to construct an external inductor than a pulse transformer to maintain the full output voltage without breakdown.

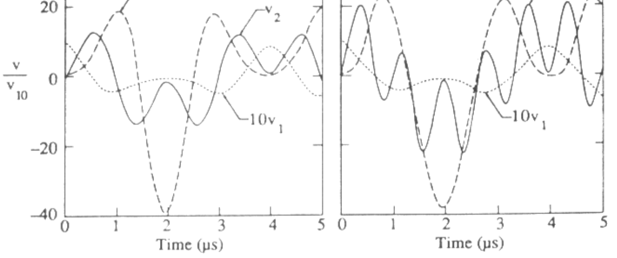

FIG. 3. Typical wave forms for triple-resonance operation.

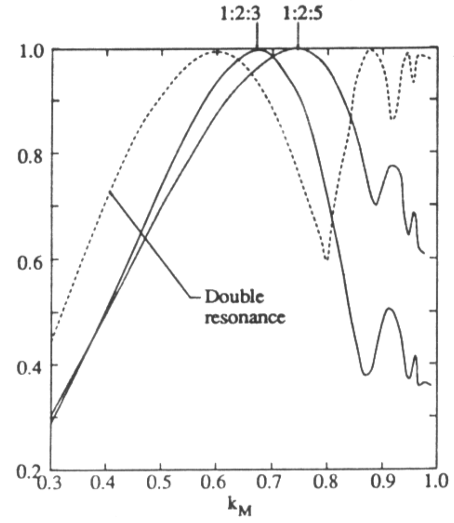

FIG. 4. Dependence of energy transfer coefficient of double- and triple-resonance circuits on transformer coupling coefficient, k_m.

The effect on efficiency of circuit component tolerances is similar to that of the double-resonance circuit. For example, the effect of variations in the transformer coupling coefficient on the energy transfer efficiency is shown in Fig. 4. Also shown for comparison is the double-resonance circuit. Note that a slightly higher coupling coefficient is required in the triple resonance cases. The width of the curve, however, is similar in all three cases. A survey of circuit components shows that, given a ± 10% variation in all component values, the theoretical efficiency remains above 78%, and in most cases is higher than 90%.

IV. DISCUSSION

Application of the circuit to the MEDEA II electron beam generator is described elsewhere.[6] An important advantage of the circuit is that peak transformer voltage is significantly lower than in the double-resonance circuit. Because of the reduction in peak transformer voltage, the circuit may have other applications in the design of resonant pulse-line charging circuits with miniaturized pulse transformers.

ACKNOWLEDGMENT

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