PHYSICALLY SYMMETRICAL AND ANTIMETRICAL LADDER FILTERS WITH FINITE TRANSMISSION ZEROS

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Abstract

Techniques are presented to generate approximations with finite transmission zeros suitable for realization as doubly terminated LC ladder filters with physically symmetrical or antimetrical structure. Such filters, with double transmission zeros, presents lower sensitivity characteristics in the transition and rejection bands when compared with filters obtained from classical approximations. Good results in passband sensitivity are also obtained from approximations presenting double attenuation zeros. Active simulations of these filters are specially attractive for fully integrated realization, because of simpler layout, and because thermal (or other) gradients over the filter structure results mainly in an easily correctable frequency shift.

Introduction

It was shown in [1] that the realization of a given transfer function as a physically symmetrical or antimetrical network results in extreme or near extreme values for the transfer function statistical deviations of gain and phase, defined as:

$$Vs(|T|)=8.686\sqrt{\sum_{i=1}^{n}(Vi\ Re\ S(T,xi))^2}\ dB \qquad (1)$$

$$Vs(\angle T)=57.30\sqrt{\sum_{i=1}^{n}(Vi\ Im\ S(T,xi))^2}\ deg. \qquad (2)$$

where xi are the elements of the network, Vi=dxi/xi are its variabilities and S(T,xi)=dLnT/dLnxi are the complex-valued sensitivities of the transfer function T to changes in xi. It was also observed that for usual all pole approximations these extremes are minima in the passband.

When all the attenuation zeros of an all pole approximation are in the jw axis, as in the Butterworth and Chebyschev approximations, the realization can be done as a symmetrical or antimetrical (for odd or even orders) LC doubly terminated network with maximum power transfer at the attenuation zeros. This results in zero sensitivities for all reactive elements in those frequencies, and low sensitivities in all the passband [2].

For other all pole approximations, such as the Bessel one, the LC doubly terminated synthesis results in assymmetrical networks. In [1] it was presented a procedure to the realization of these approximations as physically symmetrical or antimetrical RLC ladder networks, presenting better sensitivity characteristics when compared with the classical realizations. The higher losses can be easily compensated in active simulations.

The procedure is not generally applicable to the normal realization of classical non all pole approximations (elliptic, inverse Chebyschev, etc.), because of the existence of finite transmission zeros in distinct frequencies, that must be realized by different LC tanks in a ladder network, forcing physical assymmetry. With the classical realizations, passband sensitivities are good, because maximum power transfer can be made to occur at the attenuation zeros, but nothing can be said about transition an rejection band sensitivities.

Two approaches can be followed in these cases: The first one is studied in this paper, and consists in obtaining approximations with all (or all but one) double transmission zeros. If the attenuation zeros of such approximation are located all in the jw axis, it can usually be realized as a symmetrical or antimetrical LC doubly terminated ladder network. The other approach would be to obtain non ladder networks suitable for practical realization in symmetrical or antimetrical form of classical approximations, with easy active simmulation [3].

When used as passive prototypes for active RC signal-flow graph simulation realizations, symmetrical or antimetrical ladder filters results in symmetrical active structures. This simplifies the layout of fully integrated filters, and as each component out of the symmetry axis has a correspondent in the other half of the network with the same value and, by symmetry, the same sensitivity, an interesting property arises: If the temperature (or other parameter) varies across the filter structure in a direction perpendicular to the symmetry axis, and component values are dependent on it, the total error introduced in the transfer function, in first order approximation, is the same as if the whole filter were subjected to the temperature at the symmetry axis. This reduces the error to a frequency shift, easily corrected by automatic tuning techniques [10]. If the layout is made with central symmetry, the first-order effect of gradients in any direction can be cancelled.

All the approximations studied are for normalized low pass filters. Other approximations can be found by the usual frequency transformations.

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Inverse Polinomial Approximations

These approximations are obtained in a way similar to the one used for the classical Inverse Chebyschev approximation. A polinomial M(w) must be found with the following properties:

a) M(w) is an even or odd polinomial with double real roots, except for a possible pair of single roots in symmetrical frequencies for odd orders. All roots are in the interval -1, 1 and at least a single root at the origin must exist for realizability of the final transfer function as a doubly terminated LC network.

b) M(w) oscillates between -1 and 1 for $-1<=w<=1$, with at most a pair of zero crossings in symmetrical frequencies for odd orders. The extreme values of M(w) in this range are all -1, 0 or 1. $|M(w)|>1$ for $|w|>1$.

Given Amax (maximum attenuation in the passband) and Amin (minimum attenuation in the stopband) in dB, the characteristic function K(s) can be obtained from:

$$K(jw) = \frac{e.a^2}{M(1/w)} \qquad (3)$$

$$e = \sqrt{10^{0.1\,Amax} - 1} \quad (4) \qquad a^2 = \frac{1}{e}\sqrt{10^{0.1\,Amin} - 1} \quad (5)$$

This results in K(s) having poles (transmission zeros) in the jw axis in frequencies that are the inverse of the roots of M(w). The zeros of K(s) (attenuation zeros) are all at the origin, and the obtained approximation is maximally flat in the passband. From K(s), the transfer function T(s) and a doubly terminated ladder network realizing it can be obtained in the usual way [4]. Alternatively, a synthesis method similar to the one presented in [1] can be used.

As the square of M(w) in an equal ripple function between w=-1 and w=1, the final approximation presents uniform ripple in the rejection band. The double roots of M(w) are translated into double jw axis zeros in T(s), and realized by equal or dual LC tanks in symmetrical or antimetrical positions in a doubly terminated LC ladder network. The possible single roots in symmetrical frequencies for odd orders are translated into single jw axis zeros in T(s), and realized by a central LC tank in a symmetrical network. Without normalization, the rejection band begins at w=1.

Restricting the possible M(w) to polinomials with the minimum number of roots at the origin and a maximum number of double distinct roots, for better selectivity in the final approximation, a class of unique polinomials results and three cases can be identified:

The first case is of even order approximations of degree n=2(2k+1), k=1,2,... The polinomial M(w) has a double root at the origin and k double roots for 0<w<1. M(w) is simply the Chebyschev polinomial of order 2k+1 squared, and the resulting approximation has k double finite zeros for w>1 and a double zero at infinity.

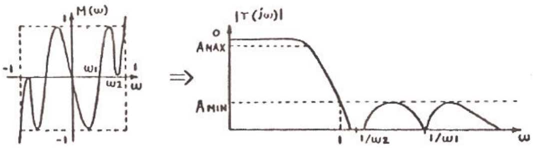

Fig. 1 Inverse polinomial approximation.

The second case is of even order approximations of degree n=2(2k+2), k=1,2,... In this case the use of a normal Chebyschev polinomial of order 2k+2 squared is not possible because such polinomial presents no roots at the origin and the inverse approximation obtained has not the essential zero at infinity necessary to the realization as a doubly terminated LC network.

The solution is to apply a Moebius transformation [4] on a Chebyschev polinomial of degree 2k+2, moving the smaller root pair to the origin. Let Cm(w) be the transformed polinomial obtained. M(w) is then Cm(w) squared and presents a quadruple root at the origin and k double roots for 0<w<1. The approximation obtained has k double finite zeros for w>1 and a quadruple zero at infinity.

The third case is of odd order approximations. M(w) must be odd and follow property (b). The graphs for possible M(w) for orders 5, 7 and 9 are in fig. 2. The single roots occur for orders n=2k+5, k=1,2,..., when k+1 different polinomials are possible. The selectivities of the approximations obtained from the alternatives are nearly equivalent, as can be observed by the comparison of the higher order coefficients for n=7 and n=11 in table 2. For other odd orders only a single polinomial exists.

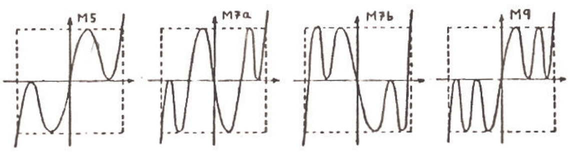

Fig. 2 M(w) for odd order approximations.

These polinomials can be found by numerical means. A procedure very alike the Remez optimization algorithm [4] can be used:

An even or odd polinomial of the desired form, of order n with p zeros at the origin is:

$$M(w) = a_n w^n + a_{n-2} w^{n-2} + \ldots + a_p w^p \qquad (6)$$

There are m=(n-p)/2+1 coefficients to determine and the same number of prescribed maxima and minima for 0<w<=1, including the value at w=1. Let fk, k=1,...,m be the prescribed maxima and minima in ascending order of w. Given an inicial approximation for M(w), that can be the Chebyschev polinomial of order n-p+1 for n odd, or n-p-2 for n even, the iteractive procedure is:

a - Find wk, k=1,...,m-1, the m-1 positive roots of M'(w), the derivative of M(w). Let wm=1.

b - Solve the system of linear equations:

M(wk)=fk, k=1,...,m

for the new coefficients of M(w).

c - Repeat a and b until convergence.

Convergence is fast, and there are no serious numerical problems. Even order polinomials can also be approximated and the only restriction upon the values of the extremes fk is that no two consecutive values can be equal and no three can be in ascending or descending order. A double transmission zero in the rejection band is produced by each fk=0 and fk=1 or -1 produces a peak of attenuation Amin between or adjacent to the zeros. For orders 5 to 14 the obtained polinomials M(w) are listed in tables 1 and 2.

n	a2	a4	a6	a8	a10	a12	a14
6	9.00000	-24.0000	16.0000				
8	0.00000	23.3137	-56.2843	33.9706			
10	25.0000	-200.000	560.000	-640.000	256.000		
12	0.00000	125.354	-810.300	1891.45	-1881.00	675.500	
14	49.0000	-784.000	4704.00	-13440.0	19712.0	-14336.0	4096.00

Table 1: M(w) for even orders.

n	a1	a3	a5	a7	a9	a11	a13
5	4.25715	-12.6409	9.38372				
7a	-6.56072	46.1922	-82.7391	44.1076			
7b	-6.11359	38.4620	-75.7526	44.4042			
9	7.60863	-74.9637	249.908	-321.502	139.949		
11a	-10.2156	176.830	-881.885	1876.41	-1789.36	629.218	
11b	-9.48827	146.070	-785.376	1777.68	-1762.98	635.092	
11c	-9.40742	142.482	-757.526	1713.03	-1720.45	632.872	
13	10.9716	-226.748	1678.95	-5552.43	9061.76	-7150.26	2178.76

Table 2: M(w) for odd orders.

Rational Approximations

When greater selectivity is needed, approximations with rational characteristic functions are mandatory. A simple approach to this objective is to obtain approximations with characteristic functions in the form:

$$K(jw) = \frac{e.a.Q(w)}{w^n Q(1/w)} = \frac{e.a.Q(w)}{Qr(w)} \qquad (7)$$

where e and a are given by (4) and (5), and Qr(w) is Q(w) with the coefficients in reverse order. Q(w) must have the following properties:

a) Identical to property (a) for M(w).

b) a.Q(w)/Qr(w) oscillates between -1 and 1 for -1<w<1, with at most a pair of zero crossings in symmetrical frequencies for odd orders. The maximum and minimum values of a.Q(w)/Qr(w) in this range are all -1, 0 or 1. |a.Q(w)/Qr(w)|>1 for |w|>1 and Q(1)=1.

The transfer function T(s) and a LC doubly terminated ladder network realizing it is obtained from K(s) in the usual way [4]. Each root of Q(w) is translated into an attenuation zero at the same frequency and into a transmission zero at the inverse frequency in T(s). As the square of a.Q(w)/Qr(w) is an equal ripple function for w between -1 and 1, the resulting transfer function presents equal ripple in the passband and in the stopband. Without normalization, the geometrical center of the transition band is at w=1.

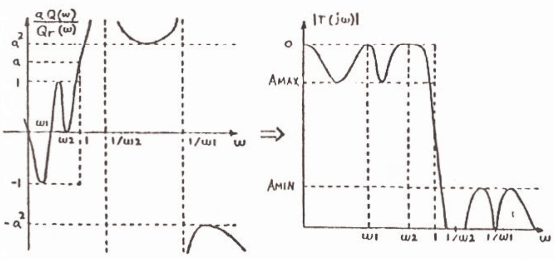

Fig. 3 Rational approximation.

Restricting the possible K(s) to those with the maximum number of distinct double zeros (and poles), K(s) for even orders can be obtained by squaring the characteristic functions of half order elliptic approximations with the specifications Amax' and Amin' given by (8) and (9), modified by an appropriate Moebius transformation [4] if necessary for realizability.

$$Amax' = 10 \, Log \, (1 + \sqrt{10^{0.1 \, Amax} - 1}) \qquad (8)$$

$$Amin' = 10 \, Log \, (1 + \sqrt{10^{0.1 \, Amin} - 1}) \qquad (9)$$

Odd order K(s) can be obtained numerically by an algorithm similar to the used for inverse polinomial approximations, also useful for even order cases. In this case, a different Q(w) is needed for each possible value of a as defined in (5).

A rational function of the type a.Q(w)/Qr(w), for an even or odd polinomial Q(w) with degree n and p zeros at the origin is of the form:

$$\frac{a.Q(w)}{w^n Q(1/w)} = a.\frac{a_n w^n + a_{n-2} w^{n-2} + \ldots + a_p w^p}{a_p w^{n-p} + a_{p+2} w^{n-p-2} + \ldots + a_n} \qquad (10)$$

There are m=(n-p)/2+1 coefficients to determine. The function (10) has m-1 prescribed extreme values for 0<w<1 and is also known that Q(1)=1. Let fk, k=1,...,m-1 be the extreme values of (10) for 0<w<1. Given an initial approximation for Q(w), which can be the same used for inverse polinomial approximations, the iteractive procedure is:

a - Find wk, k=1,...,m-1, the m-1 smaller positive

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roots of $Qr(w)Q'(w)-Q(w)Qr(w)'$, roots of the derivative of (10) for $0<w<1$.

b - Solve the system of linear equations:
$a.Q(wk)=fk.wk^{n}\,Q(1/wk)$, $k=1,\ldots,m-1$; $Q(1)=1$
for the new coefficients for $Q(w)$.

c - Repeat a and b until convergence.

Convergence in this case is also fast, but very sensible to the initial approximation. The use of Chebyschev polinomials as initial approximations is useful only when the desired approximation has a large difference between passband and stopband attenuations (large a in (5)). When this is not the case, the parameter a can be varied between iteractions, beginning with a large value and ending with the correct one. The restriction upon the values of the extremes fk is that no two consecutive values can be equal and no three can be in ascending or descending order (including an extra value, not used, fm=1). Each fk=0 produces a double transmission zero in the stopband and a double attenuation zero in the passband. fk=1 or -1 produces a peak with attenuation Amin between two transmission zeros and a valley with attenuation Amax between two attenuation zeros. If all fk are made to alternate between -1 and 1, a normal elliptic approximation is obtained.

Other approximations, with Chebyschev-like ripple in the passband can be obtained by the "general parameter" method [4], used in conjunction with an optimization algorithm to force double transmission zeros and equal ripple in the rejection band [5].

Realizability

The realizability of the studied approximations as LC doubly terminated ladders is determinable by the Fujisawa criterion [6]. For small attenuation in the rejection band, a LC doubly terminated ladder realization may be impossible.

Sensitivity Improvement

As the networks obtained are physically symmetrical or antimetrical and presents maximum power transfer in the passband, characteristics of very low sensitivity are to be expected. Conventional filters with several single transmission zeros in the stopband presents usually large gain sensitivities in the peaks between the zeros [8]. For high order and small transition band filters, with several single zeros grouped in the stopband beginning, errors due to these high sensitivities can deteriorate the filter selectivity. With double zeros, it is observed that these sensitivities are lower for the same attenuation specifications.

In the passband, the sensitivity characteristics of the inverse polinomial approximations are very alike those of inverse Chebyschev approximations, or even Butterworth approximations with the same passband. With the studied rational approximations an interesting property appears: the double attenuation zeros in the passband are also double sensitivity zeros due to maximum power transfer. These double zeros mantains the sensitivity characteristics close to zero in the passband more effectively than single zeros does for the same passband specifications. This can result in excellent sensitivity characteristics in the passband.

All pole approximations with double sensitivity zeros in the passband can be obtained by using the polinomials in tables 1 and 2 as characteristic polinomials. In this case, the final networks are symmetrical or antimetrical for any combination of single, double or higher order roots in M(w). A class of approximations intermediary between the Butterworth (all roots of M(w) equal) and Chebyschev (all roots of M(w) distinct) results. The presented algorithm must be modified if roots of order greater than two are desired. A special case of these approximations is the modified Butterworth filter [11], with multiple attenuation zeros at the same frequency. Its inverse formulation, a maximally flat approximation with multiple order transmission zeros at the same frequency [12], can be considered as a special case of the studied inverse polinomial approximations.

Examples

Ex. 1: Inverse polinomial approximation.

A 6th. order modified Inverse Chebyschev filter (the characteristic function is obtained by (3) from a 6th.order Chebyschev polinomial modified by a Moebius transformation) is compared with the physically antimetrical inverse polinomial filter obtained from the 6th. order polinomial in table 1. The specifications used are: Amax=3.0103 dB in the passband ($0<w<1$) and Amin=40 dB in the stopband. The resulting ladder networks are in fig. 4, both with the same structure. The magnitude and sensitivity characteristics obtained are in figs. 5 and 6. To eliminate the false high sensitivities in the vicinity of transmission zeros and enhance the gain statistical deviation between them, slope normalized sensitivities [8] were used in the computation of (1). The resistors sensitivies in d.c. were also subtracted from the resistors sensitivities before slope normalization, to eliminate the effect of flat gain sensitivity [1][9]. A variability of 0.05 was used for all components.

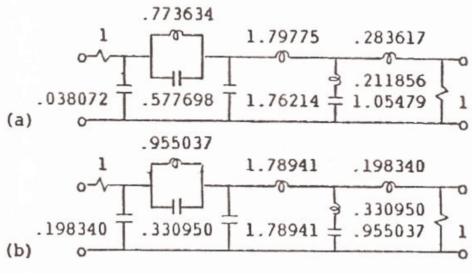

Fig. 4 (a) Inverse modified Chebyschev. (b) Antimetrical inverse polinomial.

Ex. 2: Rational approximation.

A 7th. order elliptic filter is compared with the two possible 7th. order symmetrical rational filters. The specifications used are: Amax=1 dB in the passband (0<w<1) and Amin=60 dB in the stop-band. The resulting ladder networks are in fig. 7, The magnitude and sensitivity characteristics are in figs. 8 and 9. The sensitivity measure used is the same of the first example.

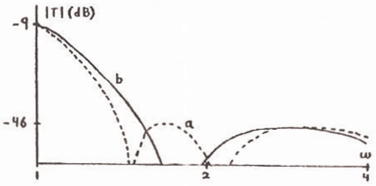

Fig. 5 Magnitude characteristics for 6th. order inverse polinomial filters.

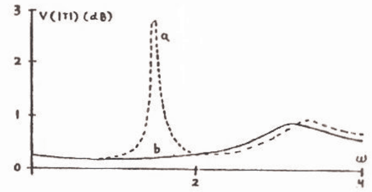

Fig. 6 Slope normalized gain statistical deviation for 6th. order inverse polinomial filters.

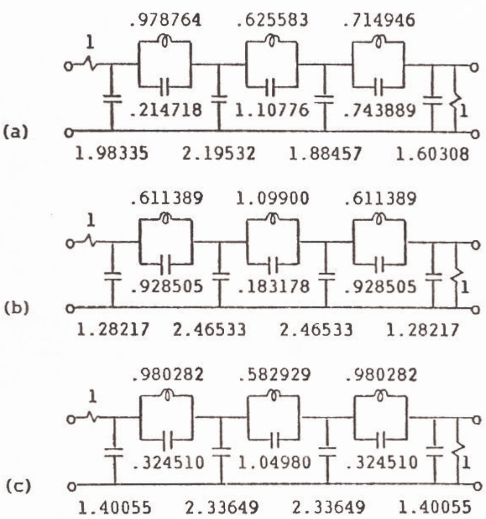

Fig. 7 (a) Elliptic. (b) Symmetrical, single zero first. (c) Symmetrical, double zero first

Conclusions

Two new approximation methods were presented, with characteristics that turns the LC doubly terminated filters obtained from them specially convenient as passive prototypes for fully integrated realizations. The selectivity characteristics of the studied approximations are slightly worse than those of the equivalent optimal (in this sense) classical approximations, but the sensitivity properties of the obtained networks make them interesting alternatives for high-order, high-selectivity filters realized in any technology.

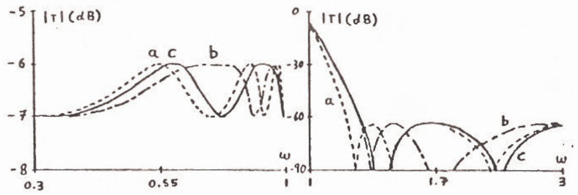

Fig. 8 Magnitude characteristics for 7th. order rational filters.

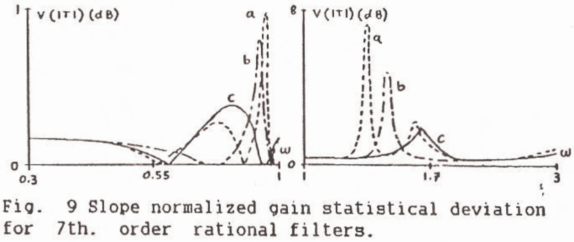

Fig. 9 Slope normalized gain statistical deviation for 7th. order rational filters.

Acknowledgment

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