Electrostatic Vibrational Energy Harvesting Using a Variation of Bennet's Doubler

Antonio Carlos M. de Queiroz
COPPE/EP/Electrical Enginnering Program
Federal University of Rio de Janeiro
Rio de Janeiro, Brazil
acmq@ufrj.br

***Abstract*—A capacitive electrostatic generator with a structure suitable for a vibrational energy harvester is described. The device is a variation of the classical "doubler of electricity". The device has only one moving part, only diodes as electronic components, and can generate any voltage, limited only by a nonlinear loading device. A simple version suitable for recharging a battery is discussed, and the results of a practical demonstration implementation are shown.**

I. Introduction

Energy harvesting devices are used to extract electrical energy from the surrounding environment, in situations where the use of a regular power supply, or a battery, is impossible or inconvenient [1][2]. One of the possibilities is the capture of vibrational energy by a capacitive device, where a capacitor is made to vary its capacitance, by changing the separation of its plates with the energy of the vibration. If the capacitor is initially charged in some way, a reduction on its capacitance by the separation of the plates converts mechanical energy into electrical energy.

One advantage of electrostatic harvesters is that they can be conveniently built by MEMS (microelectromechanical systems) [1][2] techniques directly along with integrated circuits, resulting very compact. In the basic system (Fig. 1), a variable capacitor C_a is charged from a startup voltage source V_{in}, through S_1 (or a diode) when its capacitance is maximum, and energy is harvested and stored in the storage capacitor C_s through S_2 (or another diode) when the capacitor plates are separated and the capacitance minimized.

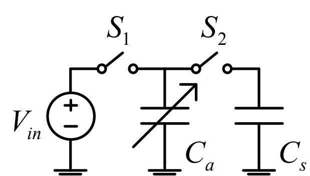

Figure 1. Basic electrostatic energy harvester.

In this work a different structure is proposed, based on the classical "doubler of electricity", the first electrostatic generator working by electrostatic induction, described originally by Abraham Bennet in 1787 [3]. Section II describes the original device and its adaptation to be used as energy harvester. Section III contains an idealized analysis of its operation that is similar to the analysis of switched capacitor networks, and considerations about the deviation from the idealized behavior caused by the use of diodes instead of contact switches. Section IV describes how to simulate the device, and shows a method implemented in a computer program for the time-domain analysis of devices containing time-varying capacitors. Section V shows how essentially the same device can be used to keep a battery charged, and Section VI describes a demonstration implementation.

This work was partially supported by the CNPq.

II. Bennet's doubler

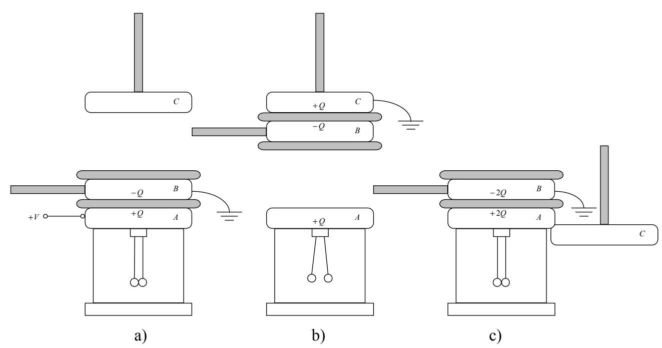

Figure 2. Bennet's doubler connected to an electroscope.

The original "doubler of electricity" [3][4] consists of three conductive plates, *A*, *B*, and *C* (Fig. 2), with suitable insulating handles or supports, and with surfaces that are to be put together forming capacitors varnished or insulated by insulating plates. The device is operated as follows:

a) An initial charge is placed on plate *A*, with plate *B* over it and grounded (by touching it or by a mechanical contact). This leaves plates *A* and *B* with opposite charges.

b) Plate *C* is put over plate *B* and grounded. This leaves plate *C* with a charge opposite to the charge in plate *B*.

c) Plates *A* and *C* are joined while plate *B* is put over plate *A* and grounded. This doubles the charge in plate A and leaves plate B with an inverted copy of the duplicated charge.

The repetition of the operations (b) and (c) ideally doubles the charge in plate A at each cycle. This device was initially used in the detection of small electric charges, by multiplying them enough to make them observable in an electroscope. Mechanized versions were soon developed. The device can be used as an electrostatic generator, because it multiplies any small initial charge imbalance, always present, to high voltage in just a few cycles.

A model for the device using lumped capacitors and switches is shown in Fig. 3. It consists of two time-varying capacitors C_a and C_b, which vary in a complementary mode, with C_a large when C_b is small and C_a small when C_b is large, three load capacitors C_1-C_3 from the three nodes of the device to ground, and C_4 to complete the capacitances. The nodes 1, 2, and 3 correspond to the plates A, B, and C, respectively. Note that in practice all the capacitors vary when C_a and C_b vary, but in a first approximation C_1-C_3 can be considered fixed and C_4 ignored. There are three contact switches, S_1 and S_2 closing when C_a is maximum and S_3 closing when C_a is minimum (and C_b is maximum). The output is at node 1.

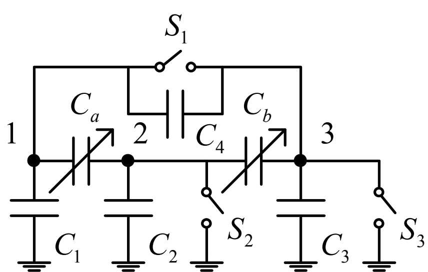

Figure 3. "Switched capacitor" model of Bennet's doubler.

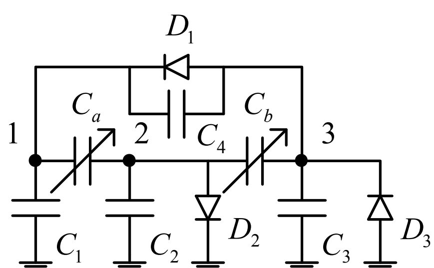

Figure 4. Bennet's doubler with diodes instead of contact switches.

An interesting observation is that the currents in the switches, if they are closed when the time-varying capacitors are at their extreme values, are always in the same directions. S_1 is always passing charge from node 3 to node 1, S_2 is always passing charge from node 2 to ground, and S_3 is always passing charge from ground to node 3, assuming a positive voltage at node 1. The three switches can then be replaced by diodes (Fig. 4). This version is also more efficient, since there is no switching of charged capacitors wasting energy.

This version of the device appears to be suitable for electrostatic energy harvesting. It can be built with plates A and C fixed, and plate B moving back and forth, being close to A and far from B in one extreme of the movement, and close to B and far from A in the other extreme. It is not a great problem to implement this scheme with MEMS techniques [1]. Three diodes complete the device. An initial voltage in C_1 is required to start the device, but it value is not critical, as the doubler will increase any sufficiently large starting voltage to any required value in a few cycles. It's even possible that the device can be made to start from electrical noise or external interferences. A device limiting the maximum voltage over C_1, that can be a battery to be charged connected through a suitable circuit, as a DC/DC converter, will limit the maximum voltages at all the nodes.

III. Analysis of the Idealized System

The circuit in Fig. 3 can be considered as a time-varying switched-capacitor network working with two phases, where the capacitors have fixed values within the phases and the switches are in different configurations at each phase. An analysis method similar to the one used in [5] can be used. Ignoring initially the switches, and using the z-transform operator "$z^{1/2}$" to indicate an advance of one phase, for the two phases a system of equations can be written, indicating charge conservation between the phases:

$$\begin{bmatrix} \mathbf{C_1} z^{1/2} & -\mathbf{C_2} \\ -\mathbf{C_1} & \mathbf{C_2} z^{1/2} \end{bmatrix} \begin{bmatrix} \mathbf{E_1}(z) \\ \mathbf{E_2}(z) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{1}$$

where $E_1(z)$ and $E_2(z)$ are the nodal voltage vectors at the two phases in z-transform, and $\mathbf{C_1}$ and $\mathbf{C_2}$ are the capacitance matrices of the circuit at both phases, structurally identical, except for the phase indices k:

$$\mathbf{C_k} = \begin{bmatrix} C_{1k} + C_{4k} + C_{ak} & -C_{ak} & -C_{4k} \\ -C_{ak} & C_{2k} + C_{ak} + C_{bk} & -C_{bk} \\ -C_{4k} & -C_{bk} & C_{3k} + C_{4k} + C_{bk} \end{bmatrix} \tag{2}$$

The effect of a switch is to short-circuit two nodes when it is closed. In the system of equations (1) a short-circuit by a switch adds the lines and columns of the system matrix corresponding to the switch terminals, on the phase where it is closed. If the switch is connected to ground, the other line and column disappear from the system. Here we consider phase 1 the phase when C_b is large, and phase 2 the phase where C_a is large. Switch S_1 then adds the lines and columns 1 and 3 in phase 2, switch S_2 eliminates the line and column 2 in phase 2, and switch S_3 eliminates the column and line 3 in phase 1. The result of these operations is shown in (3).

$$\begin{bmatrix} (C_{11}+C_{41}+C_{a1})z^{1/2} & -C_{a1}z^{1/2} & -(C_{12}+C_{a2}) \\ -C_{a1}z^{1/2} & (C_{21}+C_{a1}+C_{b1})z^{1/2} & C_{a2}+C_{b2} \\ -(C_{11}+C_{a1}) & C_{a1}+C_{b1} & (C_{12}+C_{a2}+C_{32}+C_{b2})z^{1/2} \end{bmatrix} \begin{bmatrix} E_{11}(z) \\ E_{21}(z) \\ E_{1,32}(z) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

This system of equations can be used for the calculation of the nodal voltages at one phase, from the nodal voltages on the previous phase. If the right-hand side is filed with initial nodal charges, it can be solved for the z-transforms of the nodal voltages. An important information that can be extracted from (3) is the condition for instability of the system, by equating the determinant of the system matrix to zero and finding the roots of the resulting polynomial in z. There is a single root, or natural frequency, which must be greater than 1 for instability:

$$z=\frac{\begin{array}{l}C_{11}\left(C_{12}\left(C_{21}+C_{a1}+C_{b1}\right)+C_{a2}\left(C_{21}+C_{a1}+2C_{b1}\right)+C_{b1}C_{b2}\right)+\\ C_{a1}\left(C_{12}C_{21}+C_{21}C_{a2}+C_{b1}\left(C_{a2}+C_{b2}\right)\right)+C_{41}\left(C_{a1}+C_{b1}\right)\left(C_{a2}+C_{b2}\right)\end{array}}{\begin{array}{l}\left(C_{11}\left(C_{21}+C_{a1}+C_{b1}\right)+C_{a1}\left(C_{21}+C_{b1}\right)+C_{41}\left(C_{21}+C_{a1}+C_{b1}\right)\right)\\ \left(C_{12}+C_{32}+C_{a2}+C_{b2}\right)\end{array}} \quad (4)$$

z is also the amplification factor between successive cycles. It can be seen that the ideal doubling, $z = 2$, occurs when the capacitors C_1-C_4 are null and also the minimum capacitances C_{a1} and C_{b2}. There are maximum values for C_2, C_3, and C_4, but not for C_1. For example, If $C_1 = C_2 = C_3 = C_{a1} = C_{b2} = 10$ pF, $C_4 = 0$ pF, and $C_{b1} = C_{a2} = 100$ pF, $z = 1.214$. Keeping the other capacitances, for z > 1 the maximum value of C_2 is 170 pF, the maximum value of C_3 is 37.8 pF, and the maximum value of C_4 is 18.3 pF. For C_1 there is a minimum value of 1.58 pF, and a value of 37.7 pF that maximizes z at 1.31.

The device with diodes deviates in some degree from the behavior of the ideal system, because the diodes start to conduct while the capacitances are changing. There is no simple way to analyze the circuit in this condition, other than through a complete simulation.

IV. Analysis of the doubler with diodes

Time-varying capacitances, although not a standard device in SPICE [6], can be simulated in several circuit simulators, through proper behavioral models, or by charge-controlled capacitances. Simulations of the circuits in figs. 3 and 4 were tried in the simulator LTSPICE [7], by creating charge-controlled capacitances controlled by pulsed voltage sources, with definitions as "Q=V(N001)*100e-12*x", where "N001" is the node where the controlling grounded pulsed voltage source is connected, "x" is the voltage over the capacitor, and 100e-12 is a scaling factor, in this case to make a 100 pF capacitor when the control voltage is 1 V. The results of the simulations of the circuits were close to the expected, but not exact.

To have better control on the precision of the simulation and on the models used, a special element was created for a general purpose in-house simulator. A "voltage-controlled capacitor" follows the rule:

$$\begin{aligned} q(t_0+\Delta t) &= q(t_0)+\int_{t_0}^{t_0+\Delta t} j(t)dt \\ q(t) &= Cv(t)v_1(t) \end{aligned} \quad (5)$$

where $q(t)$ is the charge, $j(t)$ is the current, $v(t)$ is the voltage over the capacitor, and $v_1(t)$ is the control voltage. C is a scaling factor that has dimension of capacitance/voltage.

Applying the backward Euler approximation to the integration, a nonlinear controlled current source is obtained:

$$j(t_0+\Delta t)=\frac{C}{\Delta t}\left(v(t_0+\Delta t)v_1(t_0+\Delta t)-v(t_0)v_1(t_0)\right) \quad (6)$$

In the simulator, in the computation of the solution at $t = t_0+\Delta t$, this current source is treated along with other nonlinear elements, with solution by the Newton-Raphson method. The backward Euler approximation ensures correct treatment of charge transfers among capacitors, essential for the correct simulation of this kind of circuit. Fig. 5 shows a comparison of simulations of the circuits with switches and with diodes, with the element values previously listed, and an initial voltage of 1 V at nodes 1 and 3, with 0 V at node 2. The capacitors C_a and C_b vary at 5 Hz, with a trapezoidal waveform, between 10 pF and 100 pF, with rise and fall times of 25 ms. The switches close for 20 ms when the capacitors are not changing. The switches and diodes are ideal.

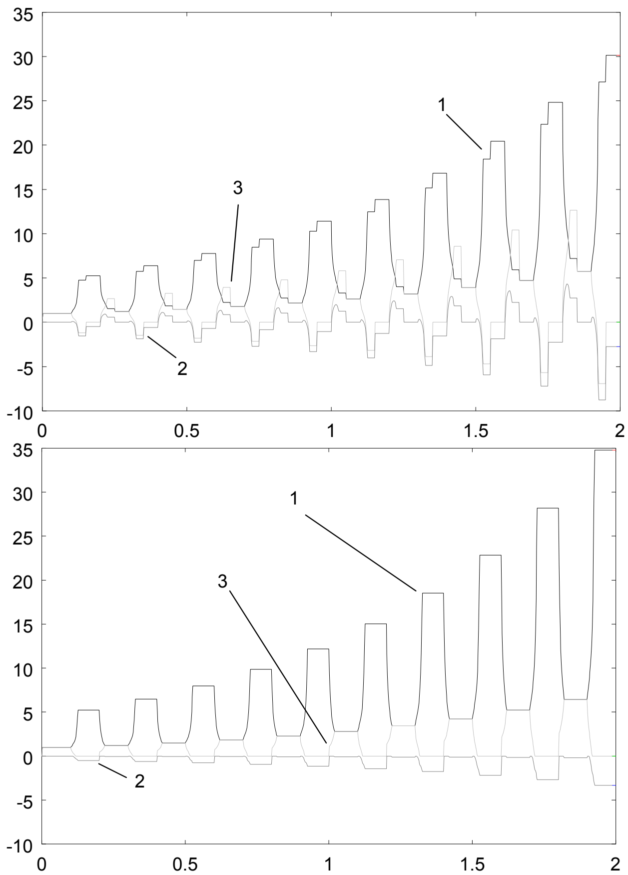

Figure 5. Simulations of the doublers with switches (above) and with ideal diodes (below), showing the three nodal voltages along the time.

There are differences, but the general behavior is the same. The version with switches shows the expected amplification factor of 1.214. The version with diodes has a somewhat larger, and not constant, amplification factor that stabilizes at 1.234 after the first cycles. It is more tolerant to the load capacitances, accepting $C_{1min} = 0.54$ pF, $C_{2max} = 185$ pF, $C_{3max} = 67.7$ pF, and $C_{4max} = 20.5$ pF with the other capacitances unchanged, to remain unstable.

When diode voltage drops and leakages are considered, the amplification factor decreases, decreasing more for low vibration frequency. Nonlinear diode capacitances make the amplification factor vary with the voltage, and may impede the startup unless a sufficiently large starting voltage is used.

V. Use as a battery charger

The doubler is not always self-starting in the version with real diodes and losses. A startup voltage must exist somewhere to cause the amplification sequence to start. This is a common problem in electrostatic energy harvesters, but is not a problem when the objective is to recharge a battery. The doubler can be used in several ways as a battery charger.

In the simplest way, C_1 is replaced by a battery (Fig. 6). With ideal diodes, no load capacitors, and the capacitors C_a and C_b varying between $C_{\max}$ and 0, when C_a is large, a charge $q = C_{\max}V_{\text{bat}}$ is drained from the battery, but returned to it when C_a decreases. C_b drains this same charge from the ground, and passes it to the battery at the next cycle. Curiously, with ideal elements the diode D_2 is not necessary, since the currents in the variable capacitors are almost identical, but with losses the circuit works better with it.

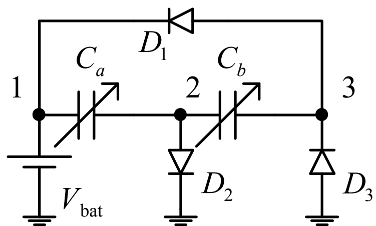

Figure 6. Simple battery charger.

The ideal energy harvested per cycle is then $E = C_{\max}V_{\text{bat}}^2$, two times the energy obtained by the technique used in [2], because both capacitors harvest energy. A simulation with V_{bat} = 3 V, real diodes, and the parameters used in the simulations in Fig. 5 results in 512 pJ of energy harvested per cycle. With ideal diodes and no load capacitances, 900 pJ would be harvested. This energy is quite small. For greater energy output, the device in the basic form (Fig. 4) can be adapted to first store energy in C_1 by charging it to as high voltage as possible, and then dump the energy to a battery through a DC-DC converter similar to the ones described in [1][2].

VI. Experimental device

An experimental device, generating high voltage, was built to test the idea with real elements [8]. It has three circular brass plates with 15 cm of diameter, with the outer plates fixed and the central plate moving between them. The diodes are series associations of five 1N4007 diodes. The fixed plates are separated by 3.6 cm, with the inner plate moving to 5 mm of distance from them. This geometry results in C_a and C_b varying between 5.5 pF and 33 pF, with the load capacitances estimated to be around C_1 = 9 pF (counting with the meter loading), $C_2 = C_3$ = 5 pF, and C_4 = 2 pF, not counting the diode capacitances. The central plate can be moved by a mechanism at a few cycles per second. The device worked as expected, reaching about 6 kV before sparking between the plates and starting over, or being limited by the increased leakage current in the diodes at high voltage. Fig. 7 shows a measurement of the voltage over C_1, initially charged with 200 V (The device was allowed to discharge to 200 V before the vibrations were started). A high starting voltage was found necessary because the too high capacitances of the diodes at low voltage turn the circuit stable. With larger C_1 added it starts with less voltage. The amplification factor also increases with time because of the nonlinear diode capacitances. Diode leakage is the main source of losses at this slow operation at ~5 Hz. The startup voltage was generated by simply approximating a conductor charged at high voltage to the device, indicating that it can be made to start using an electret. The amplitudes of the oscillations in Fig. 7 are actually higher. They were limited by the frequency response of the electrostatic voltmeter used.

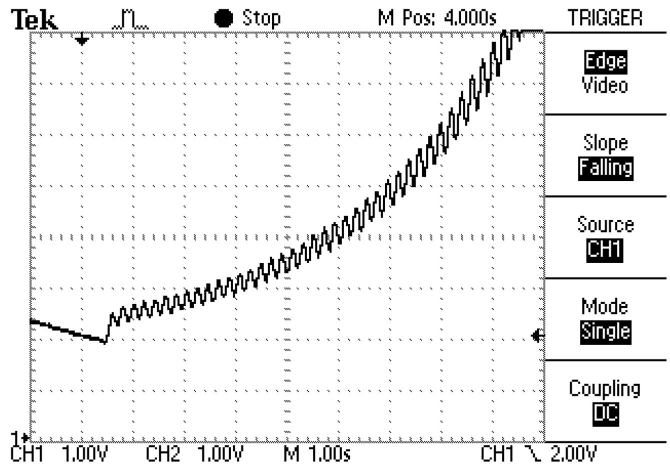

Figure 7. Measurement of the charging waveform at C_1. 100 V/div.

VII. Conclusions

A different structure for an electrostatic energy harvesting device was described, along with methods to predict its behavior. An interesting characteristic of the device is that it is an unstable nonlinear time-varying network, but with a behavior that can be approximated by a switched-capacitor network that is easier to analyze. The application of the device as a low-power battery charger seems possible. An experimental demonstration device, operating at high voltage, was described. Although it operates with voltage levels much above the levels in a MEMS device, the operation is similar. The curious problem with the nonlinear diode capacitances requiring high startup voltage is expected to be less serious in an integrated version, with smaller diodes.

References

[1] S. Meninger, J. Mur-Miranda, R. Amirtharajah, A. Chandrakasan, and J. Lang, “Vibration-to-electric energy conversion,” IEEE Tran. on Very Large Scale Integration (VLSI) Systems, vol. 9, pp. 64-76, 2001.

[2] E.O. Torres and G.A. Rincón-Mora, “Electrostatic energy-harvesting and battery-charging CMOS system prototype,” IEEE Tran. on Circuits and Systems (TCAS) I, vol. 56, no. 9, pp. 1938-1948, Sept. 2009.

[3] A. Bennet, “An account of a doubler of electricity,” Philosophical Transactions, XLVII, 1787, pp. 288-296.

[4] Antonio C. M. de Queiroz, “Doublers of Electricity,” Physics Education, 42, March 2007, pp. 156-162.

[5] Antonio C. M. de Queiroz, Paulo R. M. Pinheiro e Luiz P. Calôba, “Nodal analysis of switched-current filters,” IEEE Tran. on Circuits and Systems-II, Vol. 40, No. 1, pp. 10-18, january 1993.

[6] D. Biolek, Z. Kolka, V. Biolková, “Modeling time-varying storage components in PSpice, ” EDS 2007, IMAPS CS Int. Conf., FEEC VUT Brno, 2007, pp. 39-44.

[7] Downloaded from http://www.linear.com.

[8] Antonio C. M. de Queiroz, “Electronic version of Bennet’s doubler”. November 2009. http://www.coe.ufrj.br/~acmq/bennetd.html.