Considerations about Frequency Spectrum and Signal Feedthrough in the Analysis of Switched-Current Filters

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Abstract

The ASIZ program, written for the analysis of switched-current filters, in the original version considered only the main spectral component of the output signal, and assumed sampled-and-held input signals. This paper describes the implementation in the program of full output spectrum computation and direct signal feedthrough effects. The extension is simple, since the semi-symbolic approach of the algorithm already computes all the required quantities.

I. INTRODUCTION

The analysis of ideal switched-current (SI) filters was already shown to be very similar to the analysis of ideal switched-capacitor (SC) filters [1]-[3]. Both cases are, assuming idealized operation, particular cases of the analysis of periodically switched linear circuits (RC-active), where the circuit is assumed to reach a DC steady state soon after each change in the configuration of the switches. This assumption is the same used in the design of these filters, and so a tool that considers the same condition in the analysis is useful as a design verification tool. The ASIZ program [1]-[3], [8] was the first program specially designed for the analysis of these circuits, computing frequency-domain and time-domain responses, transfer functions and poles and zeros in z-transform, and frequency-domain sensitivities.

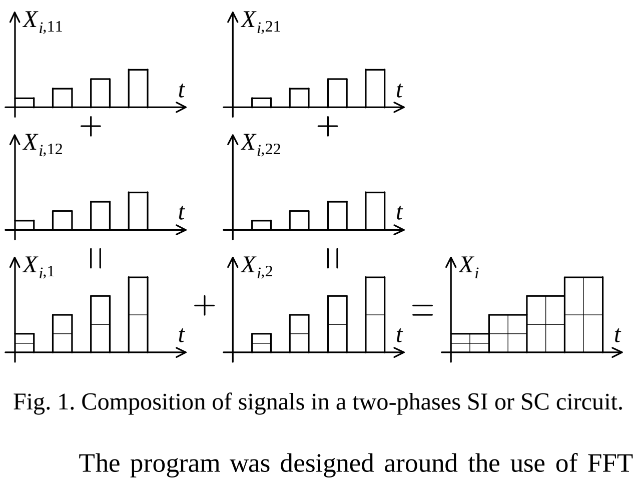

Fig. 1. Composition of signals in a two-phases SI or SC circuit.

The program was designed around the use of FFT interpolation for the computation of the transfer functions in z-transform of the circuit, from the input current or voltage to outputs that are nodal voltages. This fast and stable semi-symbolic method obtains transfer functions as ratios of polynomials of z. From these polynomials, all the other calculations are performed, including sensitivity analysis, done by the adjoint network method extended to SI and SC circuits.

In an f-phases SI or SC circuit, any signal X_i can be considered as composed by f components $X_{i,m}$, one for each of the m=1,...,f phases. Each of these components is composed by another f components $X_{i,mk}$, each for one of the k=1,...,f phases of the input signal, that add by superposition to form $X_{i,m}$. These $f\times f$ components add together to form X_i as shown in fig. 1, for the case of two phases and sampled-and-held (S/H) input.

Considering as outputs the nodal voltages E_i and their partial components $E_{i,mk}$, there are $f\times f$ transfer functions to be computed. In the original version, the program could compute these transfer functions, and combine them to generate the global transfer functions in the frequency domain as:

$$E_i(j\omega) = \frac{1}{f}\sum_{m=1}^{f}\sum_{k=1}^{f} E_{i,mk}(j\omega) \tag{1}$$

$$E_i(j\omega) = \frac{1-e^{j\omega T/f}}{j\omega T}\sum_{m=1}^{f}\sum_{k=1}^{f} E_{i,mk}(j\omega) \tag{2}$$

The double summation is computed by the evaluation of the $E_{i,mk}(z)$ for $z=e^{j\omega T}$, where T is the switching period. Case (1) considers impulse sampling at the output, what produces the usual "digital" transfer functions usually used in the design of filters. Case (2) considers S/H output signals, at each phase and is a better approximation of what is really seen at the analog output of a filter. In both cases, each clocking period T is considered divided in f phases of equal duration, and the input is considered sampled-and held at each phase.

This analysis ignores the other components of the output spectrum (it computes only the component with the same frequency of the input), and also the effects of changes in the input signals in the intervals between samples (equivalent to say that it assumes S/H input signals).

How to take these effects into account is known from the analysis methods developed for SC circuits [4] and for linear periodically switched circuits [5]. In this paper, the adaptation of these techniques to the analysis method used in the ASIZ program is discussed. In the discussion, it is still assumed "fast" stabilization of the circuit between the switching instants, and that the transfer functions from the input to the outputs within each switching phase are essentially memoryless, or instantaneous. It's shown that no additional analysis is necessary, because all the required informations are already present in the coefficients calculated by the semi-symbolic analysis.

II. TREATMENT OF DIRECT SIGNAL FEEDTHROUGH

When the input signal is allowed to vary between the sampling instants, two basic effects take place: First, what is effectively sampled by the circuit are the input values at the end of the phase intervals, that are not anymore equal to the values at the start of the intervals. Second, superimposed on the S/H output signals is a scaled copy of the input. Both effects can be taken into account, by computing $E_i(j\omega)$ as:

$$E_i(j\omega) = \frac{1-e^{j\omega T/f}}{j\omega T} e^{j\omega T/f} \left(\sum_{m=1}^{f} \sum_{k=1}^{f} E_{i,mk}(j\omega) - a_n \right) + \frac{a_n}{f} \quad (3)$$

The term a_n is simply the coefficient in the global numerator of $E_i(z)$ (numerator of the double summation of the $E_{i,mk}(z)$) that has the same degree of the denominator (a monic polynomial in z, of degree n, the same for all the $E_{i,mk}(z)$). This coefficient controls the gain for rapidly varying input signals, what is the assumed condition within each phase. The interpretation of equation (3) is: Remove from $E_i(j\omega)$ the direct feedthrough, advance the result in one phase (to take into account the sampling at the end of each phase), sample and hold the result, and add the continuous signal feedthrough. In the time domain, the equivalent expression is:

$$\begin{aligned} &E_i(t_0+\Delta t) = \\ &= E^{\#}_i(t_0+T/f) + a_{mn}(I_{in}(t_0+\Delta t) - I^{\#}_{in}(t_0+T/f)) \end{aligned} \quad (4)$$

where “#” means sampled-and-held, t_0 is the initial time of each phase m, and a_{mn} is the coefficient in the numerator of $E_{i,m}(z)$ (the summation in k of $E_{i,mk}(z)$) that has the same degree of the denominator.

The availability of the a_{mn} coefficients, obtained when the transfer function polynomials were computed by FFT interpolation, turns this analysis particularly simple.

Note that the formulation assumes “fast” circuits. The time necessary for the stabilization of the circuit after each switching instant, and the deformation of the input feedthrough due to the limited frequency response of the feedthrough path are ignored.

The effect of the signal feedthrough is significant in the rejection band of filters, because the signal passed to the output in this way can easily violate the ideal rejection band attenuation. Passband effects are usually negligible.

III. SPECTRUM OF THE OUTPUT SIGNAL

Assuming the input as a cosinusoid of frequency ω_o, the sampling operation inherent to SI or SC filters can generate at any output multiple frequency components at all $\omega_c = N\omega_s \pm \omega_o$, where $\omega_s = 2\pi/T$ is the switching frequency in rad/s, and N is an integer. The expressions in (1) and (2) can be extended to the computation of other frequency components by simply considering that the output components in phase m are sampled by impulse trains that start at $t=(m\text{-}1)T/f$. The main output is not affected, but the other components are. The modified expressions become (5) and (6) respectively, where was used the fact that $E_{i,mk}(j(N\omega_s \pm \omega_o)) = E_{i,mk}(\pm j\omega_o)$, what allows the saving of some computations. The exponents in the terms multiplying the summations in k can be simplified to $-jN(m\text{-}1)2\pi/f$.

$$\begin{aligned} &E_i(j\omega_c) = \\ &= \frac{1}{f} \sum_{m=1}^{f} e^{-j\omega_s N(m-1)T/f} \sum_{k=1}^{f} E_{i,mk}(\pm j\omega_o) \end{aligned} \quad (5)$$

$$\begin{aligned} &E_i(j\omega_c) = \\ &= \frac{1-e^{j\omega_c T/f}}{j\omega_c T} \sum_{m=1}^{f} e^{-j\omega_s N(m-1)T/f} \sum_{k=1}^{f} E_{i,mk}(\pm j\omega_o) \end{aligned} \quad (6)$$

The output spectrum considering the signal feedthrough is somewhat more complicated. The expression is given by (7). The differences from (3) are the multiplying term in the second summation, similar to what is seen in (5) and (6), but with m instead of m–1 in the exponent to compensate for the advanced sampling, and the window function (corresponding to a cosinusoidal input) replacing the input signal. The last summation reduces to a_n/f for N=0, and the equation reduces to (3).

$$\begin{aligned} &E_i(j\omega_c) = \\ &= \frac{1-e^{j\omega_c T/f}}{j\omega_c T} e^{j\omega_c T/f} \left(\sum_{m=1}^{f} e^{-j\omega_s NmT/f} \left(\sum_{k=1}^{f} E_{i,mk}(\pm j\omega_o) - a_{mn} \right) \right) + \\ &+ \sum_{m=1}^{f} a_{mn} \frac{e^{-jN2\pi(m-1)/f} - e^{-jN2\pi m/f}}{j2\pi N} \end{aligned} \quad (7)$$

IV. EXAMPLES

Example 1:

Fig. 2 shows the model for a normalized “second generation” [3] switched-current integrator, with a 1 Hz clock signal (idealized, with just two phases), receiving as input a sinusoidal current with peak value of 1 A and frequency of 0.1 Hz. Fig. 3 shows the input and output currents, computed assuming a sample-and-held input signal, with one sampling per switching phase, and the same signals when signal feedthrough due to an unsampled input is taken into consideration (eq. (4)). Fig. 4 shows the frequency responses of the integrator cell in both cases (eqs. (2) and (3)). All computed by the ASIZ program.

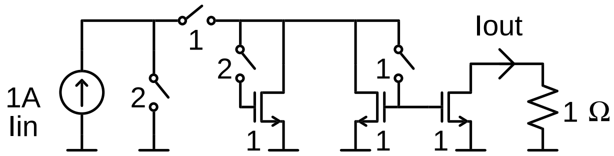

Fig. 2. Normalized “second generation” SI integrator, used as example of direct signal feedthrough and spectrum calculations.

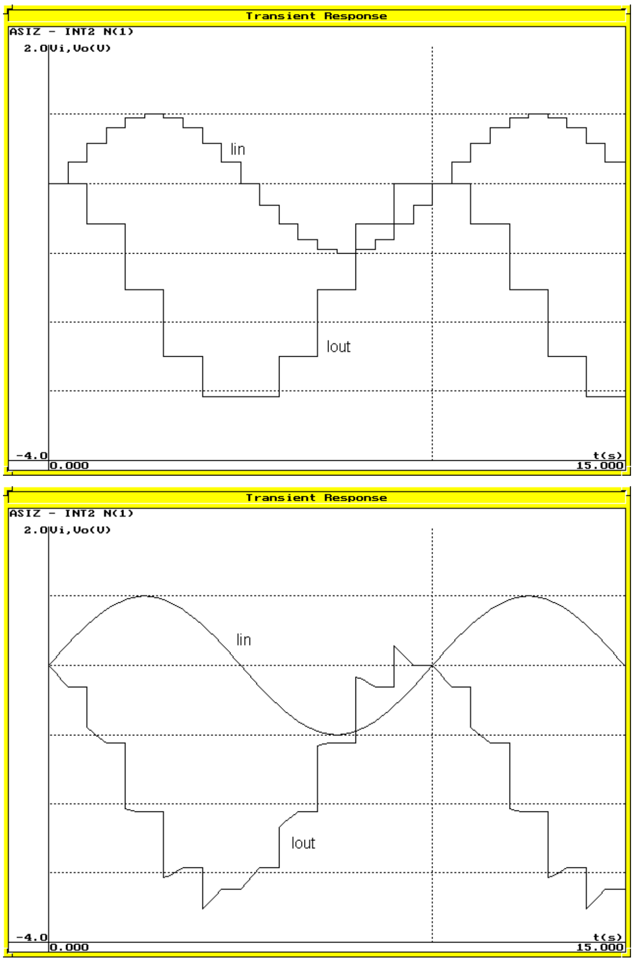

Fig. 3. Transient response I_{out} for a 0.1 Hz, 1 A, sinusoidal input I_{in}. Above: S/H input. Below: Continuous input.

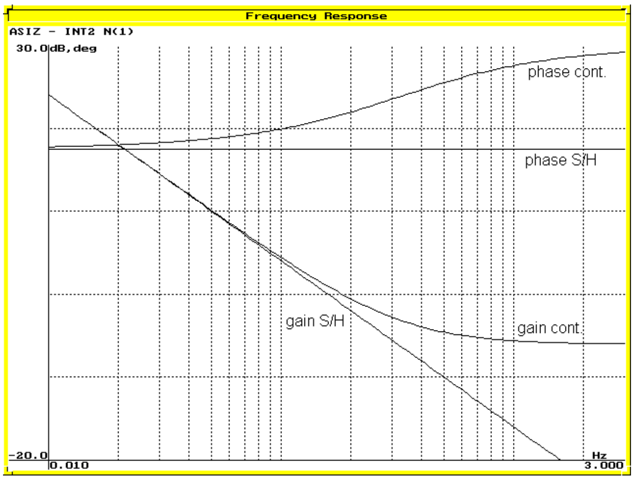

Fig. 4. Frequency responses (gain and phase) for the SI integrator, considering the input sampled and held and considering it continuous.

Example 2:

Fig. 5 shows how ASIZ plots the spectral components of the input, computed by eqs. (5) and (6). With S/H signals, the spectral components follow the main frequency response curves, because the same output waveform is obtained for input at any of the spectrum frequencies. With a continuous input, the relation is not so simple.

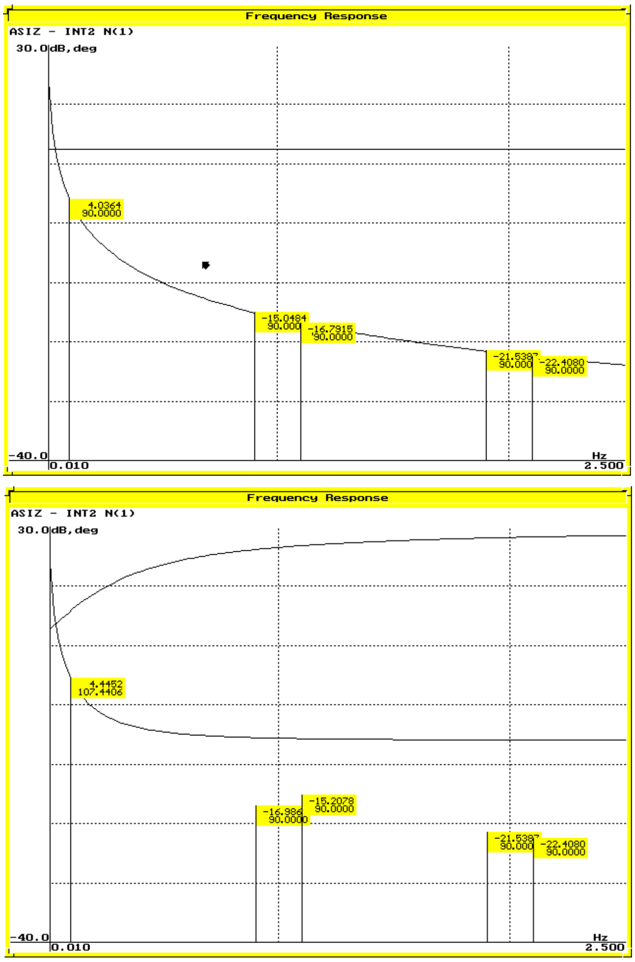

Fig 5. Frequency responses, now in linear frequency scale, with the first spectral components plotted, for a 0.1 Hz cosinusoidal input. Above: S/H signals. Below: continuous signals.

Example 3:

To see the effects of direct signal feedthrough in a more complex circuit, consider the 5th-order elliptic filter obtained by the "component simulation" technique, taken from [9], reproduced in fig. 6. This filter has a continuous direct path from input to output all the time. The filter was designed for 1 dB maximum passband attenuation and 40 dB minimum stopband attenuation, with the passband extending to 1/5 or the switching period, or 1/10 of the effective sampling rate, since the circuit operates in the same way in both phases. The element values for the normalized filter are:

C_1=2.070;	L_2= 0.7559;	C_2=0.4297;
C_3=2.264;	L_4=0.4875;	C_4=1.286;
C_5=1.583;	R=1;	T=0.6283

The frequency responses computed for the filter assuming sampled and held inputs/outputs and considering direct signal feedthrough are compared in fig. 7. The differences are only significant in the stopband, where the signal feedthrough is high enough to mask the transmission zeros and violate the minimum stopband attenuation by almost 6 dB. The solution to the problem is to just sample and hold the input at each phase, what results in the ideal response.

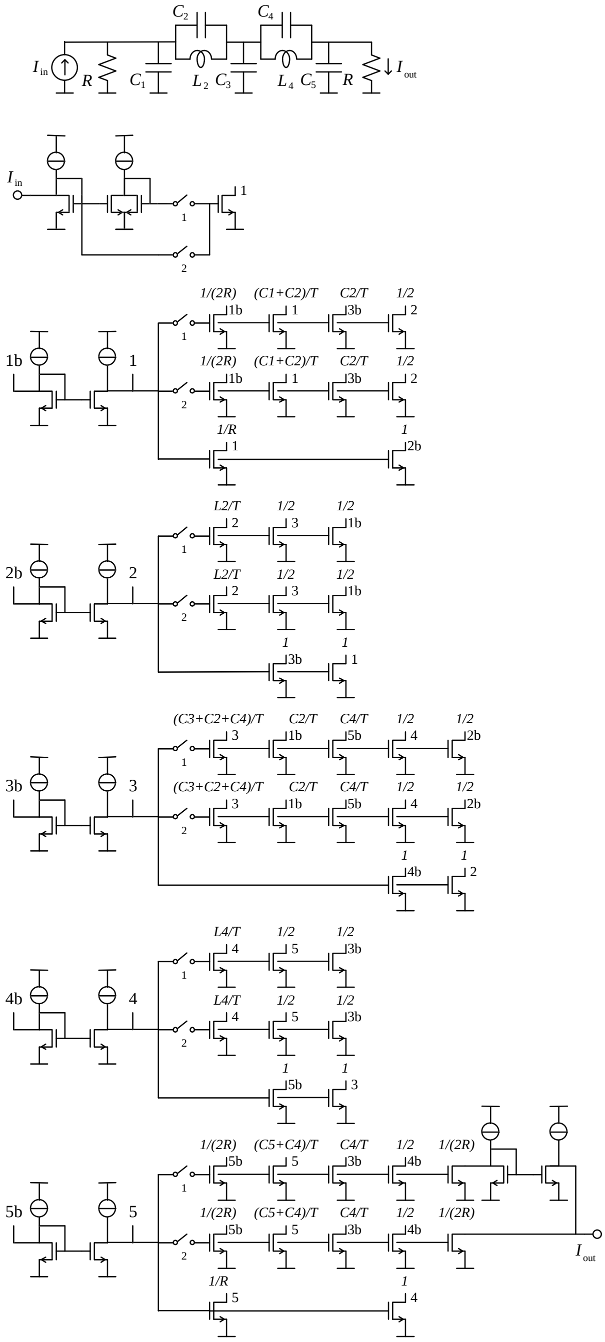

Fig. 6. Normalized 5th-order elliptic SI filter using the component simulation technique. The element values correspond to the elements of the normalized passive LC doubly terminated filter shown, and T is the normalized switching period.

V. CONCLUSION

It was shown how the complete spectrum of the output and the effects of direct signal feedthrough can be computed from the $f{\times}f$ transfer functions of an SI or SC filter, using the same data obtained from the FFT interpolation, as implemented in the ASIZ program. The availability of the transfer functions in the form of ratio of polynomials in z, as obtained by the interpolation, simplifies the computation, because it is only necessary to change the form of the evaluation for the global transfer function. It is never necessary to reanalyze the circuit. Note that the techniques presented here are just approximations of the true behavior of the circuit, even assuming perfect linearity. A more precise analysis, although still assuming linearity, considering all the effects of the limited frequency response of the continuous circuit at each phase, is also possible [5][6][7], but significantly more complex.

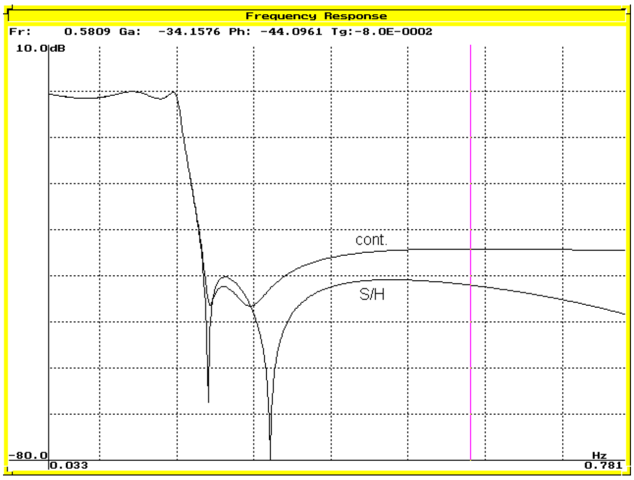

Fig. 7. Effect of direct signal feedthrough in the filter in fig. 6.

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