

A SIMPLE DESIGN TECHNIQUE FOR MULTIPLE RESONANCE NETWORKS

Antonio Carlos M. de Queiroz

COPPE/EE - Federal University of Rio de Janeiro
CP 68504, 21945-970 Rio de Janeiro, RJ, Brazil
acmq@coe.ufrj.br

ABSTRACT: The paper shows a new and simple technique for the design of “multiple resonance networks”. These networks are composed of inductors, transformers, and capacitors, and have the property of transferring all the energy initially stored in a capacitor to another capacitor in the network, through a linear transient. The new technique doesn’t require the solving of a system of equations, and is as general as another technique previously proposed by the author.

1. INTRODUCTION

Multiple resonance networks [1] is a name that generalizes the concepts of the “double resonance” and “triple resonance” networks known in the literature [2][3]. These networks are composed of ideally lossless inductors, transformers, and capacitors, and designed in such a way that some energy initially stored in a capacitor is, at the closing of a switch, transferred to another capacitor in the circuit through a linear transient. At a certain instant, all the energy is available in this other capacitor, and can be used for some purpose. Networks with this function are usually found in pulsed power systems for physics research, where the energy is transferred from a large capacitance charged at low voltage to a small capacitance, that becomes charged at high voltage, with the same energy.

The double resonance case is long known, and has found applications ranging from early radio transmitters [6] and electrotherapeutics [4] to the generation of long sparks for demonstrations about electricity [5]. The triple resonance system was developed more recently [3], for pulsed power applications. In [1], it was shown that these cases can be generalized to any order, and a relatively simple design procedure was proposed, based on the synthesis of the networks instead of in results of their analysis.

The following sections review the double and triple resonance cases, and the technique presented in [1]. Then, a simpler technique is proposed and illustrated with an example.

2. DOUBLE RESONANCE NETWORKS

Double resonance networks are usually built as a transformer with loose coupling, with a primary capacitor at one side and a secondary capacitor at the other side.

The secondary capacitor is usually formed by the distributed capacitance of the secondary windings and the distributed capacitance of a terminal, or of the device that receives the energy. The device is known as the “Tesla transformer”, and design formulas for it are long known [6] and can be found in many texts. Directly from the formulation in [1], and relative to fig. 1, the relations for optimum design can be obtained as:

$$\begin{aligned} L_1 C_1 &= L_2 C_2 \\ k_{12} &= \frac{l^2 - k^2}{l^2 + k^2} \end{aligned} \quad (1)$$

The constants k and l are two positive integers, with $l-k$ odd, that define the operation “mode”. They determine the two natural oscillation frequencies of the network with the switch closed, that appear as:

$$\begin{aligned} \omega_1 &= k\omega_0; & \omega_2 &= l\omega_0 \\ \omega_0 &= \frac{1}{kl} \sqrt{\frac{k^2 + l^2}{2L_2 C_2}} \end{aligned} \quad (2)$$

The complete energy transfer occurs in a time equal to π/ω_0 , at the “ l ”th semicycle of the output voltage, or the “ $l/2$ ”th cycle of the primary voltage. At this instant, all the currents are null, and all the energy initially stored in C_1 is in C_2 . If the energy is not used at this point, it returns to C_1 in another identical time interval, and the cycle repeats.

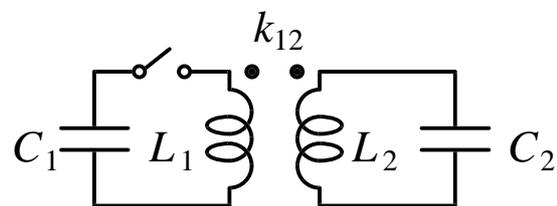


Fig. 1. Double resonance network with a transformer.

Several other equivalent circuits can be obtained, and as discussed in [1], it’s simpler to derive the design equations for a case that has no transformer (see fig. 3), that can always be reincluded by a simple circuit transformation.

3. TRIPLE RESONANCE NETWORKS

The fastest mode allowed for double resonance networks is $k=1, l=2$, that results in $k_{12}=0.6$. This mode is preferred for pulsed power systems, because the energy is transferred in just one cycle. The tight coupling, however, imposes severe construction problems for a high-voltage transformer, and so a solution found was to add an extra inductor between the transformer and the output capacitor, that supports great part of the output voltage, reducing the stress on the transformer. This idea can also be traced to the works of Tesla at the end of the XIX century (the ‘‘Tesla magnifier’’ circuit). Systems built in this way were adopted for several pulsed power applications due to the better insulation. A drawback from the structure is that the parasitic capacitance at the output of the transformer stores significant energy, that is not delivered to the output capacitance. Recently, it was shown [3] that if another capacitor is added to the circuit, there is again a set of configurations that result in complete energy transfer. Design formulas for the triple resonance network in fig. 2 can be found (by manipulating the formulas in [3] and [1]) as:

$$\begin{aligned} L_1 C_1 &= (L_2 + L_3) C_3 \\ \frac{L_2}{L_3} &= \frac{(l^2 - m^2)(k^2 - l^2)}{2k^2 m^2} \\ k_{12} &= \sqrt{\frac{L_2}{L_2 + L_3}} \\ \frac{C_2}{C_3} &= \frac{2l^4}{(l^2 - m^2)(k^2 - l^2)} \end{aligned} \quad (3)$$

Where k, l , and m are three successive integers with odd differences, that define the operating mode, and also multiply a basic frequency ω_0 to produce the three natural oscillation frequencies of the complete circuit with the switch closed:

$$\begin{aligned} \omega_1 &= k\omega_0; & \omega_2 &= l\omega_0; & \omega_3 &= m\omega_0 \\ \omega_0 &= \frac{1}{l} \sqrt{\frac{1}{L_3 C_3}} \end{aligned} \quad (4)$$

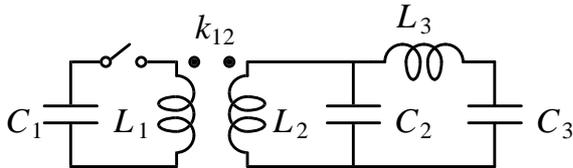


Fig. 2. Triple resonance network with a transformer and an additional inductor.

The complete energy transfer occurs again at the time π/ω_0 , at the ‘‘ $l/2$ ’’th cycle of the primary voltage. Also

for this circuit there are many equivalents, including the transformerless version used in the deductions in [1], and even a version with two transformers in cascade.

4. MULTIPLE RESONANCE NETWORKS - GENERALIZATION

In [1], it was presented a design procedure that covers the double and triple resonance cases, and cases of higher orders. The procedure was based on two observations. The first was that the transformer can be eliminated from the design problem by a simple circuit transformation, leaving a transformerless network that is just a ladder circuit with series inductors and shunt capacitors, with one shunt inductor, or more, as shown in fig. 3. This circuit has the same complexity order of the original and produces the same input and output voltage waveforms (with a scaling factor).

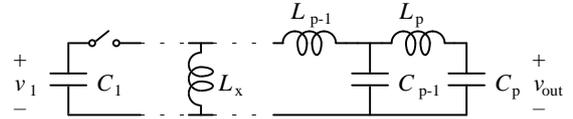


Fig. 3. Transformerless multiple resonance network.

The second observation was that, as the waveforms are symmetrical in respect to the time, the network can be designed as seen from the output side. An initial energy in C_p also makes its way back to C_1 in π/ω_0 seconds. A charged C_p can be replaced by an uncharged capacitor in parallel with an impulsive current source. In this configuration, the Laplace transform of the voltage over C_p becomes proportional to the impedance seen between the terminals of C_p . This impedance can be expressed in Foster’s first form, as:

$$Z_{out}(s) \propto V_{C_p}(s) = \sum_{j=1}^p \frac{A_j s}{s^2 + k_j^2 \omega_0^2} \quad (5)$$

Where the k_j correspond to the k, l, m, \dots of the simpler cases, also sequences of positive integers with odd differences. A set of equations was then derived for the residues A_j , by the observation that the condition of all the capacitors having zero voltages, except C_1 , at $t=\pi/\omega_0$, is equivalent to say that the output voltage and its first even derivatives, up to order $2p-4$, are null at this instant. The condition that resulted was:

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & -1 & 1 & \dots & (-1)^{p-1} \\ k_1^2 & -k_2^2 & k_3^2 & \dots & k_p^2 (-1)^{p-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ k_1^{2p-4} & -k_2^{2p-4} & k_3^{2p-4} & \dots & k_p^{2p-4} (-1)^{p-1} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_p \end{bmatrix} = \begin{bmatrix} 1/C_p \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (6)$$

The first equation comes from the behavior of (5) as $s \rightarrow \infty$, where it must reduce to a capacitance C_p . From

the residues, Z_{out} can be obtained and expanded in Cauer's first form, resulting in the transformerless multiple resonance network (fig. 3), with the shunt inductor in parallel with C_1 . The procedure can be interrupted at any step, if convenient, and the shunt inductor extracted by a total pole removal at $s=0$ and positioned in parallel with another capacitor. More than one shunt inductor can also be generated, by partial pole removals at $s=0$.

5. A SIMPLER DESIGN PROCEDURE

A careful observation of the network and its properties reveals that there is a simpler way to obtain the residues in eq. (5).

Consider the Laplace transform of the output voltage in the circuit in fig. 3, when C_1 is initially charged to a voltage $v_1(0)$. It's the same Laplace transform obtained when an impulsive current source with the value $I_{in}(t)=C_1v_1(0)\delta(t)$ is applied in parallel with an uncharged C_1 . The natural frequencies of the network are known, as $\pm jk_1\omega_0$, $\pm jk_2\omega_0$, ..., $\pm jk_p\omega_0$. The structure of the network places all the transmission zeros of $V_{out}/I_{in}(s)$ at infinity, except for a single zero at 0. At low frequency, $V_{out}/I_{in}(s)$ reduces to sL_x . $V_{out}(s)$ for $I_{in}(s)=C_1v_1(0)$ must then have the form:

$$V_{out}(s) = \frac{sC_1L_xv_1(0)\omega_0^{2p}k_1^2k_2^2\cdots k_p^2}{(s^2+k_1^2\omega_0^2)(s^2+k_2^2\omega_0^2)\cdots(s^2+k_p^2\omega_0^2)} \quad (7)$$

If this expression is expanded in partial fractions, the result is similar to eq. (5), but the residues are different because in this case the initial charge is in C_1 , not in C_p . The relation between the residues in both cases is, however, trivial: $V_{out}(t)$ is a sum of p pure cosinusoids, with amplitudes given by the residues and known frequencies. At $t=\pi/\omega_0$ it reaches the maximum value, when all the cosinusoids are at peaks with the same polarity. The absolute value of the peak output voltage is the sum of the absolute values of the residues of the partial fraction expansion of eq. (7). The considerations that lead to eq. (5) use this same idea and refer to the same waveform, with the residues being all positive. The only difference is the initial instant of the analysis. The conclusion is that:

The residues in eq. (5) are the absolute values of the residues obtained from the partial fraction expansion of eq. (7).

A difficulty is that the values of C_1 and L_x are not known, but it's known that the sum of the residues in eq. (5) reduces to $1/C_p$, and with C_p specified, arbitrary values for C_1 and L_x can be used, with the resulting residues (in absolute values) scaled to make their sum equal to $1/C_p$. Actually, the numerator of eq. (7) could be simply set to s and the same procedure followed. The following example, however, uses arbitrary values for $v_1(0)$, C_1 and L_x instead of this.

6. EXAMPLE

Consider a normalized triple resonance circuit design, with the mode 2, 3, 4, $\omega_0=1$, and $C_p=C_3=1$. Assuming $v_1(0)=1$, and arbitrarily $C_1=1$, and $L_x=L_1=1$, The output voltage is obtained as:

$$\begin{aligned} V_{out}(s) &= \frac{4 \times 9 \times 16s}{(s^2+4)(s^2+9)(s^2+16)} = \\ &= \frac{A_1's}{s^2+4} + \frac{A_2's}{s^2+9} + \frac{A_3's}{s^2+16} \\ A_1' &= \left. \frac{s^2+4}{s} V_{out}(s) \right|_{s^2=-4} = \frac{576}{60} \\ A_2' &= \left. \frac{s^2+9}{s} V_{out}(s) \right|_{s^2=-9} = -\frac{576}{35} \\ A_3' &= \left. \frac{s^2+16}{s} V_{out}(s) \right|_{s^2=-16} = \frac{576}{84} \end{aligned} \quad (8)$$

Multiplying these residues by 35/1152 their absolute values add to 1. The residues for eq. (5) are then:

$$\begin{aligned} A_1 &= \frac{35}{1152} \frac{576}{60} = \frac{7}{24} = 0.29167 \\ A_2 &= \frac{35}{1152} \frac{576}{35} = \frac{1}{2} = 0.5 \\ A_3 &= \frac{35}{1152} \frac{576}{84} = \frac{5}{24} = 0.20833 \end{aligned} \quad (9)$$

These are the same values obtained by solving the system of equations (6). The output impedance of the network is then:

$$\begin{aligned} Z_{out}(s) &= \frac{\frac{7}{24}s}{s^2+4} + \frac{\frac{1}{2}s}{s^2+9} + \frac{\frac{5}{24}s}{s^2+16} = \\ &= \frac{s^5 + 20s^3 + 81.5s}{s^6 + 29s^4 + 244s^2 + 576} \end{aligned} \quad (10)$$

The final normalized network, seen in fig. 4, is obtained by the expansion of eq. (10) in Cauer's first form. Fig. 5 shows the voltage and current waveforms that result from an initial unitary voltage in C_1 . At $t=\pi$ only the voltage over C_3 is not null. All the currents are sums of three pure sinusoids with frequencies of 2, 3, and 4 rad/s, and are null at this instant too.

The transformerless network produces a voltage gain equal to:

$$\frac{V_{out}(peak)}{V_{in}(peak)} = \sqrt{\frac{C_1}{C_3}} = 4.6571 \quad (11)$$

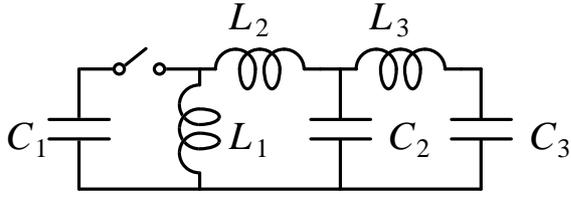


Fig. 4. Transformerless triple resonance network obtained in the example. $C_3=1$ F, $L_3=0.11111$ H, $C_2=4.62857$ F, $L_2=0.0238582$ H, $C_1=21.6890$ F, and $L_1=0.00652373$ H.

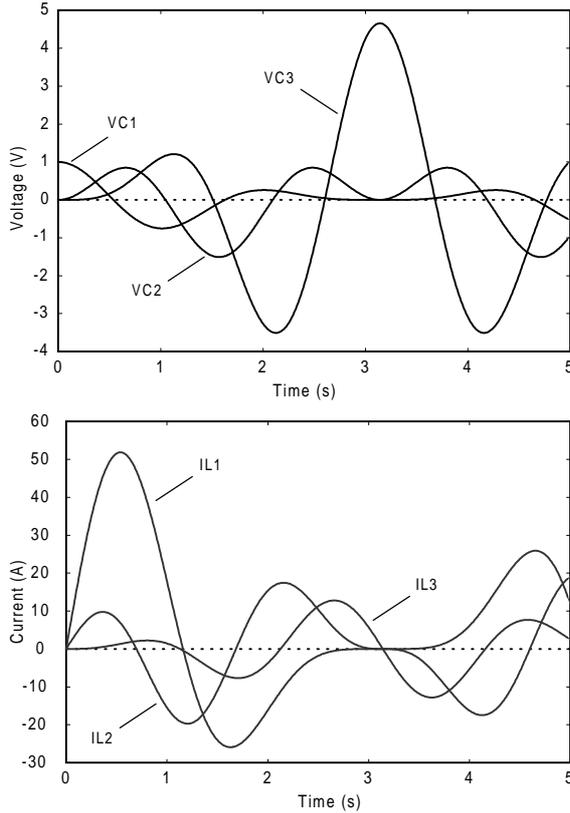


Fig. 5 Voltages at the capacitors and currents at the inductors for the normalized example circuit. Total energy transfer occurs at π seconds.

A transformer can be included at this point, by inserting an ideal transformer with turns ratio $1:n$ between the switch and L_1 , multiplying C_1 by n^2 , and transforming the circuit formed by the ideal transformer, L_1 , and L_2 into a real transformer using the equivalence (see fig. 6):

$$\begin{aligned} L_a &= \frac{L_1}{n^2} \\ L_b &= L_1 + L_2 \\ k_{ab} &= \sqrt{\frac{L_1}{L_1 + L_2}} \end{aligned} \quad (12)$$

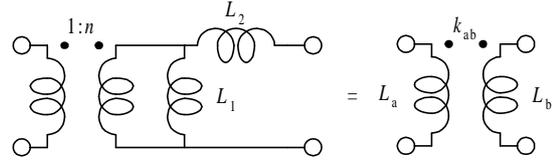


Fig. 6. Introduction of a transformer in the network.

This operation multiplies the voltage gain by n directly. In the example, the inclusion of a transformer with turns ratio $n=10$ results in a structure as in fig. 2 with the element values: $C_3=1$ F, $L_3=0.11111$ H, $C_2=4.62857$ F, $L_2=30.382$ mH, $C_1=2168.90$ F, $L_1=0.065237$ mH, and $k_{12}=0.46338$. The same values can be obtained from the equations (3). The voltage waveforms are identical to the ones in fig. 5, but 10 times larger for V_{C2} and V_{C3} . The current waveforms change in accordance with the transformation, but continue to cross zero at $t=\pi$ seconds.

7. CONCLUSIONS

A simpler design procedure for the design of multiple resonance networks of arbitrary order was presented and illustrated with an example. Exactly the same technique can be applied for networks of any order, and the networks can be designed without need of solving systems of equations. The approach that resulted in the new technique seems also to be a rather curious example of how circuit theory concepts can be applied.

8. REFERENCES

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