TRIPLE RESONANCE HIGH VOLTAGE PULSE TRANSFORMER CIRCUIT

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Abstract

A triple-resonance circuit has been devised for improving the efficiency of producing high voltage pulses by the MEDEA II accelerator. This circuit modifies the well-known dual-resonance circuit by introducing an intermediate capacitor between the pulse transformer and the load capacitor. Complete transfer of energy (except for dissipation) is achieved when the normal mode frequencies of the coupled circuit are in the ratios of certain whole numbers. In the simplest case, the normal-mode frequencies are in the ratios 1:2:3. In this case, the ratio of the peak transformer voltage to the peak output voltage is reduced to 0.36.

Introduction

Transformer-driven high voltage generators based on the principle of dual resonance of the charging circuit are a simple, compact alternative to the Marx generator for charging high voltage pulse forming lines (pfl) [Refs. 1-4]. The dual-resonance circuit for charging pfls is a special case of the circuit shown in Fig. 1. Here C_1 is the energy storage capacitor, C_2 is the internal capacitance of the transformer, C_3 is the pfl capacitance, and L_1, L_2, and L_3 are the inductances of the transformer primary, transformer secondary, and an external tuning/isolation inductor, respectively. The dual-resonance transformer is designed (assuming C_2 of Fig. 1 is zero) to meet the conditions $L_1C_1 = (L_2+L_3)C_3$ and $k_m = M/\sqrt{L_1(L_2+L_3)} = 0.6$. A complete coupling of the energy, neglecting damping, takes place from C_1 to C_3. However, for electrically short pfls in which the capacitance of the pfl is comparable to the internal capacitance of the transformer, the dual-resonance design becomes inefficient. A large fraction of the available energy remains stored in the internal capacitance of the transformer, and must be dissipated in the charging circuit, as on MEDEA II [Ref. 4]. This paper describes a design that allows complete coupling of the available energy in all such cases. The design also substantially reduces voltage stress on the transformer.

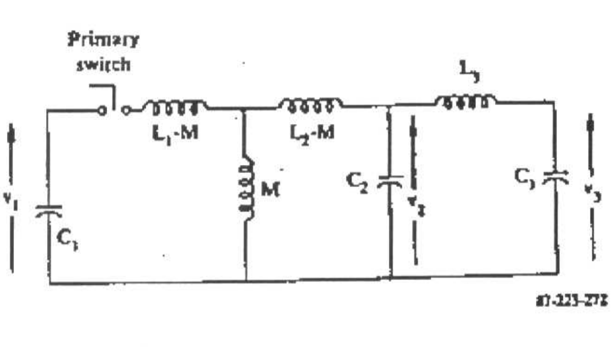

Fig. 1 Equivalent circuit of pulse-line charging circuit of MEDEA II, neglecting resistive elements.

I. Triple Resonance Principle

The circuit depicted in Fig. 1 has a purely oscillatory transient response with three natural radian frequencies ω_1, ω_2, and ω_3. If the capacitor C_1 is initially charged to voltage v_1, the voltage across the three capacitors of Fig. 1 may be described as

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} \cos \omega_1 t \\ \cos \omega_2 t \\ \cos \omega_3 t \end{bmatrix} \tag{1}$$

where t=0 corresponds to the time at which the primary switch closes. The circuit is resonant if the energy stored in the inductance and capacitance of the transformer primary and secondary circuits is zero when the pfl voltage reaches its peak. This can only occur when the three normal mode radian frequencies $\omega_1 = k\omega_0$, $\omega_2 = l\omega_0$, and $\omega_3 = m\omega_0$ are in the ratio of whole numbers k, l, and m where $l=k+1$, $k+3$, $k+5$, ... and $m=l+1$, $l+3$, $l+5$, ... The simplest resonant mode, which is also the mode of greatest practical interest, is that in which the three normal mode frequencies of the coupled circuit are in the ratios 1:2:3. Requiring 100% energy transfer, i.e., $v_1=v_2=\dot{v}_1=\dot{v}_2=\dot{v}_3=0$ at the time corresponding to $v_3 = v_{3max}$, determines several of the coefficients in Eq. 1. Scaling to v_1 (t=0) = 1, Eq. 1 becomes

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} a_1 & 1/2 & 1/2-a_1 \\ a_2 & 0 & -a_2 \\ a_3 & b_3 & -a_3-b_3 \end{bmatrix} \begin{bmatrix} \cos k\omega_0 t \\ \cos l\omega_0 t \\ \cos m\omega_0 t \end{bmatrix} \tag{2}$$

where ω_0 corresponds to the lowest order resonant radian frequency. A solution for the remaining undetermined parameters may be found by considering the coupled circuit equations, which may be written as

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = - \begin{bmatrix} L_1C_1 & MC_2 & MC_3 \\ MC_1 & L_2C_2 & L_2C_3 \\ MC_1 & L_2C_2 & (L_2+L_3)C_3 \end{bmatrix} \begin{bmatrix} \ddot{v}_1 \\ \ddot{v}_2 \\ \ddot{v}_3 \end{bmatrix} \tag{3}$$

A series of nine simultaneous equations was derived by use of Eqs. 2 and 3 and equating like frequency components. Solution of these simultaneous equations, under the constraint of 100% energy transfer, yields the conditions on circuit components listed in Table 1.

Figure 2 shows the voltage waveforms for a sample case of 100% energy transfer, calculated for an overall

Table 1 Triple resonance circuit parameters

Parameter	1:2:3 mode	General mode $k:\ell:m$ *
$\omega_0^2 L_1 C_1$	$\frac{11}{24}$	$\frac{1+\ell^2 P_3}{2}$
$\omega_0^2 L_2 C_2$	$\frac{4}{9}$	$\frac{(1-k^2 P_3/k_m^2)(P_2-P_1P_3k^2)-P_1P_3^2k^4}{P_2k^2(1+P_3k^2(1-1/k_m^2)}$
$\omega_0^2 L_3 C_3$	$\frac{1}{4}$	$\frac{1}{\ell^2}$
$k_m^2 = \frac{M^2}{L_1L_2}$	$\frac{5}{11}$	$\frac{\ell^2 P_3}{1+\ell^2 P_3}$
L_2/L_3	$\frac{5}{6}$	$\ell^2 P_3$
		$P_1 = \left(\frac{m^2-\ell^2}{m^2-k^2}\right)$
		$P_2 = P_1\left(\frac{\ell^2-k^2}{\ell^2}\right)$
		$P_3 = \frac{P_2}{2}\left(\frac{1}{k^2}-\frac{1}{m^2}\right) = \omega_0^2 L_2 C_3$

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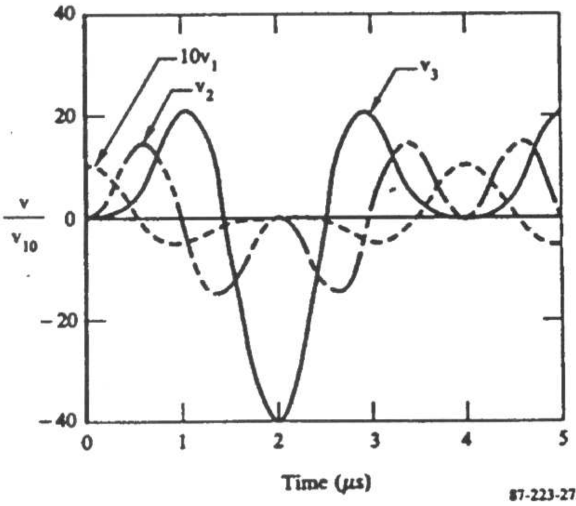

Fig. 2 Typical waveforms for triple-resonance operations.

voltage step-up of 40 and a fundamental resonant frequency of 250 kHz. A parameter of particular interest is the ratio of peak transformer voltage to peak pfl voltage. For the case of 1:2:3 resonance, this ratio is 0.36. The output voltage enhancement is obtained by transferring peak voltage stress from the transformer to the external inductor. This transfer of peak voltage stress is significant because the external inductor does not suffer from the conflicting requirements on the transformer to maintain close magnetic coupling and good electrical insulation. As the parameter m becomes large (mode numbers 1:2:m), corresponding to small C_2, the circuit parameter values and output voltage characteristics approach those of the dual-resonance design. However the transformer secondary voltage tends to exhibit rapid large amplitude oscillations as it rings with its internal capacitance.

II. Application to MEDEA II

MEDEA II has the capability to produce two independent electron beam pulses up to 1.5 MeV energy [Ref. 4]. The pulse transformers are based on a spiral-strip air-core design by Rohwein [Ref. 3]. The charging circuit was designed on the basis of the dual-resonance principle, but was generally operated with an external inductor L_3 to provide isolation between the transformer and pfl, and to yield a modest increase in pfl voltage over the peak transformer voltage. Transformer internal capacitance and pfl capacitance were comparable, and a large fraction of the energy remained in the transformer after output switch closure. This energy was dumped by a safety switch in parallel with the external inductor.

Because of the broad maximum in the response of the triple resonance circuit around the resonance peak, it was possible to modify the existing MEDEA II second-stage circuit hardware to increase the theoretical energy transfer efficiency to 87%, neglecting losses. The inductor L_3 was changed to 1050 μH and modifications were made to improve its voltage holdoff capability; an external capacitor was added to increase the effective transformer internal capacitance by 40 pF; and the primary circuit conductors were re-routed to reduce stray inductance. The safety switch was removed. Fig. 3 compares the measured pfl and transformer voltages with the calculated values. The circuit component values used are listed in Table 2. The differences between the waveforms are due to additional losses, inaccuracy in modeling circuit components, and stray coupling between components not included in the model. Fig. 4 shows the measured and calculated waveforms for a case in which the output switch breaks near the peak of the pfl voltage waveform. This shows the residual voltage ripple in the transformer after the pfl fires, which is due to lack of perfect energy coupling.

Conclusion

The circuit described here improves energy transfer and eliminates the need for a safety switch on MEDEA II. Because of the reduction in peak transformer voltage, it may have general application in miniaturizing the pulse transformer in resonant pulse-line charging circuits.