# DISC LEVITATION EXPERIMENT

## Detailed Data

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1. Drawings

1.1. Disc and Electromagnetic

\[ R = 3.50 \times 10^{-2} \text{m} \]
\[ R_1 = 0.02 \text{ m} \]
\[ R_2 = 0.03 \text{ m} \]
\[ \text{Cutout} = 0.01 \text{ m} \]
\[ H = 0.11 \text{ m} \]
\[ H_d = 1.50 \times 10^{-2} \text{m} \]

(a) \hspace{1cm} (b)

Figure 1 - Cross section of the levitating disc experiment of the (a) front view and (b) top view.

\[ L = 1.25 \times 10^{-1} \text{m} \]
\[ R' = 1.60 \times 10^{-1} \text{m} \]

Figure 2 - Displacement of the Gravity Center (GC) of the experiment.
1.2. Sensor Position

Figure 3 shows how the position measure is obtained. To obtain the gap measurement, it is necessary to make an algebraic calculation according to figure below and position reference in figure 2. To protect and immobilize the sensor, it was attached in a box that cushions the disc falling. The ultrasonic sensor has an offset of $3.50 \times 10^{-2}$ m, which means that the linear measure is $X_m$ and not $X_m'$.

\[ * = 2.09 \times 10^{-1} \text{m} \]

\[ D_1 = 13.97 \times 10^{-2} \text{m} \]

\[ D_2 = 2.17 \times 10^{-2} \text{m} \]

\[ B_1 = 3.20 \times 10^{-2} \text{m} \]

\[ B_2 = 1.10 \times 10^{-2} \text{m} \]

\[ \text{Disc} = D_1 + D_2 \]

\[ X_m' = \text{Offset} + X_m \]

\[ * + B_1 + B_2 = \text{GAP} + \text{Disc} + X_m' \]

Figure 3 – Arrangement of the sensor on the testbed.
Based on equations (1-3), it is possible to obtain the gap measure:

\[
\text{GAP} = * + B1 + B2 - \text{Disc} - \text{Offset} - Xm = K_3 - Xm = 3.40 \times 10^{-2} - Xm. \tag{4}
\]

The output of the ultrasonic sensor is a signal \( V_m \) in volts. Therefore, the gap in volts, considering the gain of the ultrasonic sensor \( K_{\text{pos}} = 248.36 \text{ Vm}^{-1} \), is:

\[
\text{GAP (V)} = K_3 - \frac{V_m}{K_{\text{pos}}}. \tag{5}
\]

1.3. Magnetic Field

In order to obtain a constant magnetic field and allow the analytical calculations in the disc article “Levitação Eletromagnética de um Disco”, the disc has cut outs. Figure 4 is presented to show the differences among the magnetic field in the region of air gap for the sphere, flat disc and the disc with cut outs.

Figure 4 – Magnetic density flux (vector direction in black and magnitude below in gradient) on half of the (a) sphere, (b) flat disc and (c) modified disc for a current of 0.50 A in the electromagnet and a gap of \( 5.00 \times 10^{-3} \text{m} \).
2. List of Materials

- 1 Data Acquisition Board PCI-1711;

- 1 **Position Sensor Circuit**:
  1 ultrasonic position sensor U-Gage S18U;
  1 source 15 $V_{dc}$;

- 1 **Current Sensor Circuit**:
  1 current sensor LA 25-NP;
  1 source 30 $V_{dc}$;
  1 resistor 100Ω;

- 1 **Power Actuation Circuit**:
  2 mosfets IRF 640N;
  2 diodes BYT79;
  1 source 30 $V_{dc}$ 180 W;
  1 Electromagnetic, 3126 turns of copper wire and 5.20 Ω;

- 1 **Protection Circuit**:
  2 optocouplers 4N35;
  2 resistors 100Ω;
  2 resistors 56Ω;
  2 resistors 2k6Ω;
3. Electronic Circuits

3.1. Position Sensor and Current Sensor Circuit

Figure 5 – Position sensor circuit.
3.2. Protection and Power Actuation Circuit

Figure 6 – Protection and power actuation circuit.

4. Hardware Information

The control used in this system is a real time digital control through the Simulink, a tool of real time simulations of Matlab. As shown in the article “Levitação Eletromagnética de um Disco”, two signals enter the computer (ultrasonic sensor output and current sensor output) and only one out (the current reference signal, to raise or decrease the current).

It is known that the system natural frequency is 53 rads\(^{-1}\) (approximately 10Hz) and to properly control the system, the sampling rate of the command signal must be
much larger (at least 10 times). Also, this determines the minimum sampling rate to the inputs. Considering the sensors response time (see data-sheets in next Section) and the natural frequency of the system, it was chosen the following sampling rates:

- Ultrasonic sensor (input) = 1.00 \times 10^{-3}s;
- Current sensor (input) = 66.67 \times 10^{-6}s;
- Command signal (output) = 66.67 \times 10^{-6}s;

4.1. Components Data-Sheets

- Ultrasonic Position Sensor
- Current Sensor
- Mosfet
- Diodes
- Acquisition Board
- Optocouplers
5. MATLAB Codes

5.1. LEAD Controller

This Matlab code generates the transfer functions (G, H, F and C) that must be imported into the rlttool to create the Root Locus of the system with the lead controller and to analyze its step response.

\[
\begin{align*}
\text{kc} &= 0.51; \quad \text{Current sensor gain} \\
K &= 0.003664; \quad \text{Plant constant} \\
m &= 1.378; \quad \text{Object mass} \\
g &= 9.81; \quad \text{Acceleration of gravity} \\
\text{x0} &= 0.01; \quad \text{Equilibrium Position} \\
\text{i0} &= ((m \cdot g) / K) \cdot (\text{x0}^2)^0.5; \quad \text{Equilibrium Current} \\
\text{Kx} &= (2 \cdot K \cdot \text{i0}^2) / (\text{x0}^3); \\
\text{Ki} &= (2 \cdot K \cdot \text{i0}) / (\text{x0}^2); \\
\text{K1} &= \text{Ki} / m; \quad \text{Plant Parameter} \\
\text{K2} &= \text{Kx} / m; \quad \text{Plant Parameter} \\
\text{gn} &= \text{kc} \cdot [\text{K1}]; \quad \text{Plant numerator} \\
\text{gd} &= [1 \ 0 \ -\text{K2}]; \quad \text{Plant denominator} \\
G &= \text{tf}([\text{gn} \ \text{gd}]); \quad \text{Plant = Transfer function of the system} \\
\text{kpos} &= 248.36; \\
F &= 0.001 \cdot \text{kpos}; \quad \text{Necessary to transform step to 0.001m}
\end{align*}
\]
5.2. Controller to Reject Steady State Error

This Matlab code generates the transfer functions (G, H, F and C) that must be imported into the rltool to create the Root Locus of the system with the lead controller with additional integral term and to analyze its step response.
K1=Ki/m; % Plant Parameter
K2=Kx/m; % Plant Parameter

gn=kc*[K1]; % Plant numerator
gd=[1 0 -K2]; % Plant denominator
G=tf(gn,gd); % Plant = Transfer function of the system

kpos=248.36;
F=0.001*kpos; % Necessary to transform step to 0.001m

H=kpos; % Ultrasonic sensor gain

Kp=6.94; % Proportional gain of C(s)
z=56; % Absolut value of the zero of LEAD
p=280; % Absolut value of the pole of LEAD
ki=0.2; % Integral gain
cn=Kp*[1 (Ki+z) ki*p]; % Controller numerator
cd=[1 p 0]; % Controller denominator
C=tf(cn,cd); % Transfer function of the Controller

rltool % open rltool
6. Simulink Diagrams

6.1. Position Simulation

To analyze the system and compare to the step response of the rltool, the Simulink Diagram below can be made. According to the reference in figure 2, the position is negative. The initial vertical velocity and position of the disc can be adjusted in the integral blocks of the plant (those two in the right).

![Simulink Diagram](image)

Figure 7 – Offline Position Simulation.

6.2. Gap Simulation

To simulate the gap variation, it is important to pay attention to the system reference in figure 2 (gap is always positive) and in the relation below:

\[ \Delta GAP = -\Delta X. \quad (6) \]

Equation (6) above changes the equation (17) of the article “Levitação Eletromagnética de um Disco” to:

---

*Note: The diagram appears as an image.*
Also, to include real measures to compare to real results, it is necessary to obey the relations of equation (4). Therefore, to analyze the gap and its variation over time, some changes must be made in the diagram in figure 7, including the addition of an offset current, resulting in the figure below.

![Diagram](image)

Figure 8 – Offline Gap Simulation.

### 6.3. Real Time Control Interface

The real time (online) diagram has a few differences in relation to these offline simulations. As explained in Section 4, there are two inputs (the outputs of the ultrasonic sensor and the output of the current sensor) and one output (the current reference, positive or negative). This online approach change some features:
- The plant is removed from the diagram;
- The ultrasonic gain is removed, since the reference is in Volts;
- A current offset must be added, so that when the position measured is equal to the position reference, the controller output won’t be zero;
- To control the current, an extra control loop had to be added;
- The current sensor signal must be multiplied by the current gain $kc$ to compare to the controller output in Amper;
- As the command signal (the output of the interface) must be positive or negative (high or low), it was added a digital relay before it;

Figure 9 show how is the real-time Simulink interface.

![Diagram of the control system](image)

Figure 9 – Online control interface.

### 7. Adaptative Control

#### 7.1. Plant Variation

According to equations (11, 12 and 17) of the article “Levitação Eletromagnética de um Disco”, the plant parameters vary, because they depend on the equilibrium
position \((x_0)\). Table 1 shows how these parameters vary. The equilibrium current \((i_0)\) was calculated by equation (8).

\[
i_0 = \sqrt{\frac{P x_0^2}{K}} = \sqrt{\frac{8 \, m \, g \, x_0^2}{\mu_0 \, A \, N^2}}
\]  

(8)

Table 1 – Plant’s parameters variations.

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<th>(x_0 , (m))</th>
<th>(i_0 , (A))</th>
<th>(Kx)</th>
<th>(Ki)</th>
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7.2. Scheduler Gain Controller

Under Construction…