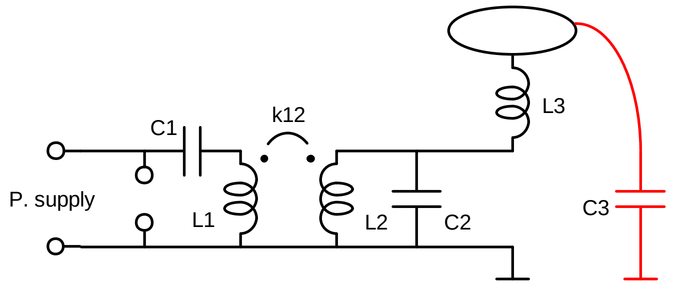
Designing a Tesla Magnifier



The idea is to start from this primary coil:
L_1: 859.1nH (calculated, 50mm x 3 mm copper strip)
This is the top load::
Toroid with:
D: 600mm (measured)
d: 170mm (measured)
With these dimensions:
C_3 = 26.33 pF (not 40.38 pF)
And this is the third coil:
389mH (measured Meterman 37XR)
Diameter: 200mm
Length: 860mm
Medhurst self-capacitance: 13.89 pF
Toroid 100mm above top turn.
Probable primary capacitor:
C_1: 15.51µF (measured)
Initial voltage in C_1: 1000 V
Maximum primary current: 4500 A

First, some verifications:
C_1 and L_1 resonate at 43.6 kHz
Since L_3 and the topload were used in a Tesla coil, L_3 and C_3 shall also resonate at 43.6 kHz.

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{(2\pi f)^2 L} = 34.25\text{pF}$$

Consistent with the calculations above.

The system has 8 parameters to determine: L_1, C_1, L_2, C_2, L_3, C_3, k_{12}, and w, from the equations:

1) $w^2L_1C_1 = (2m^2k^2+(m^2-l^2)(l^2-k^2))/(2k^2l^2m^2)$
2) $w^2L_2C_2 = l^2/(k^2m^2)$
3) $w^2L_3C_3 = 1/l^2$
4) $L_2/L_3 = ((l^2-m^2)(k^2-l^2))/(2*k^2m^2)$
5) $k_{12}^2 = ((k^2-l^2)(l^2-m^2))/(k^2(l^2+m^2)-l^2(l^2-m^2))$

The ratio k:l:m determines the mode of operation. With 8 unknowns and 5 equations, 3 elements can be chosen at will. The other are consequences.

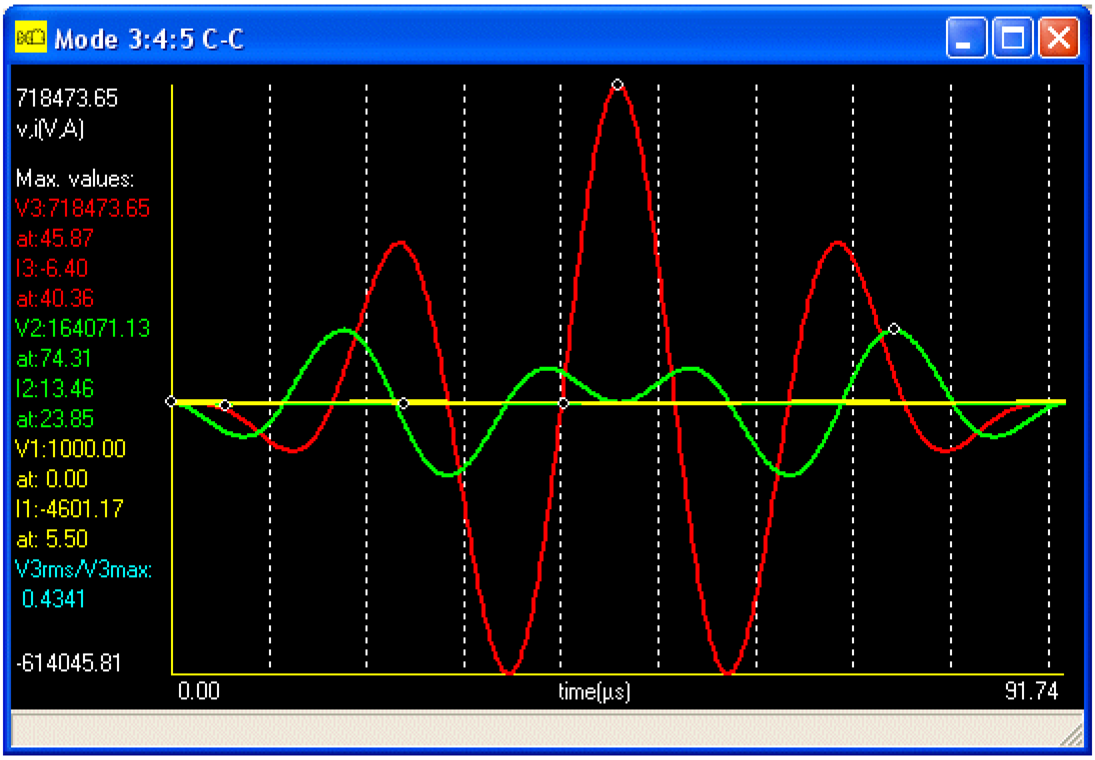
Let's fix L_1, L_3, and C_3.
L_1 = 859.1µH
L_3 = 389 mH
C_3 = 34.25 pF

Using the program Mrn6:

Mode 3:4:5:

C1=17680.0000000000 nF
L1= 0.8590783371 µH
C2= 278.3492063492 pF
L2= 54.4600000000 mH
C3= 34.2500000000 pF
L3= 389.0000000000 mH
k12= 0.3504383220

A simulation results in:

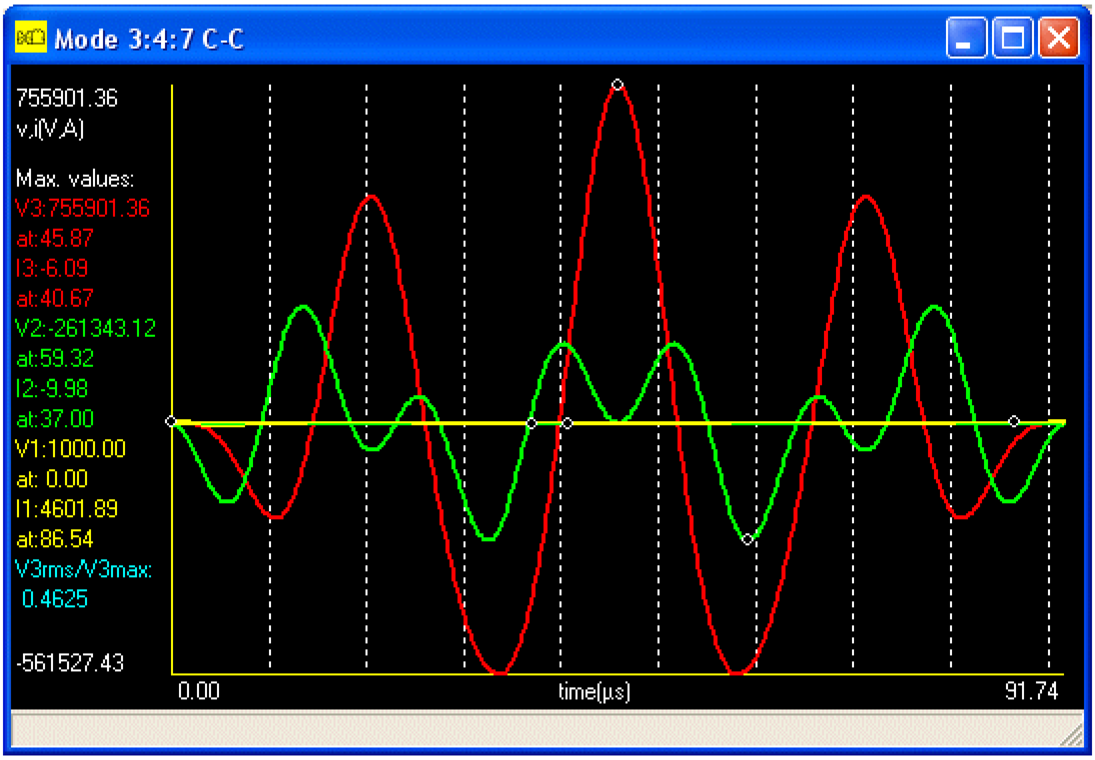


Maximum output voltage: 718 kV
Maximum input current: 4601 A
Maximum voltage over C_2: 164 kV
The input current is excessive, C_2 is quite large, and the voltage over it is large too.
This will be the reference design.

Trying the next mode, 3:4:7:

C1=19570.0000000000 nF
L1= 0.8591043750 µH
C2= 75.9134199134 pF
L2= 101.8809523810 mH
C3= 34.2500000000 pF
L3= 389.0000000000 mH
k12= 0.4555734516

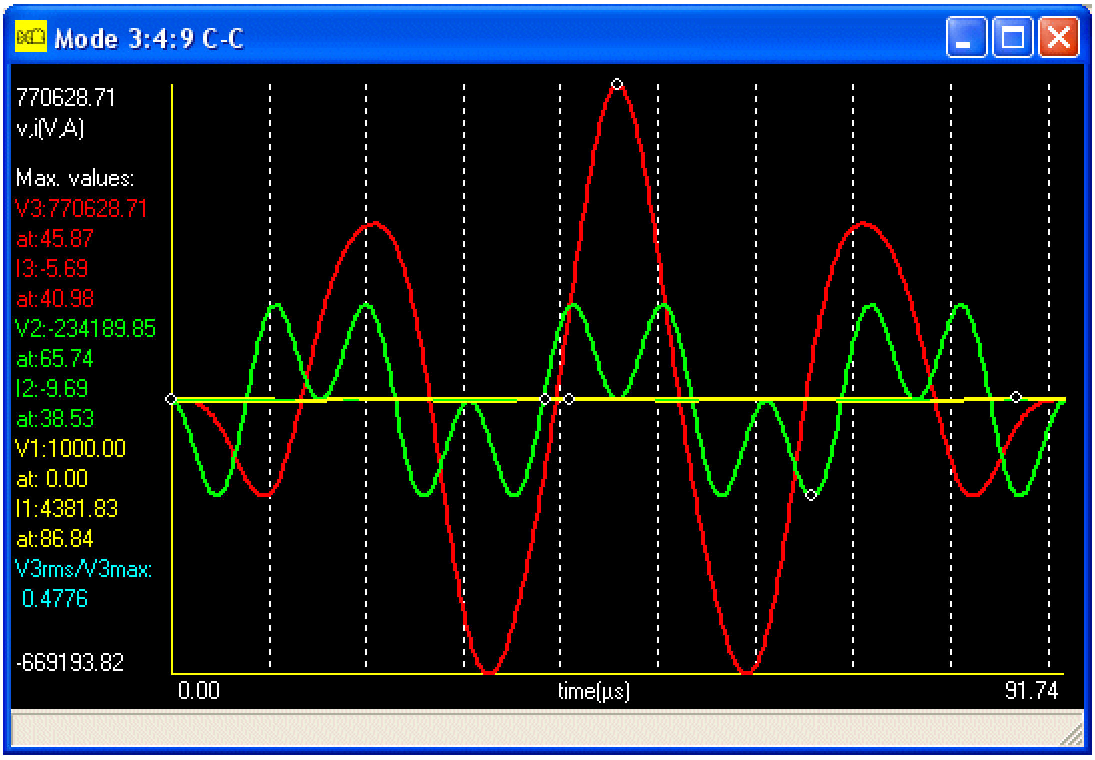
A simulation results in:



The voltage over C_2 is even greater, but C_2 is smaller. The input current is the same.

Next mode: 3:4:9:

C1=20340.0000000000 nF
L1= 0.8594422004 µH
C2= 38.5406593407 pF
L2= 121.3957475995 mH
C3= 34.2500000000 pF
L3= 389.0000000000 mH
k12= 0.4876948992

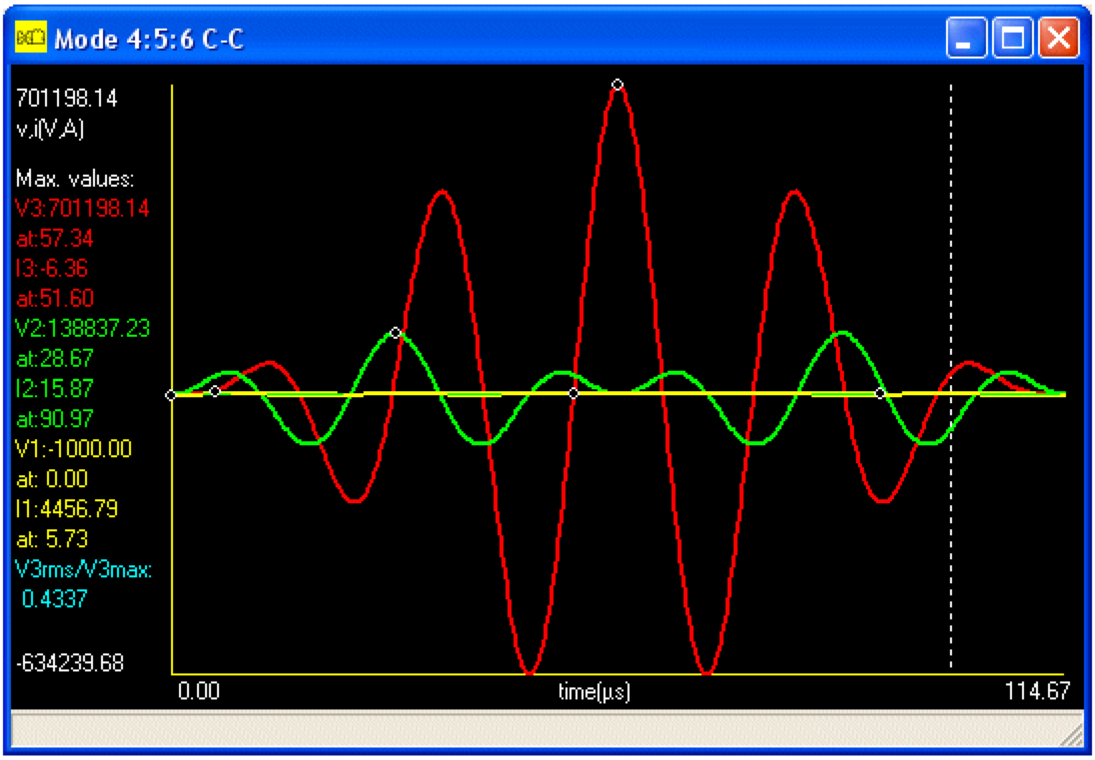


This mode has something very interesting. C_2 is a bit larger than C_3. For reasons of symmetry, the capacitance "seen" at the bottom of L_3 must be similar to the Medhurst capacitance of L_3. With this design a distributed capacitor probably can be used for C_2. Modes with higher "m" will be unrealizable because C_2 will be too small. The input current is also reasonable. But k12 is too high.

Trying then the next series:

Mode 4:5:6:

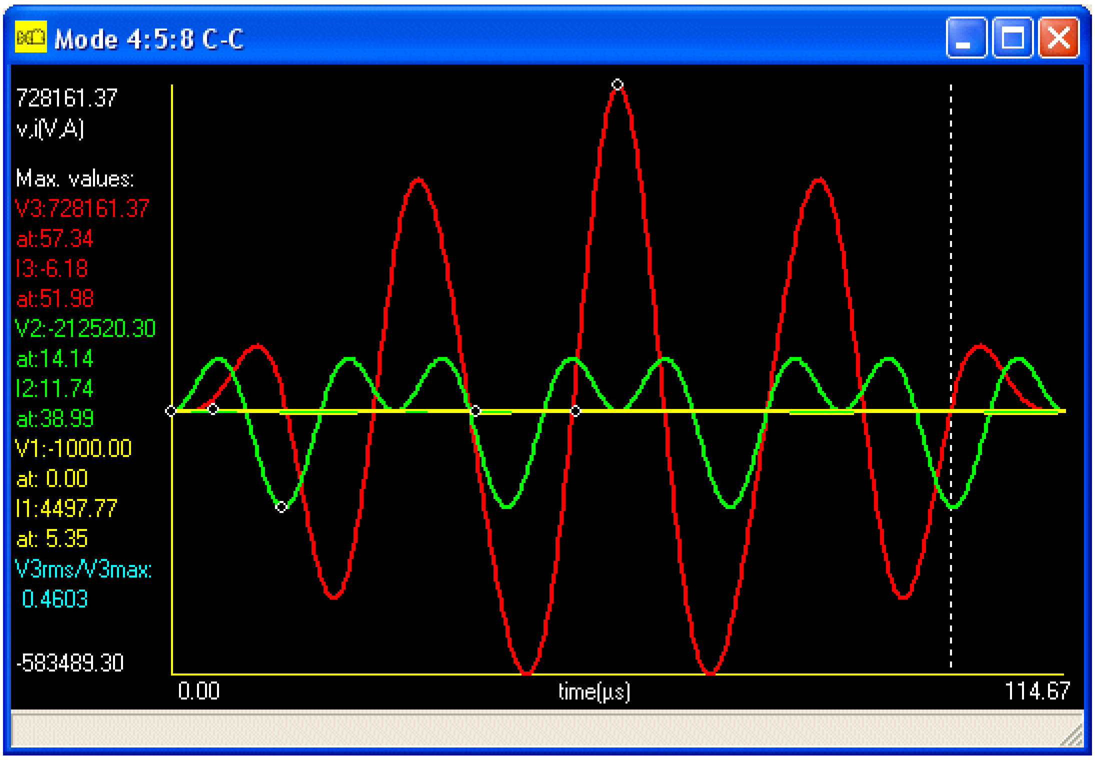
C1=16840.0000000000 nF
L1= 0.8591577670 µH
C2= 432.4494949495 pF
L2= 33.4296875000 mH
C3= 34.2500000000 pF
L3= 389.0000000000 mH
k12= 0.2813124431



C_2 is too big. The input current is within the limit. Note that the output is inverted.

Mode 4:5:8:

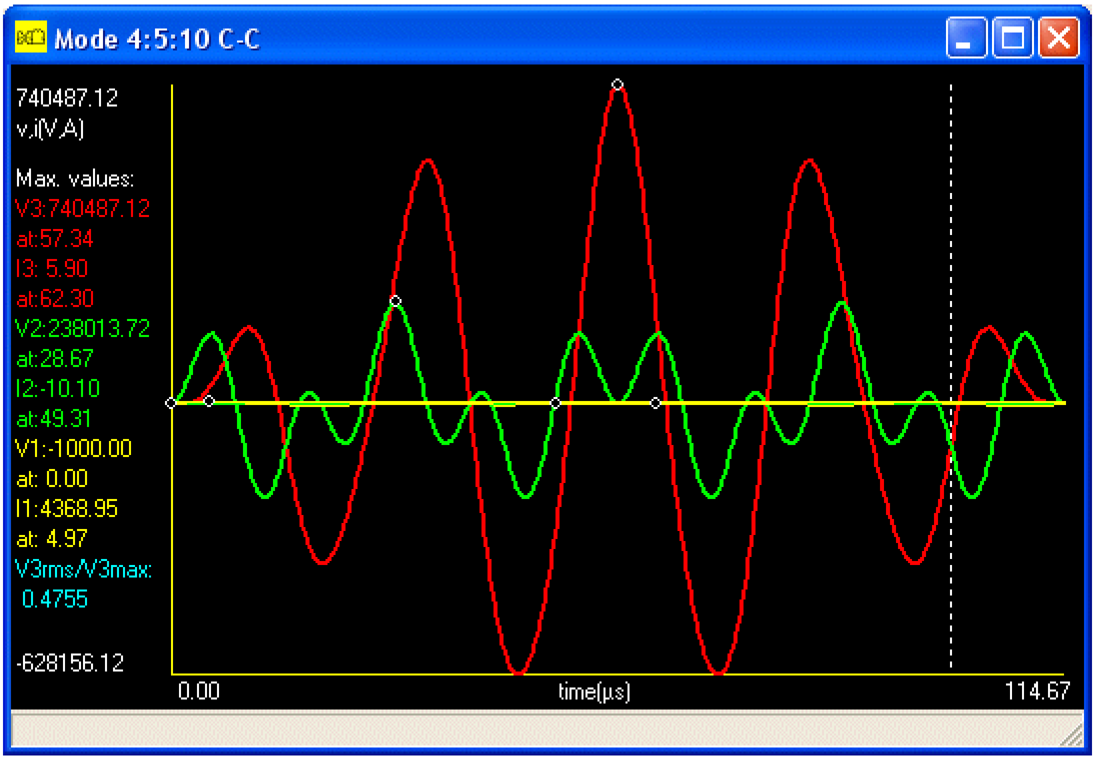
C1=18160.0000000000 nF
L1= 0.8593985738 µH
C2= 121.9729344729 pF
L2= 66.6694335938 mH
C3= 34.2500000000 pF
L3= 389.0000000000 mH
k12= 0.3825061606



C_2 is smaller, k_{12} is quite high.

Mode 4:5:10:

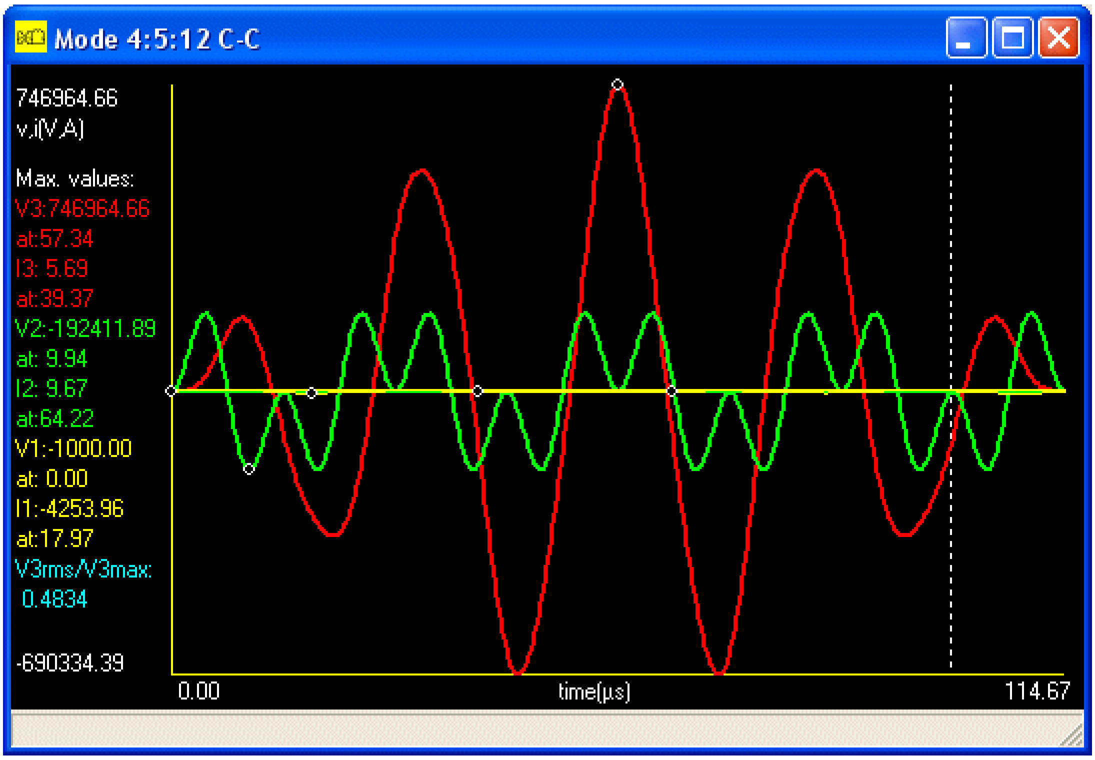
C1=18780.0000000000 nF
L1= 0.8590853593 µH
C2= 63.4259259259 pF
L2= 82.0546875000 mH
C3= 34.2500000000 pF
L3= 389.0000000000 mH
k12= 0.4173650062



Too high k_{12}, too high voltage on C_2. C_2 may be distributed.

Mode 4:5:12:

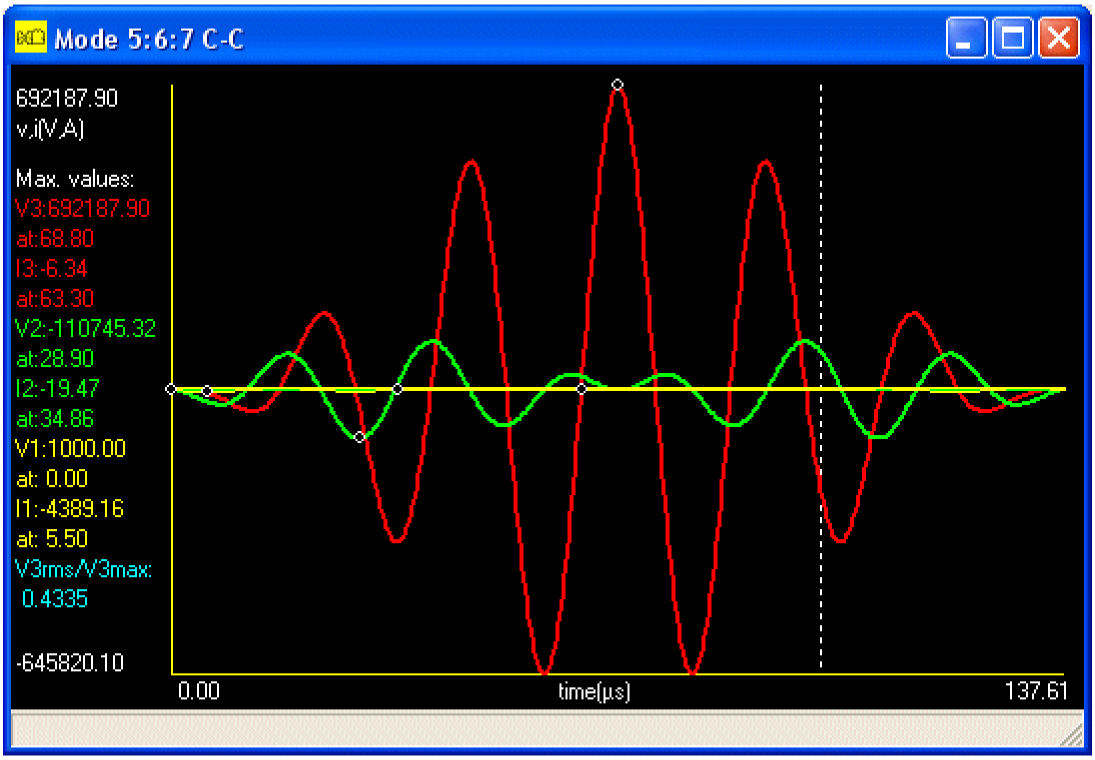
C1=19110.0000000000 nF
L1= 0.8592289244 µH
C2= 39.9743230626 pF
L2= 90.4121093750 mH
C3= 34.2500000000 pF
L3= 389.0000000000 mH
k12= 0.4342689724



C_2 is possibly too small. k_{12} is too high.

Trying the next series, that has the output with the original polarity:
Mode 5:6:7:

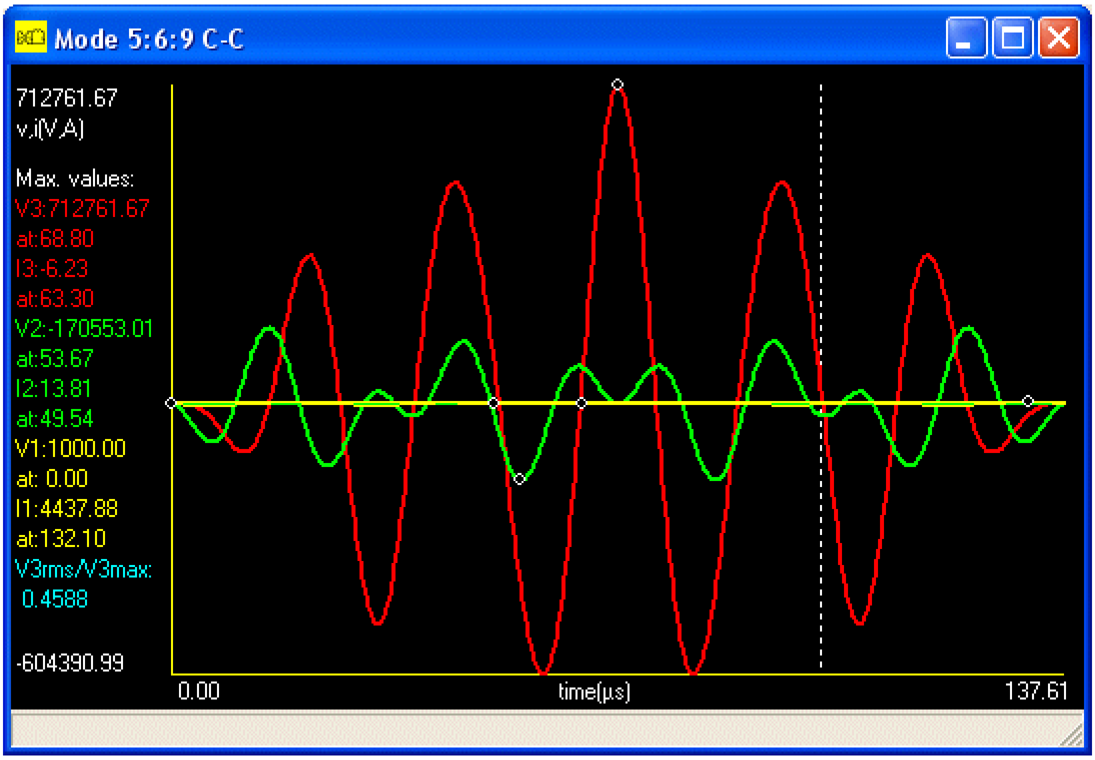
C1=16410.0000000000 nF
L1= 0.8592865786 µH
C2= 620.8111888112 pF
L2= 22.7048979592 mH
C3= 34.2500000000 pF
L3= 389.0000000000 mH
k12= 0.2348371280



Huge C_2, smaller voltage, good input current, good k_{12}.

Mode 5:6:9:

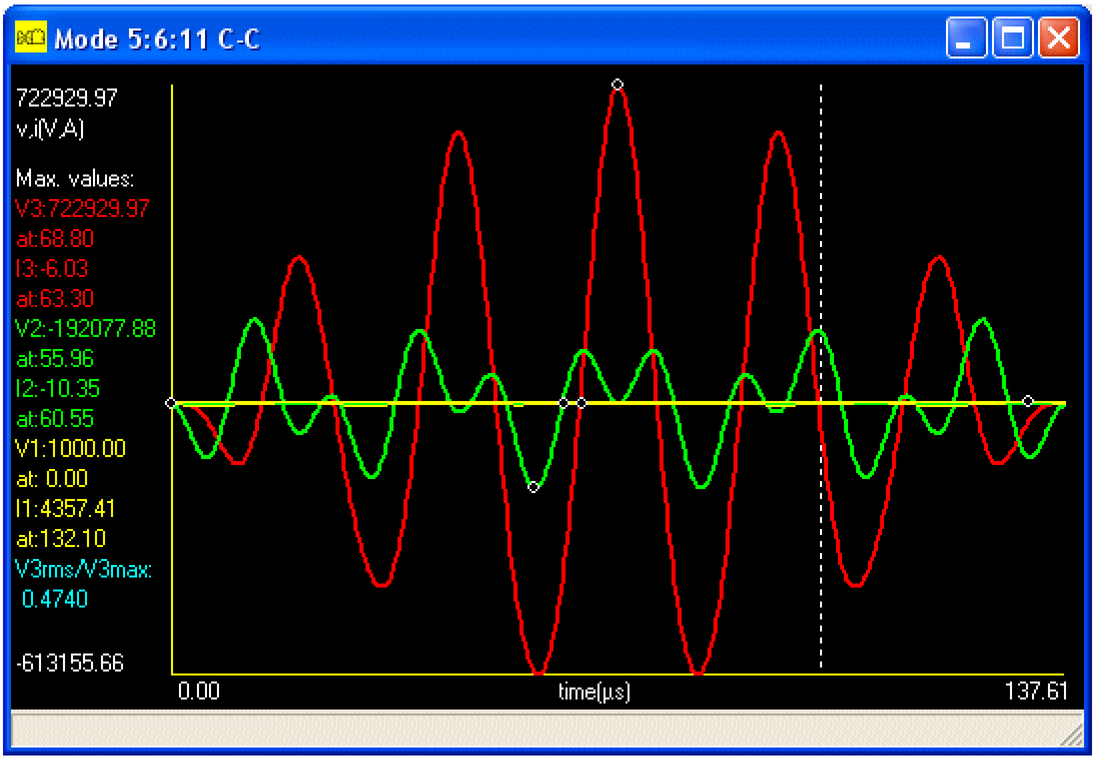
C1=17400.0000000000 nF
L1= 0.8592900702 µH
C2= 179.3454545455 pF
L2= 47.5444444444 mH
C3= 34.2500000000 pF
L3= 389.0000000000 mH
k12= 0.3300165012



Not too bad values.

Mode 5:6:11:

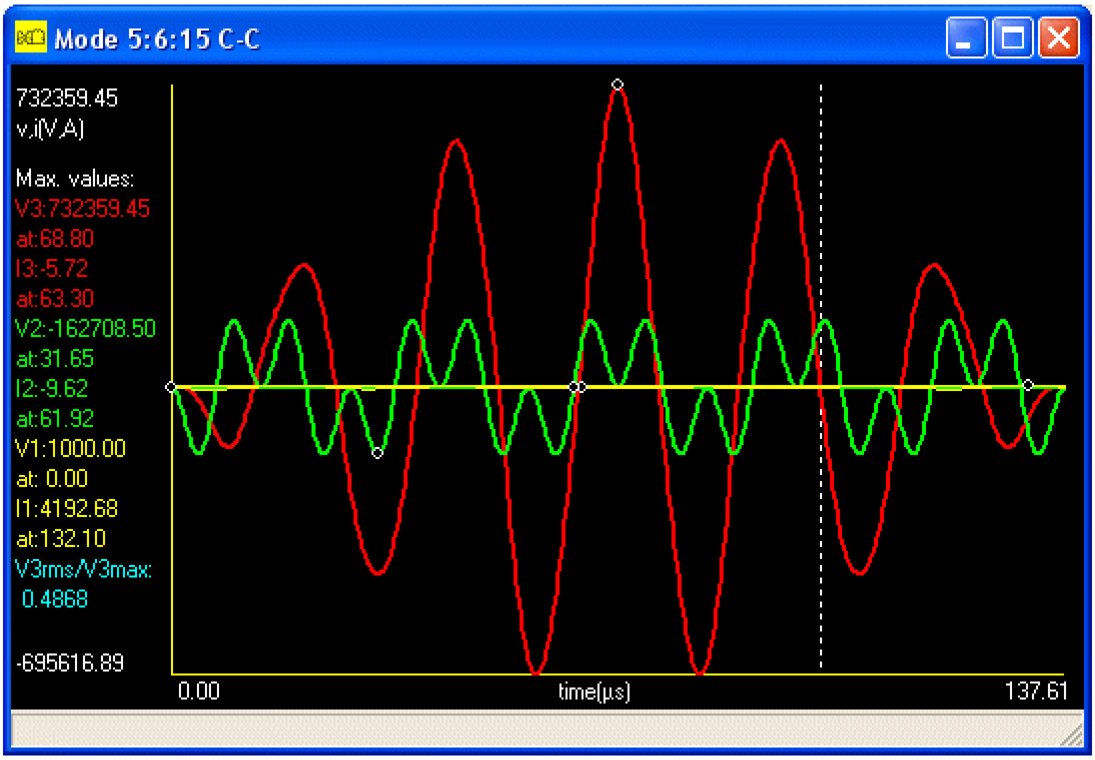
C1=17900.0000000000 nF
L1= 0.8593462417 µH
C2= 94.9475935829 pF
L2= 60.1181818182 mH
C3= 34.2500000000 pF
L3= 389.0000000000 mH
k12= 0.3658664616



Smaller C_2, greater voltage, k_{12} increased a bit.

Jumping to mode 5:6:15, to explore the nice symmetry on the voltage over C_2:

C1=18370.0000000000 nF
L1= 0.8593024823 µH
C2= 42.7012987013 pF
L2= 71.8872000000 mH
C3= 34.2500000000 pF
L3= 389.0000000000 mH
k12= 0.3949375800



C_2 can be distributed, the voltage over it is quite high, the input current is ok, k_{12} is probably at the limit.

It's clear what will happen in higher-order designs. The next with this regular waveform over C_2 is mode 7:8:21, or 6:7:18 with inversion. These modes are about the last that can be constructed with a positive extra capacitance at the "transmission line" between L_2 and L_3. These low-order designs exploit better the fast energy transfer possible with a magnifier. Higher modes will be progressively less difficult to build, but in terms of number of cycles required for the energy transfer will not be very different from a Tesla coil.
So far, the modes of the type k:k+1:3k look the most interesting.

There are other possibilities, as increasing the difference between "k" and "l" instead of between "l" and "m".

21/2/2010 Antonio Carlos M. de Queiroz