CAPACITIVELY COUPLED MULTIPLE RESONANCE NETWORKS

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ABSTRACT

This paper extends the idea of "multiple resonance networks" to networks where a series of grounded inductors are coupled through a network of capacitances. The networks discussed have the property of transferring all the energy initially stored in a set of capacitors connected to a node of the network, or in an inductor, to another set of capacitors connected to another node, or to another inductor, through a lossless linear transient.

1. INTRODUCTION

In previous publications, the concept of "multiple resonance networks" was explained [1]-[4]. These networks are specially designed LC structures, where the energy initially stored in a set of elements (in the case studied here always at least two capacitors) is, in the absense of losses, completely transferred to another set of elements of the network after some time, in the linear perpetual transient that starts when the elements that contains the energy are connected to the unenergized network. The multiple resonance networks then generalize the ideas of the "double resonance networks", that have their best known form in the Tesla resonant transformer [5], and of the recently described "triple resonance networks" [6]. In all the considered cases, the network took the form of a low-pass LC ladder structure, where a transformer could be inserted, and it was only hinted that other structures were possible.

In this paper, a different structure is presented, that takes the form of a high-pass LC ladder, where a set of grounded inductors are interconnected by a network of grounded and floating capacitors. Fig. 1 shows the 4^{th}-order case (double resonance), the 6^{th}-order case (triple resonance), and the general case of the high-pass multiple resonance networks, with input voltage over C_1 and output voltage over C_{2a}, C_{3a}, or generally C_{pa}.

Ideally, the capacitances C_{ia} could not exist, but this would result in an unrealistic network with nodes that don't have capacitances to ground. In a practical circuit, they are always present, at least as the "self-capacitances" of the inductors, and so must be considered. Only the first inductor will be left without associated capacitance, since it can be absorbed by C_1. This also turns the structures symmetrical (although usually with very different element values at both sides).

Networks with this function find applications in pulsed power systems, for physics research, radar systems, ignition systems, etc., where the energy is transferred from a large capacitance charged at low voltage, or from an inductor where a high current passes, to a small capacitance, that becomes charged at high voltage, ideally with the same energy.

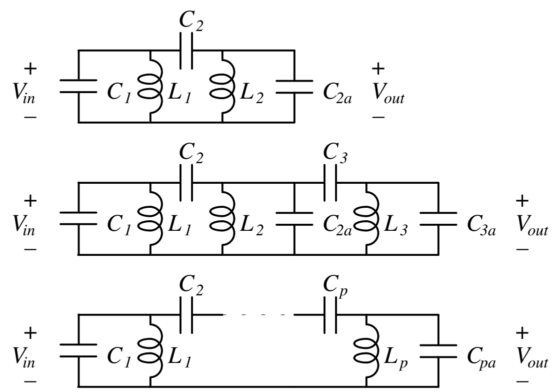

Figure 1. Capacitively coupled double resonance (a), triple resonance (b), and general p-resonance, networks.

The following sections describe the design procedure, that is an adaptation of the procedures already described in [1]-[4]. Only the case of energy transfer between capacitors will be detailed in this paper, but energy transfers between inductors or between a capacitor and an inductor are also possible [4]. An example of a curious experimental circuit is described at the end.

2. ENERGY TRANSFER BETWEEN CAPACITORS

This case is similar to the cases already studied in [1]-[3], and can be treated almost in the same way. The differences are that if C_1 is charged to V_{in}, depending on how the charging is performed, at least C_2, and maybe the other capacitors, receive some charge too, and when the energy arrives at the last capacitor C_{pa}, at least C_p becomes charged too. Following the procedures in [1]-[4], we consider that, as the waveforms are symmetrical in relation to the time, the network can be designed as seen from the output side, with the specification that an initial voltage over C_{pa} eventually passes through zero, with at the same time all the currents in the network being null, and the capacitor voltages, except for V_{in}, being as small as possible, null too if possible.

2.1 First design - asymmetrical operation

A first design approach is to consider that a charged C_{pa} can be replaced by an uncharged capacitor in parallel with an impulsive current source. When C_{pa} and the current source are connected to

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the network at t=0, the current source charges C_{pa} and distributes some charge over the remaining capacitor network instantaneously, with the energy transfer transient starting from this configuration. Due to this impulsive form of excitation, the Laplace transform of the voltage over C_{pa} becomes proportional to the impedance seen between the terminals of C_{pa}. This impedance can be expressed in Foster's first form, as:

$$Z_{out}(s) \propto V_{out}(s) = \sum_{j=1}^{p} \frac{A_j s}{s^2 + k_j{}^2 \omega_0{}^2} \tag{1}$$

$V_{out}(t)$ is a sum of pure cosinusoids, all with maximum values A_j at t=0. The condition of all the currents being null after some time forces the natural frequencies of the network to be multiples $k_j\omega_0$ of a base frequency ω_0, where the k_j form a sequence of positive integers with odd differences [1][3]. All the currents then become null at $t=\pi/\omega_0$. Since $V_{out}(0)$ is assumed to be the maximum value of $V_{out}(t)$, the residues A_j, all positive due to the correspondence with the residues of an LC impedance expanded in Foster's first form, add to $V_{out}(0)$. The behavior of $Z_{out}(s)$ at high frequency causes its A_j to add to $1/C_\infty$, where $C_\infty = C_{pa}+C_p//(C_{p-1a}+C_{p-1}//\ldots)$ is the output capacitance seen at infinite frequency. If $V_{out}(0) = C_\infty = 1$, a normalized network can then be designed by a proper expansion of $Z_{out}(s)=V_{out}(s)$.

The residues A_j are easily found by a procedure similar to the one described in [2]. Consider C_1 and C_2 initially charged to a voltage $V_{in}(0)$, now with the time origin shifted to an instant when $V_{in}(t)$ is maximum, and that there is no other element with stored energy in the network. Assuming that complete energy transfer to the output is possible, $V_{out}(s)$ calculated from this configuration must assume the form discussed above when the origin of time is shifted back to when $V_{out}(t)$ is maximum. The charged capacitors can be replaced by uncharged capacitors in series with step voltage sources $V_{in}(0)/s$. The voltage source in series with C_2 can then by relocated in the direction of L_1 and C_1, where it cancels the voltage source that was in series with C_1, and appears in series with L_1 (fig. 2). The structure of the network and the known natural frequencies force $V_{out}(s)$ caused by $V_{in}(0)$ to take the form:

$$V_{out}(s) = \frac{\alpha s^{2P-3}}{(s^2 + k_1{}^2\omega_0{}^2)(s^2 + k_2{}^2\omega_0{}^2)\cdots(s^2 + k_p{}^2\omega_0{}^2)} \tag{2}$$

where α is a constant. If this expression is expanded in partial fractions, it must have the form of eq. 1, with each term corresponding to one of the perpetual cosinusoids that form the output, shifted in time by π/ω_0 seconds. The A_j in eq. 1 are simply the residues of the partial fraction expansion of eq. 2, taken as absolute values, and renormalized so they add to 1.

With $Z_{out}(s)$ obtained from eq. 1, The shunt capacitors C_{ia} are obtained by partial extractions of admittance poles at infinity, with the fractions of the pole residues being extracted chosen so practical elements are obtained. The series capacitors and shunt inductors are extracted by total pole extractions at zero, in impedances and admittances respectively, as in Cauer's second form.

This synthesis produces a network that generates the correct output voltage waveform and correct current waveforms all going to zero at $t=\pi/\omega_0$, starting from when $V_{out}(t)$ is maximum, but because of the assumed impulsive charging of the output node all the capacitors receive some charge, and so the starting voltages on them, even in the first capacitors, are not exactly null. Identical waveforms, shifted in time, are produced by charging C_1 and C_2 with the same input voltage $V_{in}(0)$. At $t=\pi/\omega_0$, $V_{out}(t)$ will be maximum, but there will be some voltage, and energy, remaining in all the capacitors. This is not bad, when it's considered that a sudden discharge of the output node at this time releases all the energy stored in the capacitors to the output, what is the usual objective of these networks. See fig. 4b.

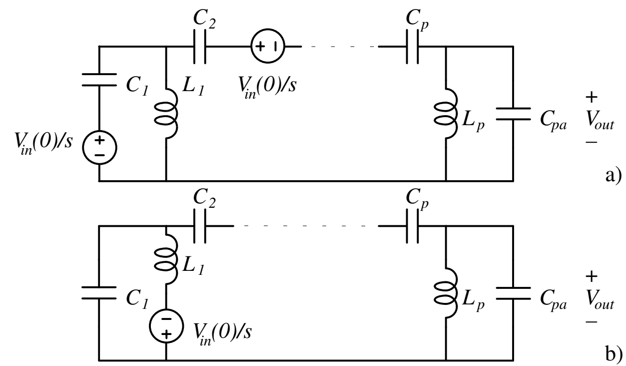

Figure 2. a) Initial conditions for charging of the input node only. b) Relocating the input voltage sources.

It's also possible to design and operate the network backwards, charging one side by a current impulse, and transferring the energy to the two capacitors at the other end. In this case, C_{pa} would be at the input, and would account for the greatest part of the input admittance pole at infinity. The impulsive input can be obtained by charging C_{pa} and connecting it to the network. Some energy is lost in the initial charge redistribution in this case.

2.2 Second design - symmetrical operation

Another approach, more curious if not more practical, is to force a symmetrical operation by specifying that when the output voltage is maximum there is voltage only over C_{pa} and C_p, and that when it is null, with all the currents null too, there is voltage only over C_1 and C_2.

As the situation when $V_{out}(t)$ is maximum doesn't correspond to impulsive charging of the output in this case, $Z_{out}(s)$ must be different from $V_{out}(s)$. From the discussion in the previous section $V_{out}(s)$ must be the same because the input was not changed and eq. 2 still holds. Fig. 3 illustrates how the new output impedance can be obtained. The model (a) including step voltage sources to represent the maximum output voltage (normalized to 1 V) over C_{pa} and C_p is first transformed by shifting the voltage source (b) in series with C_p to the two output branches, what results in a single voltage source in series with L_p. A Norton equivalent is then applied to leave only a current source driving the network (c). The desired output impedance is given by the ratio between the transformed output voltage and current:

$$Z_{out}'(s) = L_p\left(s - s^2 V_{out}(s)\right) \tag{3}$$

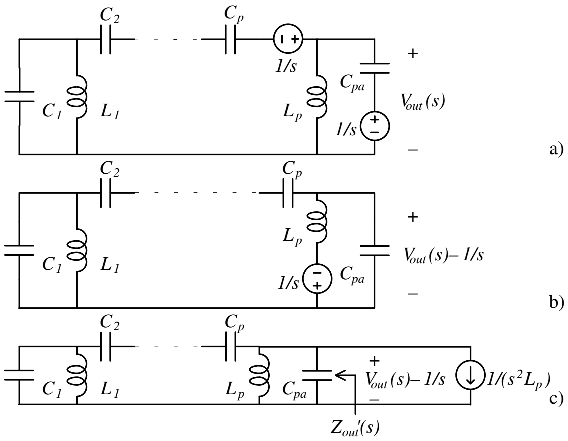

Figure 3. Forcing symmetrical operation. a) Charge in the output capacitors only. b) Equivalent circuit. c) Obtaining $Z_{out}'(s)$.

The obtained $Z_{out}'(s)$ is can then be scaled as in the first design, to reduce it to $1/s$ when $s\rightarrow\infty$, and expanded as before.

It's however worthwhile to investigate the form taken be eq. 3. Considering that $V_{out}(s)$ is identical to $Z_{out}(s)$ of the first design, eq. 3 can be expanded in partial fractions as:

$$\begin{aligned} Z_{out}'(s) &= L_p\left(s-\frac{A_1s^3}{s^2+k_1^2\omega_0^2}-\frac{A_2s^3}{s^2+k_2^2\omega_0^2}-\cdots-\frac{A_ps^3}{s^2+k_p^2\omega_0^2}\right) \\ &= k_\infty s+\frac{B_1s}{s^2+k_1^2\omega_0^2}+\frac{B_2s}{s^2+k_2^2\omega_0^2}+\cdots\frac{B_ps}{s^2+k_p^2\omega_0^2} \end{aligned} \quad (4)$$

where:

$$\begin{aligned} k_\infty &= \left.\frac{Z_{out}'(s)}{s}\right|_{s\rightarrow\infty} = L_p(1-A_1-A_2-\cdots-A_p)=0 \\ B_i &= \frac{s^2+k_i^2\omega_0^2}{s}\left.Z_{out}'(s)\right|_{s^2=-k_i^2\omega_0^2} = L_pA_ik_i^2\omega_0^2 \end{aligned} \quad (5)$$

where the fact that the residues A_i were normalized to add to 1 was used. Observing how the residues A_i were obtained, it can be recognized then that the new residues B_i can be obtained by first expanding in partial fractions the expression:

$$V_{out}'(s)=\frac{\beta s^{2P-1}}{(s^2+k_1^2\omega_0^2)(s^2+k_2^2\omega_0^2)\ldots(s^2+k_p^2\omega_0^2)} \quad (6)$$

where β is a constant, and then taking the residues as absolute values and scaling them so they add to 1. The result is exactly the same that results from eq. 3.

It's also interesting to observe that $V_{out}'(s)$ in this form corresponds to the output voltage obtained when the input side of the network is charged by an impulsive current source (fig. 4). The output impedance $Z_{out}'(s)$ is then proportional (identical with the normalization used) to the output voltage, with the origin of time shifted to when $V_{out}'(t)$ is maximum. This reasoning shows that the same network can be operated with only the first two capacitors charged, transferring energy to the last two capacitors, or with the network charged by an impulsive current source at the input side, transferring energy to the output side, where all the energy can be extracted by a current impulse (fig. 4b), as in the asymmetrical design.

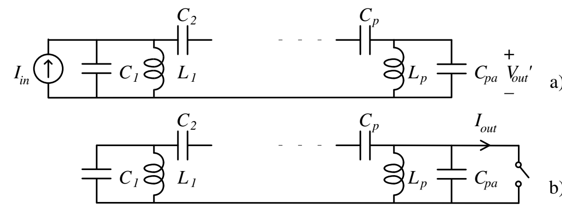

Figure 4. Impulsive charging of the input (a), and impulsive discharge of the output (b).

3. EXAMPLES

3.1 formulas for the 4th-order system

With the two multipliers being k_1=k, k_2=l, and ω_0=1, for the 4th-order asymmetrical design, $Z_{out}(s)$ is obtained as:

$$Z_{out}(s)=\frac{\frac{1}{2}s}{s^2+k^2}+\frac{\frac{1}{2}s}{s^2+l^2}=\frac{s^3+\frac{1}{2}\left(k^2+l^2\right)s}{s^4+\left(k^2+l^2\right)s+k^2l^2} \quad (7)$$

With "a", $0\leq a\leq 1$, being the fraction of the admittance pole at infinity realized by C_{2a}, the elements of the structure in fig. 1a, normalized for ω_0=1 and unitary capacitance at infinite frequency, are obtained as:

$$\begin{aligned} C_{2a} &= a \\ L_2 &= \frac{k^2+l^2}{2k^2l^2} \\ C_2 &= \frac{2\left(k^4+l^4\right)-a\left(k^2+l^2\right)^2}{\left(k^2+l^2\right)^2} \\ L_1 &= \frac{2\left(k^2+l^2\right)\left(k^4-2k^2l^2+l^4\right)}{\left(a\left(k^2+l^2\right)^2-2\left(k^4+l^4\right)\right)^2} \\ C_1 &= \frac{\left(2\left(k^4+l^4\right)-a\left(k^2+l^2\right)^2\right)(1-a)}{k^4-2k^2l^2+l^4} \end{aligned} \quad (8)$$

The voltage gain, defined as the ratio between the maximum output voltage and the initial input voltage, can be calculated by considering that the initial energy in C_1 and C_2 ends in the capacitance of the output node. Using energy conservation it is obtained as eq. 9.

$$Av=\sqrt{\frac{C_1+C_2}{\frac{C_1C_2}{C_1+C_2}+C_{2a}}} \quad (9)$$

For the 4th-order symmetrical design, $Z_{out}'(s)$ is obtained as:

$$Z_{out}'(s)=\frac{\frac{k^2}{k^2+l^2}s}{s^2+k^2}+\frac{\frac{l^2}{k^2+l^2}s}{s^2+l^2}=\frac{s^3+\frac{2k^2l^2}{\left(k^2+l^2\right)}s}{s^4+\left(k^2+l^2\right)s+k^2l^2} \quad (10)$$

and the normalized elements, also for the structure in fig. 1a, are:

$$
\begin{aligned}
C_{2a} &= a \\
L_2 &= \frac{2}{k^2+l^2} \\
C_2 &= \frac{k^4-2k^2l^2(2a-1)+l^4}{4k^2l^2} \\
L_1 &= \frac{2(l-k)^2(k+l)^2(k^2+l^2)}{\left(k^4-2k^2l^2(2a-1)+l^4\right)^2} \\
C_1 &= \frac{(1-a)\left(k^4-2k^2l^2(2a-1)+l^4\right)}{(k+l)^2(l-k)^2}
\end{aligned}
\tag{11}
$$

Note that for the symmetrical design the relation below is always valid:

$$L_1(C_1+C_2)=L_2(C_2+C_{2a}) \tag{12}$$

The voltage gain, using eq. 12, is:

$$Av=\sqrt{\frac{C_1+C_2}{C_2+C_{2a}}}=\sqrt{\frac{L_2}{L_1}} \tag{13}$$

Explicit formulas for the 6th-order case become too complex (in comparison to what happens with the low-pass versions of these circuits [1][4]), due to the presence of the C_{ia} capacitors.

3.2 Experimental realization

An experimental device was built and tested [7], designed for the mode k:l = 20:21 with the symmetrical design. The final designed element values are: C_{2a} = 6.0484 pF, C_2 = 5.1791 pF, L_2 = 28.2 mH, C_1 = 1000 pF, and L_1 = 314.98 μH. The circuit then ideally resonates at 276.2 kHz and 290.0 kHz. C_{2a} and C_2 are two distributed capacitances, between the top of L_2 (a long air-core coil with an antenna mounted above it) to ground and to an "influence ring" mounted around L_2, as shown in fig. 5. L_1 is an air-core inductor, and C_1 a high-voltage capacitor. C_1 is charged by a current-limited high-voltage transformer and connected to the network through a multiple spark gap. The values of the distributed capacitances can be adjusted by moving the influence ring and adjusting the length of the antenna at the top of L_2. The device was used for demonstrations about electricity, producing sparks at the output terminal, as a variation of the classical "Tesla coil" demonstration device. Note that with the connections shown, Only C_1 is charged by the power supply. The system then works as if the input node were charged by a current impulse when C_1 is connected to ground by the spark gap, with a small loss of energy. Fig. 6 shows the ideal voltage waveforms, ignoring losses, assuming that C_1 is charged to 5000 V and connected to the circuit at t=0. Complete energy transfer occurs in 36.2 μs, after 10.5 cycles of V_{out}.

4. CONCLUSIONS

The concept of multiple resonance networks was extended to structures with the form of a high-pass LC ladder. The peculiarities of the structure result in four basic modes of energy transfer between capacitances, depending on what capacitors are charged at the start and at the end of the energy transfer cycle. Two design procedures cover the possibilities. Only the 4th-order cases were detailed and exemplified in a curious application, but the described design techniques can be directly applied to networks of any even order. Energy transfer to and from inductors was not discussed, but is also possible, essentially with the same modifications in the design procedure described in [4].

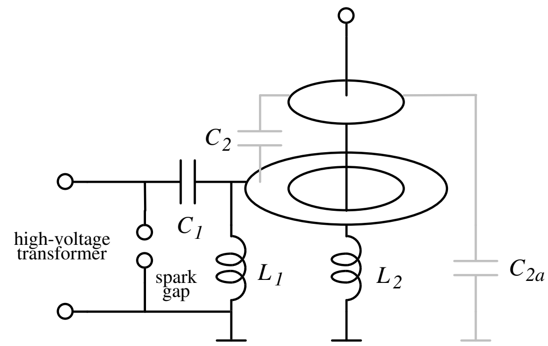

Figure 5. 4th-order experimental circuit with distributed capacitances C_2 and C_{2a}.

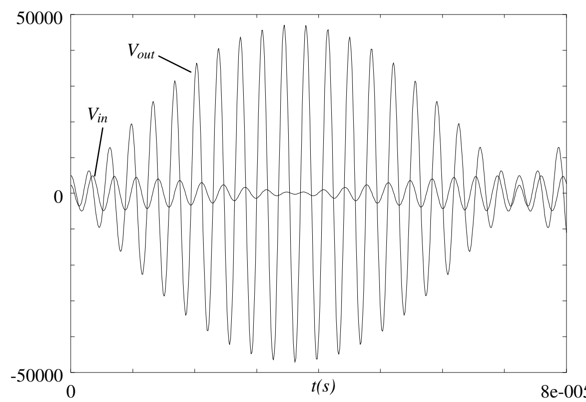

Figure 6. Ideal waveforms after C_1 is charged to 5000 V and connected to the circuit.

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