

SIGNAL FLOW GRAPH OTA-C INTEGRATED FILTERS

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Abstract

Techniques are presented to the synthesis of OTA-C (Operational transconductance amplifiers and capacitors) filters as "leapfrog" simulations of some kinds of passive prototypes. Problems as dynamic range scaling, and symmetrical layout to maximize immunity to temperature and process gradients over the filter structure in fully integrated realizations, are discussed.

Introduction

In the last few years, the use of OTA-C filters turned to be a practical alternative for the construction of fully integrated continuous-time filters in MOS technology [1]. In this work, techniques for the synthesis of these filters as signal-flow graph ("leapfrog") simulations of passive RLC prototypes are presented.

Three kinds of simulations for low-pass filters are presented: for all-pole ladder filters, for ladder filters with finite transmission zeros, and for symmetrical non-ladder filters obtained from unbalanced lattice structures.

All-pole low-pass realization

In this case, in the original prototype there are no all capacitor loops nor all inductor cut-sets. The synthesis procedure is analogous to the one used for Op. Amp. RC signal-flow graph ("leapfrog") realizations [2][3]. The normalized OTA-C circuit can be obtained from the state equations of the passive network, as in the example below:

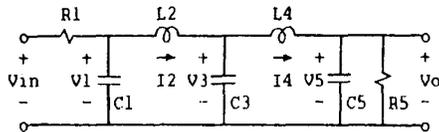


Fig. 1. A 5th. order LC doubly terminated all-pole filter.

Taking the capacitor voltages and the inductor currents as state variables, the following equation set can be obtained for the circuit in fig. 1:

$$s.C1.V1 = Vin/R1 - I2 - V1/R1 \quad (1a)$$

$$\begin{aligned} s.L2.I2 &= V1 - V3 & (1b) \\ s.C3.V3 &= I2 - I4 & (1c) \\ s.L4.I4 &= V3 - V5 & (1d) \\ s.C5.V5 &= I4 - V5/R5 & (1e) \end{aligned}$$

Identifying the terms on the left side as currents in grounded capacitors, connected at the outputs of OTAs controlled by voltages corresponding to the terms on the right side, the circuit in fig. 2 is obtained [10]. Five OTAs are used as differential integrators and two as resistive terminations. If in the original circuit the output termination is chosen to be equal to 1 Ohm, the last OTA can be eliminated, as equation (1e) becomes  $s.C5.V5 = I4 - V5$ , and the inverting input of the output OTA can be connected to the output  $V5$  to simulate it. If the terminations are equal, all the OTAs can be made equal. Convenient impedance scaling gives the final circuit.

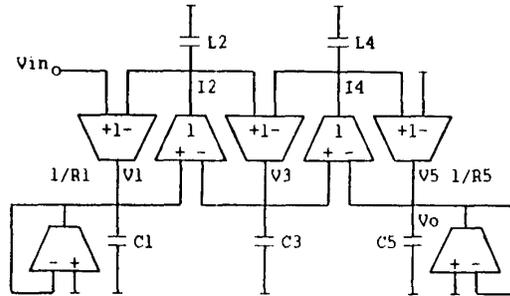


Fig. 2. OTA-C "leapfrog" realization of the filter in fig. 1.

The impedance level of each circuit corresponding to the equations (1) can be set independently. This can be used to make all the capacitors equal, at the expense of different transconductances. The voltage level at an OTA output can be divided by a factor k without alteration of other voltages, by dividing the transconductance of the OTA with output connected to the node by k, and splitting each OTA with an input connected to the node in two OTAs with one input grounded. The OTAs with one input connected to the changed voltage have then their transconductances multiplied by k. The terminations are not changed. Fig. 3 illustrates the process. If this is done for all outputs, for dynamic range equalization, the voltage levels at the OTA inputs

are also equalized, as each OTA has now its inputs connected between one capacitor voltage and ground. The number of OTAs is increased, but the input level for each one can be assured to be within the linear range.

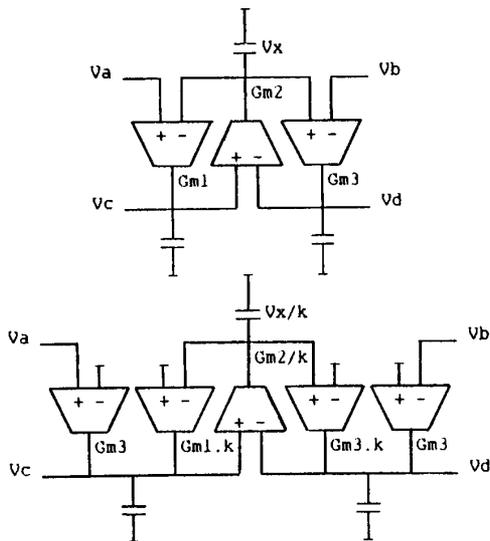


Fig. 3b. Changing  $V_x$  to  $V_x/k$  without changing  $V_a, \dots, V_d$ .

**Low-pass realizations with finite transmission zeros**

The simulation of prototype filters of this kind can be easily done if the capacitor bridges or shunt inductors in the prototype filter are transformed in equivalent controlled sources, as in fig. 4.

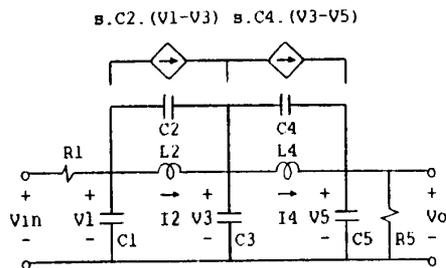


Fig. 4. Mid-series ladder with bridging capacitors replaced by controlled sources.

The state equations can now be written including the effect of the controlled sources:

$$\begin{aligned}
 s.C1.V1 &= V_{in}/R1 - I2 - V1/R1 - s.C2.(V1-V3) & (2a) \\
 s.L2.I2 &= V1 - V3 & (2b) \\
 s.C3.V3 &= I2 - I4 + s.C2.(V1-V3) - s.C4.(V3-V5) & (2c) \\
 s.L4.I4 &= V3 - V5 & (2d) \\
 s.C5.V5 &= I4 - V5/R5 + s.C4.(V3-V5) & (2e)
 \end{aligned}$$

Following the usual procedure to the realization of "leapfrog" filters with Op. Amp. RC circuits, the equations (2) are rewritten as:

$$\begin{aligned}
 V1 &= (V_{in}/R1 - I2 - V1/R1) / (s.(C1+C2)) + V3.C2 / (C1+C2) & (3a) \\
 I2 &= (V1 - V3) / (s.L2) & (3b) \\
 V3 &= (I2 - I4) / (s.(C2+C3+C4)) + V1.C2 / (C1+C2+C3) + V5.C4 / (C2+C3+C4) & (3c) \\
 I4 &= (V3 - V5) / (s.L4) & (3e) \\
 V5 &= (I4 - V5/R5) / (s.(C4+C5)) + V3.C4 / (C4+C5) & (3f)
 \end{aligned}$$

This set of equations can be simulated by the circuit in fig. 5. The five OTAs in the upper row act as differential integrators. The terminations are simulated by the rightmost and leftmost OTAs. The three central OTAs act as adders of voltage and current quantities. The four OTAs at the bottom implement the controlled source couplings. IF  $R5=1$ , the rightmost OTA can be eliminated, and the point  $x$  connected to  $V5$ .

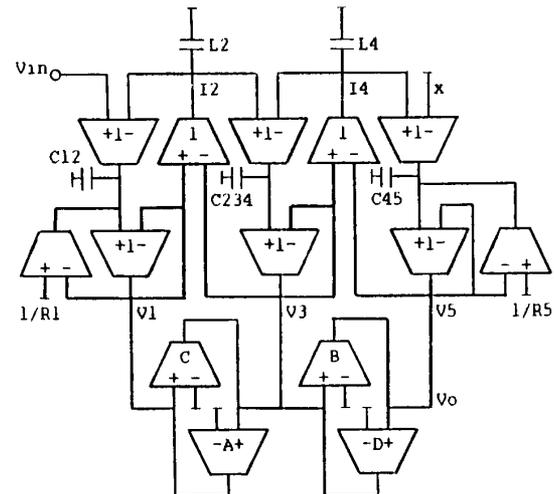


Fig. 5. Normalized OTA-C simulation of the filter in fig. 4. Version with OTAs coupling the states.  $C12 = C1+C2$ ,  $C234 = C2+C3+C4$ ,  $C45 = C4+C5$ ,  $A = C2 / (C1+C2)$ ,  $B = C4 / (C4+C5)$ ,  $C = C2 / (C2+C3+C4)$  and  $D = C4 / (C2+C3+C4)$ .

All capacitors can be made equal by changing the impedance level of the integrators (and terminations). Dynamic range equalization can be done as described above.

An economical solution is obtained directly from the equation set (2), by the use of capacitors to implement the controlled sources, exactly as is done in the original passive circuit. The resulting circuit is in fig. 6. As in the other cases, if  $R5=1$ , the rightmost OTA can be eliminated. The only possible inconvenience of this structure is the presence of floating capacitors, turning the filter more sensitive to parasitic capacitances at the floating capacitor terminals.

Due to the capacitive couplings, it is no more possible to make all the capacitors equal, and

dynamic range equalization is somewhat more complex. In this circuit, to the voltage levels be changed, the impedance levels must also be changed. It can be shown that the rules for dividing a node voltage by  $k$  without changing other voltages are:

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- Multiply by  $k$  the admittance level of the node in the  $C$  subnetwork connected to the node [6].
- Multiply by  $k$  the transconductances of all the OTAs with outputs or inputs connected to the node, in the last case, splitting the OTA in two if the other input is not grounded.

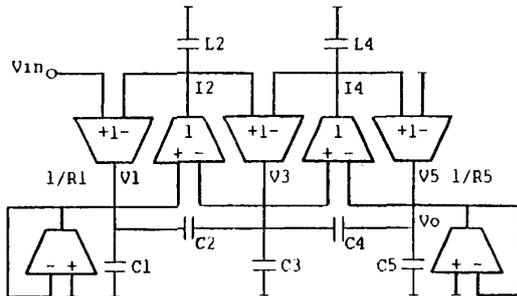


Fig. 6. Economical OTA-C simulation of the filter in fig. 4.

The process is illustrated in fig. 7, for the output node in fig. 6. Positive values for all capacitors are obtained only for a limited range of  $k$  values.

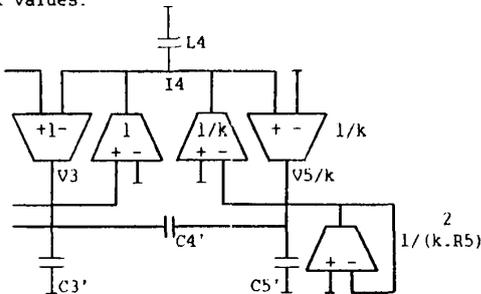


Fig. 7. Voltage level scaling in the circuit in fig. 6.  $V5$  is changed to  $V5/k$  without any change in other voltages.  $C3' = C3 + (1-k) \cdot C4$ ,  $C4' = k \cdot C4$ , and  $C5' = k \cdot (k \cdot C5 + (k-1) \cdot C4)$ .

#### Gradient-cancelling symmetrical structures

Physically symmetrical or antisymmetrical reciprocal passive networks presents an interesting property: all the components in corresponding positions in opposite sides of the network presents equal sensitivities of the network transfer function to its variation (with the exception of the terminations) [4]. If this property is retained in the active simulation, it is only necessary to dispose correspondent elements in symmetrically opposite positions around a common center to obtain a first-order gradient-insensitive layout to the filter [5]. There is no need to separate critical

components in two or more parts placed in opposite sides of the layout. Temperature or process gradients over the filter structure will result mainly in a frequency shift, easily corrected by automatic tuning techniques. Another important characteristic of physically symmetrical or antisymmetrical structures is its naturally low sensitivities [4].

In a "leapfrog" simulation that retains the coupling among capacitor voltages and inductor currents as in the original network, the sensitivity characteristics of LC elements in the prototype filter are directly copied to the capacitors in the simulation. If the prototype is physically symmetrical or antisymmetrical, the gradient-cancelling property is valid for the capacitors and terminations. The other OTAs are non-reciprocal components without correspondent in the original circuit, but it is observed that the sensitivity symmetry property can be also valid for them. For non all-pole filters, if all the OTAs are split in two, as shown above for dynamic range scaling, they form exact correspondent pairs with equal sensitivities. With non-split OTAs the formation of pairs is not exact. For an all-pole filter the OTAs form pairs even if not split. Dynamic range scaling is found to break the sensitivity symmetry for capacitors, (only in non all-pole filters), but not to affect the OTAs sensitivities.

Physically symmetrical or antisymmetrical ladder prototypes are easily obtained for all-pole approximations with all the attenuation zeros at the  $j\omega$  axis (Butterworth, Chebyshev, etc.). For approximations with attenuation zeros out of the  $j\omega$  axis (Bessel, etc.), the LC doubly terminated realization is assymmetrical. In [4], a method was presented to the realization of these approximations as symmetrical or antisymmetrical RLC ladders with very low sensitivity. These networks can be used as prototypes, but dynamic range scaling will be needed, as well as some extra OTAs to implement resistors in parallel with all capacitors.

Usual non all-pole approximations (elliptic, inverse Chebyshev, etc.) results also in assymmetrical LC doubly terminated ladder networks. Special approximations [5] can be used to obtain good passive prototypes for highly selective filters, as symmetrical or antisymmetrical LC doubly terminated ladders. If the usual approximations are to be used (for higher selectivity), and physical symmetry of the active realization is desired, alternative prototype structures should be used.

#### Symmetrical non-ladder filters obtained from unbalanced lattice structures.

Perfectly symmetrical passive structures can be obtained for most usual approximations with finite transmission zeros, in the form of unbalanced lattice networks [6]. A structure very convenient for OTA-C simulation is shown in fig. 8 [7], for the case of a 5th. order low-pass filter. This structure retains properties of LC doubly terminated filters, as zero gain sensitivities for all reactive elements at the frequencies of attenuation zeros, and in general low sensitivities in all the

passband [9]. Stopband sensitivities are not as good as for ladder realizations, due to the larger number of elements involved in the formation of the transmission zeros.

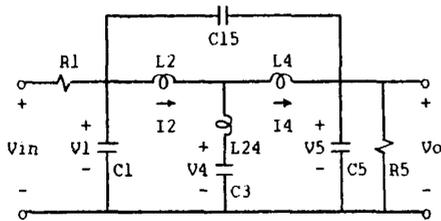


Fig. 8. Unbalanced lattice realization for a 5th-order low-pass filter with finite transmission zeros.

Substituting  $C15$  by a controlled current source  $s.C15.(V1-V5)$  and  $L24$  by a controlled voltage source  $s.L24.(I2-I4)$ , the following set of state equations can be written:

$$\begin{aligned} s.C1.V1 &= V_{in}/R1 - I2 - V1/R1 - s.C15.(V1-V5) & (4a) \\ s.L2.I2 &= V1 - V3 - s.L24.(I2-I4) & (4b) \\ s.C3.V3 &= I2 - I4 & (4c) \\ s.L4.I4 &= V3 - V5 + s.L24.(I2-I4) & (4d) \\ s.C5.V5 &= I4 - V5/R5 + s.C15.(V1-V5) & (4e) \end{aligned}$$

The circuit in fig. 9 is obtained from (4) as the one in fig. 6. A realization analogous to the one in fig. 5 is also possible. The terminations are always equal, so the output termination can be eliminated and all the OTAs can be made equal if  $R1=R5=1$ . Dynamic range equalization can be performed as shown for the circuit in fig. 6.

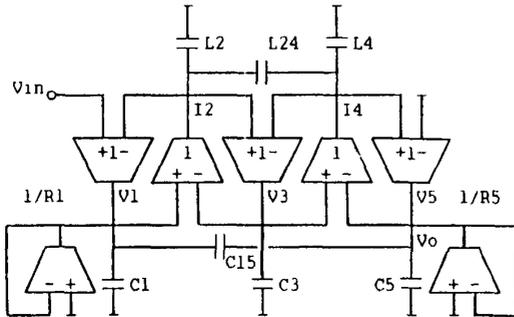


Fig. 9. OTA-C simulation of the circuit in fig. 8.

Even order filters can be obtained from antimetrical unbalanced lattice prototypes. Although even order lattices are complex RLCX circuits [8], the unbalanced forms can be transformed in real passive networks, that for most usual approximations take the form of two identical ladders coupled by gyrators [7]. OTA-C simulation is easy, but the implementation of the gyrators increases the number of OTAs in the circuit.

The unbalanced lattice realization usually results in passive networks with all positive ele-

ments even in cases in which the usual ladder realization is not possible, due to violation of Fujisawa criterion [8]. These realizations are more practical in the vicinity of these cases, when the transition band of the filter is very narrow, passband ripple very small, and stopband attenuation not very high. In cases where the ladder gives a good solution, the unbalanced lattice realization shows a tendency to result in high capacitor value dispersion, with some floating capacitors becoming too small. This problem can be minimized by the use of capacitor T circuits in place of the these elements. The problem exists also for even order realizations, in the form of gyrators with high gyration resistances.

## Conclusion

Methods for OTA-C "leapfrog" simulation of passive low-pass filters with three different structures were studied. All retain most of the sensitivity properties of the passive prototypes, and can be used for the realization of gradient-insensitive fully integrated filters, if obtained from physically symmetrical or antisymmetrical passive prototypes.

## References

- [1] R. L. Geiger and E. Sánchez-Sinencio, "Active filter design using operational transconductance amplifiers: a tutorial", *IEEE Circuits and Devices Magazine*, pp. 20-32, March 1985.
- [2] F. E. J. Girling and E. F. Good, "The leapfrog or active-ladder synthesis", *Wireless World*, pp. 341-345, June 1970.
- [3] P. O. Brackett and A. S. Sedra, "Direct SFG simulation of LC ladder networks with applications to active filter design", *IEEE Trans. Circ. and Systems*, vol. CAS-23, pp. 61-67, February 1976.
- [4] A. C. M. de Queiroz and L. P. Calóba, "Symmetrical RLC filters: an alternative to low sensitivity", *Proc. 26th. MWSCAS*, Puebla, Mexico, 1983.
- [5] A. C. M. de Queiroz and L. P. Calóba, "Physically symmetrical and antisymmetrical ladder filters with finite transmission zeros", *Proc. 30th. MWSCAS*, Syracuse, EUA, 1987.
- [6] E. A. Guillemin, "Synthesis of Passive Networks", John Wiley & Sons, 1957.
- [7] A. C. M. de Queiroz and L. P. Calóba, "Passive symmetrical RLC filters suitable for active simulation". *Proc. IEEE/ISCAS*, Espoo, Finland, 1988.
- [8] D. S. Humpherys, "The Analysis, Design, and Synthesis of Electrical Filters", Prentice-Hall, 1970.
- [9] H. J. Orchard, "Inductorless filters", *Electronics Letters*, vol. 2, June 1966, pp. 224-225.
- [10] R. Nawrocki, "Electronically tunable all-pole low-pass leapfrog ladder filter with operational transconductance amplifier", *Int. J. Electronics*, vol. 62, no. 5, 1987, pp. 667-672.