Band-Pass Multiple Resonance Networks

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Abstract—The idea of “multiple resonance networks” is reviewed and extended to networks with the structure of band-pass LC ladders. This realization combines the properties of the previously described low-pass and high-pass realizations. It allows the generation, for example, of triple resonance networks with a transformer at one side and capacitive coupling at the other, overcoming the limitations in voltage gain of high-pass realizations. A simple transformation allows the generation of networks with single input and balanced output.

I. INTRODUCTION

Multiple resonance networks are a class of LC networks that can completely transfer all the energy initially stored at a set of capacitors or inductors of the network to another set of elements. One of the simplest cases is the well-known Tesla transformer, one of the first nontrivial linear circuits to be analyzed. Ref. [1] contains a review of some early works. These circuits have found applications in many areas, when the fast lossless conversion of low voltage to high voltage is desired. Examples range from early radio transmitters [1] to modern pulsed power systems. In [2], a 6th-order triple resonance network was described for this kind of application, showing that there are justifications for the study of more complex versions of these networks, that besides practical applications have also interesting properties in the point of view of linear circuit theory. This author developed methods to extend the design of multiple resonance networks to any order, using a synthesis approach instead of an analysis approach, what lead to quite simple design procedures for several different structures. Refs. [3][5] introduced the low-pass versions (figs. 1a and 1b), that include the Tesla transformer and the triple resonance network mentioned in [2] as the simplest cases of fig. 1b, transferring energy between the capacitors \( C_1 \) and \( C_{pa} \). Ref. [4] presented a very simple design method for these circuits that avoids the solving of systems of equations. [6] extended the idea to networks where the energy is transferred from or to inductors, what allows optimized design of induction coils and generalizations of them (energy transfer between \( L_1 \) and \( C_p \) in figs. 1a and 1b).

Ref. [7] presented high-pass versions, that have the structure of a high-pass ladder network (fig. 1c). The transformerless low-pass networks (fig. 1a) allow just a single design possibility. The high-pass versions, however, allow variations in the mechanism of energy transfer, depending on which elements hold the energy at the start and at the end of the energy transfer cycle. Four cases were identified for the energy transfer between capacitors and two for the energy transfer between inductor and capacitors:

- The “symmetrical design” transfers energy between \( C_1 \) and \( C_2 \) (fig. 1c) charged to same voltage to \( C_p \) and \( C_{pa} \) only, that also get charged to the same voltage. The same network also works if the starting energy is applied to the “high-frequency input capacitance” of the network, through a current impulse across \( C_1 \), transferring the energy to the “high-frequency output capacitance” of the network, where it can be completely extracted by a current impulse caused by short-circuiting the output.

The “asymmetrical design” transfers energy between \( C_1 \) and \( C_2 \) and the high-frequency output capacitance of the network. The network can also be designed in a reverse form, that transfers energy between the high-frequency input capacitance and \( C_p \) and \( C_{pa} \) only.

The introduction of a low-pass section in a high-pass network allows the inclusion of a transformer, overcoming the...
association between the operating mode and the voltage gain that limits the application of pure high-pass networks. As examples of these structures, fig. 2 shows the possibilities for 6th-order networks, without and with a transformer. In all cases, energy transfer between two sets of capacitors or an inductor at both sides is possible. In this paper, only the energy transfer from a set of capacitors, or from \( L_t \), to another set of capacitors at the output side will be considered. Designs with energy transfer to the output inductor, or inductors, can be obtained by simple dualization.

II. DESIGN PROCEDURE FOR BAND-PASS MULTIPLE RESONANCE NETWORKS

The simple design method described in [4] departs from the observation that the output voltage of one of these networks can be predicted, from the natural frequencies of the network, and from the number of transmission zeros at zero that the structure of the network places between an impulsive source that has the same effect or the initial conditions (in figs. 1 and 2, a current source in parallel with \( C_1 \) or a voltage source in series with \( L_t \) and the output). Supposing that the circuit resonates at frequencies \( \omega_0 = \frac{1}{\sqrt{k_j \omega_0^2}} \), where \( p \), is the number of conjugate pairs of natural frequencies, in all the cases the output voltage \( V_{out}(s) \) must have the form:

\[
V_{out}(s) = \frac{B s^n}{(s^2 + k_1^2 \omega_0^2)(s^2 + k_2^2 \omega_0^2) \cdots (s^2 + k_p^2 \omega_0^2)} \quad (1)
\]

For complete energy transfer after a finite time, the multipliers \( k_j \) must be successive integers with odd difference for the case of energy transfer between capacitors [5] (or in the dual case between inductors), or must be all odd with double odd differences for energy transfer between inductors and capacitors [6]. The power of \( s, m \), is odd for energy transfer between elements of the same kind, and even for energy transfer between inductors and capacitors. The expansion of (1) in partial fractions results in a sum of pure sinusoids (2a) in the first case, and in a sum of pure sinusoids (2b) in the second case. With the condition imposed on the multipliers \( k_j \), the waveform components add destructively (or are null in the second case) at the start of the energy transfer from \( C_1 \) or \( L_t \), at \( t = 0 \), and add constructively when the energy transfer is complete, at \( t = \pi/\omega_0 \) in the first case, and at \( t = \pi/(2\omega_0) \) in the second case.

\[
V_{out}(s) = \sum_{j=1}^{p} \frac{A_j s^j}{s^2 + k_j^2 \omega_0^2} \quad (2a)
\]

\[
V_{out}(s) = \sum_{j=1}^{p} \frac{B_j s^j k_j \omega_0^2}{s^2 + k_j^2 \omega_0^2} \quad (2b)
\]

In a pure high-pass network with energy transfer between capacitors [7], the power of \( s \) in the numerator of (1) is \( m = 2p-3 \) for the asymmetrical design and \( m = 2p-1 \) for the symmetrical design. For the energy transfer from \( L_t \), considerations similar to the ones in [7] lead to \( m = 2p-2 \) for the asymmetrical design and \( m = 2p \) for the symmetrical design (in this case, a constant term appears in the partial fraction expansion of (1), but it is an artifact of the special calculation for the symmetrical design and is ignored).

The next consideration [4] is that the same waveforms can be obtained, shifted in time by the total energy transfer time, if the input is considered as being an impulsive current source in parallel with the output. With this excitation, the output voltage is proportional to the output impedance of the network. This consideration works for the symmetrical design in energy transfer between capacitors too, because one of the possibilities is energy transfer to the high-frequency output capacitance [7]. As this is the impedance of an LC network, it has the form (2a), but the residues are all positive. All that has to be done to obtain this impedance is then to calculate the expansions (2a) or (2b) for an arbitrary constant \( \beta \) in (1), as 1, and then use the absolute values of the obtained residues \( A_i \), or \( B_i \) as residues for the expansion of the impedance in Foster’s first form, multiplying it by a convenient constant \( \lambda \):

\[
Z_{out}(s) = \lambda \sum_{j=1}^{p} \frac{A_j s^j}{s^2 + k_j^2 \omega_0^2}, \quad A_j = |A_i| \quad \text{or} \quad A_j = |B_i| \quad (3)
\]

The constant \( \lambda \) can be simply the inverse of the sum of the \( A_i \). This results in a normalized impedance with a high-frequency capacitance of 1 F. The normalized network is then obtained by the expansion of \( Z_{out}(s) \) in a ladder with the required structure. A transformer with arbitrary turns ratio can be inserted where an “L” of inductors appears [1].

The inclusion of a low-pass section has the effect of decreasing by 2 the power of \( s \) in (1), by the addition of two transmission zeros at infinity. From this point the design proceeds as before, with the transformerless network being obtained by a proper expansion of the output impedance. There are always two possible designs:

The symmetrical design, transferring energy from the input capacitors, or the input inductor, to the output capacitors \( (C_1 \) and \( C_m \) in fig. 2a, or \( C_1, C_2n, \) and \( C_2 \) in fig. 2c). The symmetrical design also transfers energy correctly from the high-frequency input capacitance \( (C_1 \) in fig. 2a, \( C_1+C_2//C_{2n} \) in fig. 2c) to the high-frequency output capacitance

The asymmetrical design, transferring energy from the input capacitors \( (C_1, C_2, \) and \( C_3 \) in fig. 2a or \( C_1 \) and \( C_2 \) in fig. 2c, charged to the same voltage) or from the input inductor \( (L_t) \) to the high-frequency output capacitance \( (C_{3a}+C_3//C_2) \) in fig. 2a or \( C_3 \) in fig. 2c). The network can also be designed or operated in the reverse direction.

The inclusion of a transformer keeps the same capacitors charged at both ends of the energy transfer cycle, and so the asymmetrical design doesn’t make sense in fig. 2b for energy transfer between capacitors, since this network is obtained from fig. 2a, where \( C_2 \) and \( C_3 \) would be charged along with \( C_1 \), that ends at the other side of the included transformer. The same problem occurs with the inverted asymmetrical design in the case of fig. 2d.

It is always possible to expand the networks in the reverse order if convenient. For example, it may be required that the network in Fig. 2a shall transfer energy from \( C_1 \) (that is the high-frequency input capacitance) to \( C_1 \) and \( C_{3a} \) only (not to the high-frequency output capacitance \( C_{3a}+C_3//C_2 \)). This can be obtained by expanding the structure in Fig. 2a using the asymmetrical design, starting from the input side. \( C_2 \) would be then a selectable fraction of the capacitance seen at infinite frequency after the extraction of \( C_1, L_1, \) and \( L_2 \). The other elements are all determined. The transformation to the
structure with transformer, fig. 2b, is then possible, by the replacement of the “L” \( L_1, L_2 \), with correct operation.

### III. Examples

Consider the design of a network with the structure in fig. 2a, with the specifications: Mode: \( 5.6:7 \), \( L_1 = 30 \text{ mH} \), \( C_1 = 1 \text{ nF} \), symmetrical design with energy transfer between capacitors. The normalized resonance frequencies are 5, 6, and 7 rad/s, and the power of \( s \) in (1) is \( 2p-1=2 = 3 \). The expansion in partial fractions is then:

\[
\frac{s^3}{(s^2+25)(s^2+36)(s^2+49)} = \frac{-25}{264} \frac{s}{(s^2+25)} + \frac{36}{143} \frac{s}{(s^2+36)} + \frac{-49}{312} \frac{s}{(s^2+49)}
\]

(4)

Taking the residues in absolute value and scaling them so they add to 1, the normalized output impedance is obtained as:

\[
Z_{\text{out}}(s) = 0.1880787037 \frac{s}{(s^2+25)} + 0.5s \frac{s}{(s^2+36)} + 0.3119212963 \frac{s}{(s^2+49)}
\]

(5)

This impedance is expanded with the structure in fig. 2a, starting from the extraction of, say, one half of the high-frequency output capacitance, as \( C_3a \). The values are then denormalized for \( C_1 \) and \( L_1 \) as specified. See Table I.

#### Table I

<table>
<thead>
<tr>
<th>Element: ( C )</th>
<th>( nF )</th>
<th>( pF )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_3 )</td>
<td>0.5</td>
<td>5.463487</td>
</tr>
<tr>
<td>( L_1 )</td>
<td>0.02777777778 H</td>
<td>30 mH</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>0.558367346939 F</td>
<td>6.101265</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>4.78321673217 F</td>
<td>52.266082</td>
</tr>
<tr>
<td>( L_2 )</td>
<td>0.005501692829 H</td>
<td>5941.828255</td>
</tr>
<tr>
<td>( L_3 )</td>
<td>0.000303527035 H</td>
<td>327.809198</td>
</tr>
<tr>
<td>( C_3a )</td>
<td>91.516651180987 F</td>
<td>1 nF</td>
</tr>
</tbody>
</table>

This circuit transfers energy in 10.79 \( \mu \text{s} \), and resonates at 231.65 kHz, 277.98 kHz, and 324.31 kHz. The final capacitances \( C_3 \) and \( C_3a \) are small enough to be distributed capacitances, allowing a realization as the example in [7]. Fig. 3 shows the voltage waveforms when the input capacitors are charged to 10 kV, as if they were slowly charged by a voltage source with nonzero series resistance in series with \( L_1 \), and then the source had its output short-circuited. The output voltage reaches 95.7 kV. Observe that the voltages over \( C_1 \) and \( C_2 \) when the energy transfer is complete. A short-circuit at the output at this time would extract all the energy, discharging completely all the capacitors. The asymmetrical design would result in an initial state as in fig. 3 and a final state as in fig. 4. A reverted asymmetrical design would result in the reverse.

Fig. 4 shows the alternative operating mode for this same circuit. Only \( C_1 \) is initially charged (as if it were slowly charged by a negative voltage source with nonzero series resistance inserted at its connection with the ground and then the source had its output short-circuited). The output voltage is almost the same, but there is some voltage over \( C_2 \) when the energy transfer is complete. A short-circuit at the output at this time would extract all the energy, discharging completely all the capacitors. The asymmetrical design would result in the reverse.

Consider now the same structure (fig. 2a) but with initial energy in \( L_1 \), A mode that results in voltage waveforms similar to the ones in figs. 3 and 4 is mode 7-9:11. The normalized resonance frequencies are 7, 9, and 11 rad/s, and the power of \( s \) in (1) is \( 2p-2 = 4 \). The expansion in partial fractions, now with sinusoids instead of cosinusoids, is obtained as:

\[
\frac{s^4}{(s^2+49)(s^2+121)} = \frac{2401}{16128} \frac{s}{(s^2+49)} + \frac{-6561}{31680} \frac{s}{(s^2+81)} + \frac{14641 \times 11}{31680} \frac{s}{(s^2+121)}
\]

(6)

Now, taking the amplitudes of the sinusoids in absolute value and scaling them so they add to 1, the normalized output impedance, corresponding to an output voltage in sum of cosinusoids, is obtained as:

\[
Z_{\text{out}}(s) = 0.1261029412 \frac{s}{(s^2+49)} + 0.4824264706s \frac{s}{(s^2+81)} + 0.3914705882s \frac{s}{(s^2+121)}
\]

(7)

The expansion of this impedance in the structure of fig. 2a, again extracting first one half of the high-frequency capacitances, results in the values listed in Table II. As the normalized \( C_1 \) resulted smaller than in the previous case, The final \( C_1 \) to be smaller, in order to keep the final capacitances similar to what was obtained before, suitable for distributed realization. \( C_1 = 350 \text{ pF} \) was used. Complete energy transfer from \( L_3 \) to \( C_3a \), and \( C_1 \) occurs in 8.27 \( \mu \text{s} \). The circuit resonates at 211.60 kHz, 272.05 kHz, and 332.51 kHz. Figs. 5 and 6 shows the resulting voltage and current waveforms for a starting current of 10 A in \( L_1 \) (currents down and to the right in fig. 2a). The output voltage reaches \( -92.6 \text{ kV} \). Note the inversion of the energy transfer cycle after the energy returns to \( L_1 \). The asymmetrical design of this circuit would result in some voltage in \( C_2 \) at the end of the energy transfer cycle, as in fig. 4, and again all the energy could be extracted by a short-circuit at the output.
TABLE II
ELEMENTS FOR INDUCTOR TO CAPACITORS ENERGY TRANSFER.

| Element | normalized
c| final |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C3a</td>
<td>0.5 F</td>
</tr>
<tr>
<td>L3</td>
<td>0.011764705882 H</td>
</tr>
<tr>
<td>C3</td>
<td>0.603896103896 F</td>
</tr>
<tr>
<td>C2</td>
<td>2.906250000000 F</td>
</tr>
<tr>
<td>L2</td>
<td>0.003882558320 H</td>
</tr>
<tr>
<td>L1</td>
<td>0.000403382683 H</td>
</tr>
<tr>
<td>C1</td>
<td>32.195266544118 F</td>
</tr>
</tbody>
</table>

Fig. 5. Voltage waveforms in inductor to capacitor energy transfer, symmetrical mode.

Fig. 6. Current waveforms in inductor to capacitor energy transfer, symmetrical mode.

A transformer could be inserted in this circuit replacing \( L_1 \) and \( L_2 \), resulting in the structure of fig. 2b. This would reduce the voltage over \( C_1 \), at the expense of an increase in the required input current. The circuit could then be scaled in impedance and frequency to compensate for the current increase, increasing the inductance and keeping the capacitance. Eventually a structure similar to conventional induction coils, with high output inductance and low-frequency operation would be obtained.

IV. STRUCTURES WITH BALANCED OUTPUT

An interesting possibility with these circuits is to generate a balanced output. The last three elements in figs. 2a or 2b, \( C_3 \), \( L_3 \), and \( C_{3a} \), can be duplicated and the circuit arranged as in fig. 7. With the circuit operating in the modes shown in figs. 3 or 5, at the end of the energy transfer the voltage over the tank \( L_3-C_{3a} \) is identical to the voltage over \( C_3 \). If the copy of these elements is assembled reversed, two opposite copies of the output voltage are obtained (observe the signs). If \( C_{3a} \) is chosen so \( C_{3a} = C_3 \), two similar structures with distributed capacitance, as the one shown in [7], can be used, and the total output voltage is doubled. This can also be done with the pure high-pass realizations, but the band-pass realization allows the use of a transformer to set the voltage gain independently of the operating mode.

Fig. 7. Band-pass 6th-order multiple resonance networks with balanced output. a) transformerless. b) with transformer.

V. CONCLUSIONS

The concept of multiple resonance networks was extended to polynomial bandpass ladder structures, that mix low-pass and high-pass sections. The simple design procedure previously developed was shown to be effective in these cases too. In all cases where high-pass sections are present, four possible designs are possible for the same structure in the case of energy transfer between capacitors, and two designs are possible with energy transfer from inductors to capacitors. A curious form of network with balanced output was also proposed.

REFERENCES