Wireless Energy Transfer

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Introduction

This document will relate a precise analysis and experiments of a "wireless energy transfer" system using a system similar to the one proposed by Nikola Tesla in his patents 649621 "Apparatus for Transmission of Electrical Energy" and 645576 "System of transmission of electrical energy", both initially from 1897.

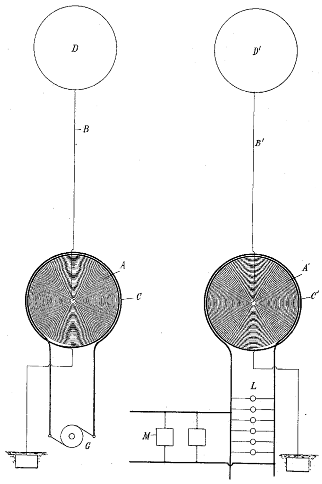

Figure 1. Tesla's wireless system.

The system (Fig. 1) consists of two identical air-core transformers, the transmitter (left) having a primary coil with few turns and a secondary coil with many turns, and the receiver (right) with the opposite. In the drawing the coils as depicted as flat, but solenoidal coils can be used in the same way. The large coils are grounded and have (optional) terminals with distributed capacitance. The primary coil of the transmitter is powered from an AC power supply at a frequency ideally identical to the resonance frequency of the secondary coil with its capacitive load. A conventional "Tesla coil" circuit could be used as transmitter too, by the addition of a primary capacitor, a spark gap, and a medium high-voltage power supply. The receiver is shown connected directly to loads as lamps (and motors, but this would require some signal conditioning as rectification). The system operates with the displacement current between the terminals, that may be very small, but with both devices operating at resonance with high voltages, it allows energy transmission at a distance similar to the sizes of the transmitter and receiver with reasonable efficiency (Tesla imagined that it would be possible to transmit significant energy at long distances, but this is actually infeasible due to energy dispersion with distance and finite quality factor of the coils). Energy return is through the ground. It's also possible to use similar systems in balanced mode, without a ground connection, or to replace the ground at one or both sides by "counterpoises", relatively large conductors with distributed capacitance to ground much larger than the one of the coils and terminals (note that the return current continues to pass through the ground).

Implementation

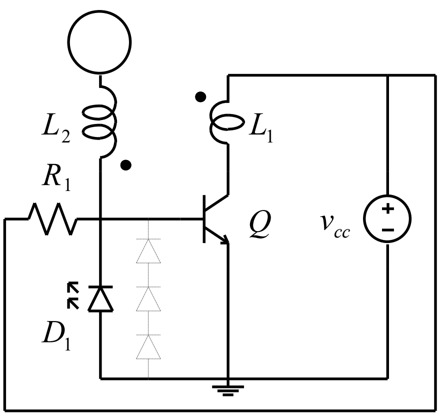

Figure 2. Blocking oscillator transmitter.

For the transmitter, a good sinusoidal high-voltage generator is needed. A convenient possibility, although not very efficient, is a blocking oscillator known as "Slayer exciter", shown in Fig. 2. L_1 and L_2 form the transformer with low coupling coefficient (k = 0.1-0.2) of the Tesla transmitter. L_2 usually is an air-core solenoidal coil with many turns and L_1 can be solenoidal of flat, with just a few turns. The terminal at the upper end of L_2 forms a distributed capacitance, along with the self-capacitance of the coil L_2, resonating with L_2 in the hundreds of kHz or a few MHz. While in normal operation, the current in L_2 is essentially sinusoidal, due to the high quality factor Q of the resonant circuit made of L_2 and its distributed capacitances, and turns the transistor on and off once in each oscillation cycle. When the transistor is turned on, current flows through L_1, what transfers some energy to L_2. When the transistor is turned off, the LED D_1 lights up, limiting the negative voltage at the base of the transistor and providing a convenient indication that the oscillator is working. The three diodes limit possible excessive current in the LED. A similar protection could be added for the transistor, with two diodes in the opposite direction. The resistor R_1 is only for startup, providing a small current for the base of the transistor. The circuit is reasonably efficient when transistor conducts saturated, but the interruption of the primary current when the secondary current crosses zero consumes significant power. The output voltage at the terminal depends on the quality factor of the output resonant circuit. It stabilizes in a condition where the losses in L_2 due to wire resistance, energy consumed by the receiver, irradiation, and loss by voltage drop at the base of the transistor and the LED balance the energy transferred from L_1. Usually a few volts of v_{cc} result in up to a few thousands of volts at the output.

For the receiver, a copy of L_2 and its terminal can be used, with a copy of L_1 for low-voltage output. The coils can also be different, but the resonance frequency of L_2 with its distributed capacitance and the terminal capacitance must be around the same of the transmitter.

Approximate analysis of the transmitter

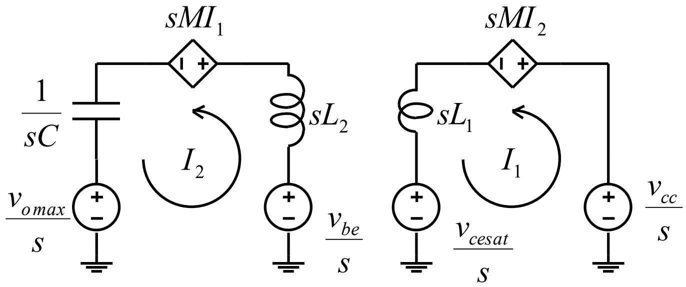

Figure 3. Model for the first semicycle.

An approximate analysis of the transmitter behavior can be done by replacing the transistor by a switch, possibly with a voltage drop v_{cesat}, controlled by the base current. It is assumed that at the beginning of a cycle there is no current in the coils, but a high voltage v_{omax} is present at the output terminal, modeled as a capacitance C. Fig. 3 shows the model, using Laplace transforms. Resistances were ignored to avoid exponential terms in the solution. The currents in the coils can be calculated by solving the mesh system:

$$\begin{bmatrix} sL_1 & sM \\ sM & sL_2 + \frac{1}{sC} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{v_{cc} - v_{cesat}}{s} \\ \frac{v_{be} - v_{omax}}{s} \end{bmatrix}$$

Resulting in:

$$I_1(s) = \frac{s^2 C(L_2(v_{cc} - v_{cesat}) + M(v_{omax} - v_{be})) + v_{cc} - v_{cesat}}{s^2(s^2 C(L_1 L_2 - M^2) + L_1)}$$

$$I_2(s) = -\frac{C(L_1(v_{omax} - v_{be}) + M(v_{cc} - v_{cesat}))}{s^2 C(L_1 L_2 - M^2) + L_1}$$

$I_1(s)$ can be expanded in partial fractions as:

$$I_1(s) = \frac{CM(L_1(v_{omax} - v_{be}) + M(v_{cc} - v_{cesat}))}{L_1(s^2 C(L_1 L_2 - M^2) + L_1)} + \frac{v_{cc} - v_{cesat}}{s^2 L_1}$$

Returning to the time domain it's then seen that $i_1(t)$ is a sinusoid added to a ramp and $i_2(t)$ is a pure sinusoid.

$$i_1(t) = \frac{CM(L_1(v_{omax} - v_{be}) + M(v_{cc} - v_{cesat}))\sqrt{\frac{L_1}{C(L_1 L_2 - M^2)}}}{L_1^2} \sin\sqrt{\frac{L_1}{C(L_1 L_2 - M^2)}}t + \frac{v_{cc} - v_{cesat}}{L_1}t$$

$$i_2(t) = -\frac{C(L_1(v_{omax} - v_{be}) + M(v_{cc} - v_{cesat}))\sqrt{\frac{L_1}{C(L_1 L_2 - M^2)}}}{L_1} \sin\sqrt{\frac{L_1}{C(L_1 L_2 - M^2)}}t$$

The oscillation occurs at the resonant frequency with the primary coil short-circuited:

$$f=\frac{1}{2\pi}\sqrt{\frac{L_1}{C(L_1L_2-M^2)}}=\frac{1}{2\pi}\sqrt{\frac{1}{CL_2(1-k^2)}}$$

Note that the coupling coefficient is defined as $k=M/\sqrt{L_1L_2}$, and is always smaller than 1. The output voltage can be calculated as:

$$V_o(s)=\frac{v_{omax}}{s}+\frac{1}{sC}I_2(s)=$$
$$=\frac{s(L_1(v_{omax}-v_{be})+M(v_{cc}-v_{cesat}))}{L_1\left(s^2+\frac{L_1}{C(L_1L_2-M^2)}\right)}+\frac{L_1v_{be}+M(v_{cesat}-v_{cc})}{sL_1}$$

Returning to the time domain, $v_o(t)$ is a cosinusoid added to a constant:

$$v_o(t)=\frac{L_1(v_{omax}-v_{be})+M(v_{cc}-v_{cesat})}{L_1}\cos\sqrt{\frac{L_1}{C(L_1L_2-M^2)}}t+\frac{L_1v_{be}+M(v_{cesat}-v_{cc})}{L_1}$$

The minimum value of $v_o(t)$ happens when the base current falls to zero, and the cosine reaches −1:

$$v_{omin}=-v_{omax}+2v_{be}-\frac{2M}{L_1}(v_{cc}-v_{cesat})$$

Energy is gained if $v_{omin}<-v_{omax}$, but the losses in the remaining of the cycle must be accounted too. Ideally the transistor ceases to conduct at the end of the first semicycle. In practice it takes some time to switch off, first leaving the saturation when the ratio of the collector and base currents equals the HFE of the transistor and then ceasing to conduct after some time. Normally a large voltage pulse appears at the collector due to the interruption of the collector current, dissipating some energy in the transistor. A simplified model for the second semicycle is shown in Fig. 4. The transistor is considered as an open circuit, and the time origin is shifted to the beginning of the semicycle (it could be to some time later to account for the time taken by the transistor to switch off). Considering the beginning of the semicycle, $i_{12}(0) = i_{1final}$ and $i_{22}(0) = 0$, where i_{1final} is the final collector current in the first semicycle:

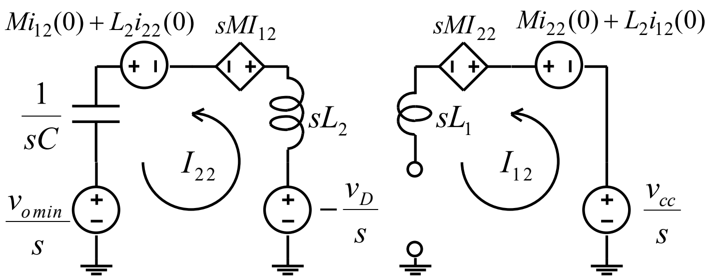

Figure 4. Model for the second semicycle.

$$i_{1final}=\frac{v_{cc}-v_{cesat}}{L_1}\pi\sqrt{CL_2(1-k^2)}$$

From Fig. 4 the current $I_{22}(s)$ in the second semicycle is obtained (Note that $I_{12}(s) = 0$):

$$I_{22}(s)=\frac{\frac{-v_D}{s}+Mi_{1final}-\frac{v_{omin}}{s}}{\frac{1}{sC}+sL_2}=\frac{s\frac{M}{L_2}i_{1final}}{s^2+\frac{1}{L_2C}}+\frac{\sqrt{\frac{1}{L_2C}}\sqrt{\frac{C}{L_2}}\left(-v_D-v_{omin}\right)}{s^2+\frac{1}{L_2C}}$$

This is a cosinusoid added to a sinusoid. The frequency is slightly different from the one in the first semicycle, and there is a small step at the beginning due to the cosine, meaning that some extra energy is transferred to the secondary when the transistor cuts off.

$$i_{22}(t)=\frac{M}{L_2}i_{1final}\cos\sqrt{\frac{1}{L_2C}}t+\sqrt{\frac{C}{L_2}}\left(-v_D-v_{omin}\right)\sin\sqrt{\frac{1}{L_2C}}t$$

The voltage at the collector when the transistor cuts off comes from:

$$V_{ce}(s)=\frac{v_{cc}}{s}+Mi_{22}(0)+L_1i_{12}(0)-sMI_{22}(s)=\frac{v_{cc}}{s}+L_1i_{1final}-sMI_{22}(s)=$$

$$=\frac{sM(v_{omin}+v_D)}{L_2\left(s^2+\frac{1}{L_2C}\right)}+\frac{i_{1final}M^2\sqrt{\frac{1}{L_2C}}}{\sqrt{L_2C}\left(s^2+\frac{1}{L_2C}\right)}+\frac{v_{cc}}{s}+i_{1final}\left(L_1-\frac{M^2}{L_2}\right)$$

This a cosine, a sine, a DC level, and an impulse:

$$v_{ce}(t)=\frac{M(v_{omin}+v_D)}{L_2}\cos\sqrt{\frac{1}{L_2C}}t+\frac{i_{1final}M^2}{\sqrt{L_2C}}\sin\sqrt{\frac{1}{L_2C}}t+v_{cc}+i_{1final}\left(L_1-\frac{M^2}{L_2}\right)\delta(t)$$

The output voltage at the second semicycle comes from:

$$V_o(s)=\frac{v_{omin}}{s}+\frac{1}{sC}I_{22}(s)=$$

$$=\frac{\pi M(v_{cc}-v_{cesat})\sqrt{\frac{C(L_1L_2-M^2)}{L_1}}\sqrt{\frac{1}{L_2C}}}{L_1\sqrt{\frac{1}{L_2C}}\left(s^2+\frac{1}{L_2C}\right)}+\frac{s(L_1(2v_{be}+v_D-v_{omax})+2M(v_{cesat}-v_{cc}))}{L_1\left(s^2+\frac{1}{L_2C}\right)}-\frac{v_D}{s}$$

This is a sinusoid, as cosinusoid, and a constant level:

$$v_o(t)=\frac{\pi M(v_{cc}-v_{cesat})\sqrt{\frac{C(L_1L_2-M^2)}{L_1}}}{L_1\sqrt{\frac{1}{L_2C}}}\sin\sqrt{\frac{1}{L_2C}}t+$$
$$+\frac{L_1(2v_{be}+v_D-v_{omax})+2M(v_{cesat}-v_{cc})}{L_1}\cos\sqrt{\frac{1}{L_2C}}t-v_D$$

All these waveforms are plotted in Fig. 5, for the setup:

C = 2.5 pF; k = 0.16; L_1 = 4 μH; L_2 = 1.4 mH; v_{cc} = 6 V; v_{cesat} = 0; v_{omax} = 1000 V; $v_D = v_{be}$ = 0.6 V; M = 11.97 μH.

There is an error in this model in the collector waveform, because the voltage can't drop to -2.84 V as shown when the transistor cuts off, because the diode from the base to the collector conducts. There is little difference caused by this, however, when a simulation is observed. The values used correspond approximately to the transmitter shown in Fig. 7, where the waveforms at the collector and at the base are shown in the oscilloscope. The simulation does not agree well with the experimental measurement, because there is a delay in the switching of the transistor, it has junction capacitances, ignored, and the stray capacitances in the breadboard used are significant. This causes a significant shortening of the time when the transistor does not conduct, transforms the impulse in a large pulse containing ringing, and causes ringing also in the base voltage. A LED was used as the base diode, with a larger voltage drop, and there is a load added by the receiver. The operation is at 2.7 MHz, quite high for the BC337 transistor used. The general behavior, however, is correct, and if coils resonating at a lower frequency are used, the waveforms get closer to the ideal. Figs. 8 and 9 show the same setup with different coils, operating at lower frequencies. In Fig. 8 the frequency is 1.58 MHz, and in Fig. 9 approximately 1 MHz, but the waveforms are irregular, not repeating at every cycle. This is quite common for this circuit, caused by the delays of the transistor and stray capacitances. In the last circuit small adjustable antennas were added to allow precise tuning. In the other cases an alligator clip was used as top load in the receiver, also to improve the tuning.

Energy consumed

The energy balance in the circuit can be evaluated by considering that at the beginning of the first semicycle all the energy was concentrated at the capacitance C charged to v_{omax}, and that at the end of the second semicycle the energy is again concentrated there. The final voltage can be obtained by taking the magnitude of the sine-cosine combination in $v_o(t)$ in the second semicycle and subtracting v_D:

$$v_{ofinal} = \sqrt{\left(\frac{\pi M(v_{cc}-v_{cesat})\sqrt{\dfrac{C(L_1L_2-M^2)}{L_1}}}{L_1\sqrt{\dfrac{1}{L_2C}}}\right)^2 + \left(\frac{L_1(2v_{be}+v_D-v_{omax})+2M(v_{cesat}-v_{cc})}{L_1}\right)^2} - v_D$$

The numerical evaluation of the expression gives 1035 V. This value can be made equal to v_{omax}, and the resulting equation can be solved for v_{omax}. The result, using k, is:

$$v_{omax} = \frac{4k(v_{cc}-v_{cesat})(2v_{be}+v_D)\sqrt{L_1L_2} + \pi^2k^4L_2(v_{cc}-v_{cesat})^2 - k^2L_2(\pi^2+4)(v_{cc}-v_{cesat})^2 - 4L_1v_{be}(v_{be}+v_D)}{4(k(v_{cc}-v_{cesat})\sqrt{L_1L_2} - L_1(v_{be}+v_D))}$$

Due to the subtractions, this equation has a limited range of valid results. v_{omax} must be positive. If a negative value is obtained the meaning is that there is no limit in v_{omax}, or that the oscillator does not work with these values due to excessive losses. In the simulated case, v_{omax} is obtained as -63.63 V, meaning that there is no limit. The elements causing losses are v_{be} and v_D only. Using just v_D, for example, v_{omax} = 1000 V is obtained with v_D = 18.12 V. The range that produces valid results are from 17.36 V to 62.03 V. Unrealistic values, of course.

No resistors were added to the coils to avoid excessively complicated formulas. The most important resistance to add would be one in series with L_2. It is possible to adjust the values of v_{be} and v_D to emulate the action of a resistor. The expression for v_{omax} can be modified by replacing v_{be} by v_{be}+v_R and v_D by v_D+v_R, v_R being the voltage drop due to a nonlinear resistance (a dead space operator) equivalent to a linear R in series with L_2. The expression can then be solved for v_R, resulting in a quite complicated expression due to the second-degree equation:

$$v_R = -\frac{\sqrt{-L_1(2k(v_{be}-v_D-2v_{omax})(v_{cc}-v_{cesat})\sqrt{L_1L_2} - L_1(v_{be}{}^2 - 2v_{be}(v_D+2v_{omax}) + (v_D+2v_{omax})^2) - k^2L_2(v_{cc}-v_{cesat})^2(2\pi^2k^2-2\pi^2+1))} + 3k(v_{cesat}-v_{cc})\sqrt{L_1L_2} + L_1(3v_{be}+v_D-2v_{omax})}{4L_1}$$

Calculated v_R, it can be substituted in the expression for the magnitude of $i_2(t)$ in the first semicycle (here, and in the calculation of v_R, it is assumed that the the current in the second semicycle follows the same sinusoid, what is not exact), giving i_{2max}. From v_R and i_{2max}, the equivalent resistor in series with L_2 is:

$$R = \frac{4}{\pi}\frac{v_R}{i_{2max}}$$

The term $4/\pi$ appears because the voltage drop in the approximation is a square wave, not a sinusoid. For example, with the listed values, to keep $v_{omax} = 1000$ V v_R is obtained as 8.764 V, $i_{2max} = 43.18$ mA, and the equivalent $R = 258.5\ \Omega$. Simulations using v_R and using R agree almost perfectly.

This resistance is partially due to the resistance of the wire in L_2, and partially due to energy transmission to the load. Note that these values correspond to a power dissipation $P \approx Ri_{2\max}^2/2 = 240.9$ mW.

It is simple to calculate v_{omax} as function of v_R (just add v_R to v_{be} and v_D), but not from R, because i_{2max} depends on v_{omax}, and the resulting equation for v_{omax} becomes too complex. Numerical solutions are possible, however, plotted in Fig. 6. For example, $R = 100\ \Omega$ results in $v_{omax} = 2521$ V. The solutions are valid as long as $i_2(t)$ is approximately sinusoidal, what in the example goes until $R \approx 17.5$ kΩ, generating $v_{omax} \approx 30.4$ V.

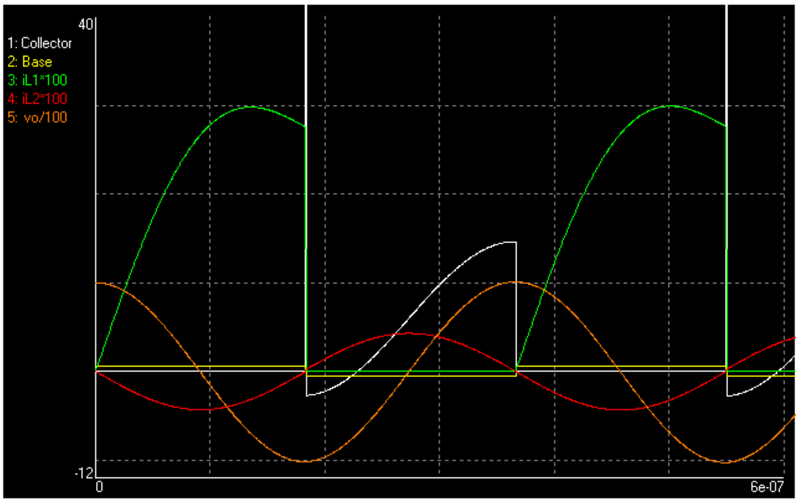

Figure 5. Simulated ideal waveforms.

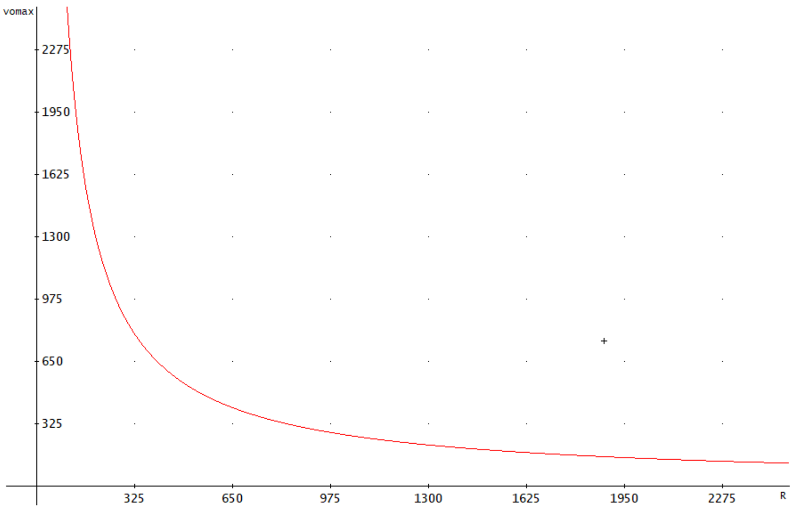

Figure 6. v_{omax} as function of the load resistance R.

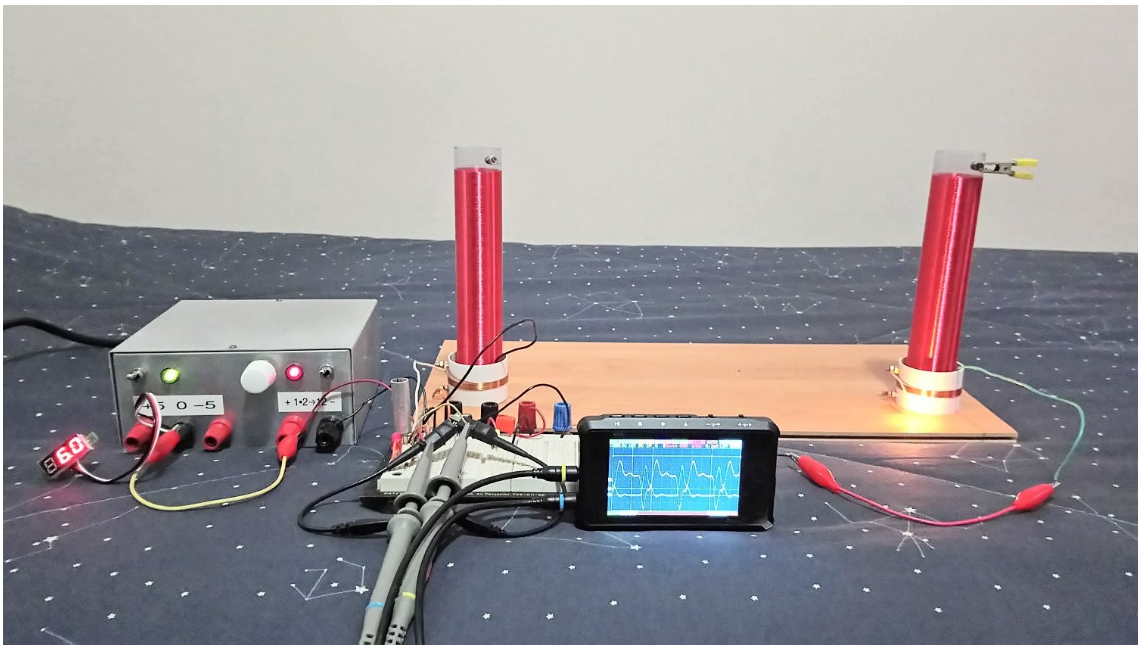

Figure 7. Experimental setup with values close to the ones of the simulations.

A problem with this driver is the power dissipated in the transistor. With the model used, power is dissipated when the transistor conducts, when it ceases to conduct due to the impulse, and at the *be* junction. All can be calculated.

When the transistor is conducting, in the first semicycle, the power dissipated at the collector is simply:

$$P(t) = v_{cesat} i_1(t)$$

As the numerical evaluations so far were assuming $v_{cesat} = 0$, this dissipation may be ignored.

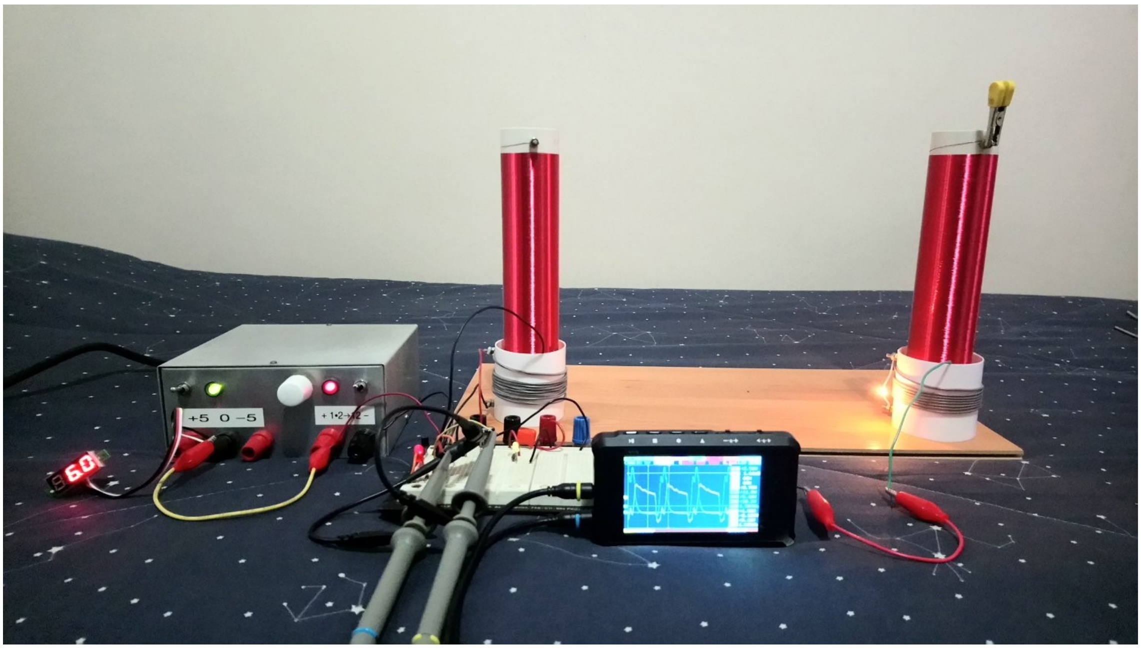

Figure 8. Setup with larger coils.

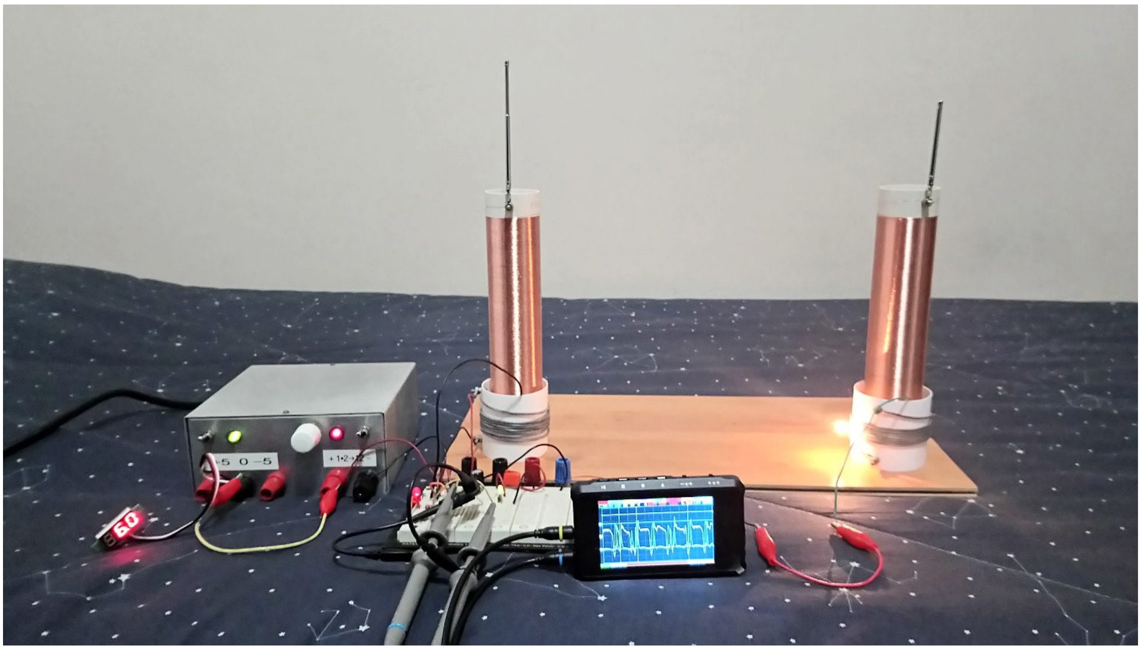

Figure 9. Setup with coils of thin wire (#36) and adjustable top loads.

The power dissipated by the impulse is more interesting. If a voltage impulse $\alpha\delta(t)$ happens at the same time of a current step $\beta u(t)$, or a step going down $\beta-\beta u(t)$, as is the case, the energy dissipated can be evaluated as:

$$E_{impulse}=\frac{1}{2}\alpha\beta$$

Using the calculated values:

$$E_{impulse}=\frac{1}{2}i_{1final}\left(L_1-\frac{M^2}{L_2}\right)i_{1final}=\frac{\pi^2 CL_2(k^2-1)^2(v_{cc}-v_{cesat})^2}{2L_1}$$

As there is one impulse per cycle, the average power is, considering the times T_1 and T_2 from the two semicycles, the second approximated:

$$\overline{P}_{transistor}=\frac{E_{impulse}}{T_1+T_2}=\frac{\pi^2 CL_2(k^2-1)^2(v_{cc}-v_{cesat})^2}{2L_1\left(\dfrac{\pi}{\sqrt{\dfrac{1}{CL_2(1-k^2)}}}+\dfrac{\pi}{\sqrt{\dfrac{1}{CL_2}}}\right)}$$

Using the numerical values, these expressions result in $E_{impulse}$= 0.1476 μJ and $P_{transistor}$ = 399.6 mW. Note that these values are independent of the load, and very significant. It is interesting to note that if the transistor delays its switching off, i_{1final} can be smaller because the sine term in $i_1(t)$ becomes negative, reducing the dissipated power.

The exact time of the second semicycle can be found by making $i_{22}(t) = 0$ in the second semicycle. As i_{22} depends on v_{be} and v_D, it also depends on the load in the secondary coil, that can be represented by v_R, as discussed above. With the substitutions of M, i_{1final} and v_{omin}:

$$i_{22}(t)=\frac{\pi k(v_{cc}-v_{cesat})\sqrt{L_2C(1-k^2)}}{\sqrt{L_1L_2}}\cos\sqrt{\frac{1}{L_2C}}t+$$

$$+\frac{(2k(v_{cc}-v_{cesat})\sqrt{L_1L_2}-L_1(2v_{be}+v_D-v_{omax}+3v_R))\sqrt{\dfrac{1}{L_2C}}}{L_1}\sin\sqrt{\frac{1}{L_2C}}t$$

Making $i_{22}(t)=0$ and solving for t results in infinite solutions, but the one that matters is the one at the end of the semicycle, slightly smaller than the approximation:

$$t=T_2=\frac{\pi}{\sqrt{\dfrac{1}{L_2C}}}-\frac{1}{\sqrt{\dfrac{1}{L_2C}}}\arctan\frac{\pi k(v_{cc}-v_{cesat})\sqrt{L_1L_2}\sqrt{1-k^2}}{2k(v_{cc}-v_{cesat})\sqrt{L_1L_2}-L_1(2v_{be}+v_D-v_{omax}+3v_R)}$$

The numerical value is T_2 = 0.1826 µs. The approximate expression $\pi\sqrt{L_2C}$ gives 0.1859 µs. With the exact value, the power dissipated by the impulse evaluates as 403.2 mW. The system oscillates at $1/(T_1+T_2)$ = 2.732 MHz. Considering just L_2 and C the frequency would be 2.725 MHz.

There is energy dissipated in the base too. Using the average value of $i_2(t)$ in the first semicycle, that is a complete sinusoidal arc, and dividing by the total period, without simplifications:

$$\overline{P}_{v_{be}}=\frac{\dfrac{C(L_1(v_{omax}-v_{be}-v_R)+k\sqrt{L_1L_2}(v_{cc}-v_{cesat}))\sqrt{\dfrac{1}{CL_2(1-k^2)}}}{L_1}v_{be}\dfrac{2}{\sqrt{\dfrac{1}{CL_2(1-k^2)}}}}{\dfrac{\pi}{\sqrt{\dfrac{1}{CL_2(1-k^2)}}}+T_2}$$

The expression evaluates to 8.266 mW, or to 8.193 mW with T_2 approximated.

The power in the diode can be evaluated by integrating $i_{22}(t)$ in the second semicycle, multiplying the result by v_D, and dividing by the full period. Curiously, with $v_{be}=v_D$ the result is the same obtained for v_{be}. This happens because the average values if $i_2(t)$ in both semicycles must be identical, otherwise a crescent constant voltage would accumulate at the output.

$$\overline{P}_{v_D}=\frac{\dfrac{C(L_1(v_{omax}-v_{be}-v_R)+k\sqrt{L_1L_2}(v_{cc}-v_{cesat}))\sqrt{\dfrac{1}{CL_2(1-k^2)}}}{L_1}v_D\dfrac{2}{\sqrt{\dfrac{1}{CL_2(1-k^2)}}}}{\dfrac{\pi}{\sqrt{\dfrac{1}{CL_2(1-k^2)}}}+T_2}$$

The power coming from the power supply v_{cc} shall be the sum of the computed powers, but it can be calculated directly by integrating the product of v_{cc} and $i_1(t)$ in the first semicycle and dividing by the total period.

$$\overline{P}_{v_{cc}} = \frac{\dfrac{Cv_{cc}(4k(v_{omax}-v_{be}-v_R)\sqrt{L_1L_2}+L_2(v_{cesat}-v_{cc})(k^2(\pi^2-4)-\pi^2))}{2L_1}}{\dfrac{\pi}{\sqrt{\dfrac{1}{CL_2(1-k^2)}}}+T_2}$$

The power evaluates to 661.2 mW. The sum of the previously computed powers adds to 660.6 mW. A simulation gives precisely 661.2 mW. The error is due to the approximation made in the power dissipated in the resistor.

It is possible to improve the evaluation of the power in v_R, by integrating $i_2(t)$ in both semicycles, multiplying by v_R, and dividing by the total period. The integrations are identical in both semicycles, as cited above. The result is, without simplification:

$$\overline{P}_{v_R} = \frac{\dfrac{C(L_1(v_{omax}-v_{be}-v_R)+k\sqrt{L_1L_2}(v_{cc}-v_{cesat}))\sqrt{\dfrac{1}{CL_2(1-k^2)}}}{L_1}\,2v_R\,\dfrac{2}{\sqrt{\dfrac{1}{CL_2(1-k^2)}}}}{\dfrac{\pi}{\sqrt{\dfrac{1}{CL_2(1-k^2)}}}+T_2}$$

The numerical value of the expression is 241.5 mW, and with this the sum of powers adds to 661.2 mW, as expected.

It is interesting to note how these powers were simulated. The power over the given element was computed by a multiplication of voltage and current, integrated by transforming it into a current in an 1 F capacitor, and divided by the time, generated by integrating 1 A by an 1 F capacitor. The division at the end of the first period gives the average power. Fig. 10 show the corresponding circuit in the Edfil editor and the simulation in the MNAE simulator, for the case of the power in v_{cc}. The switch and the diodes are ideal. v_R is generated by the nonlinear resistor. The diode in parallel with the switch has a drop of 100000 V, to allow calculation of the power in the impulse over the transistor, approximated as a large and short pulse.

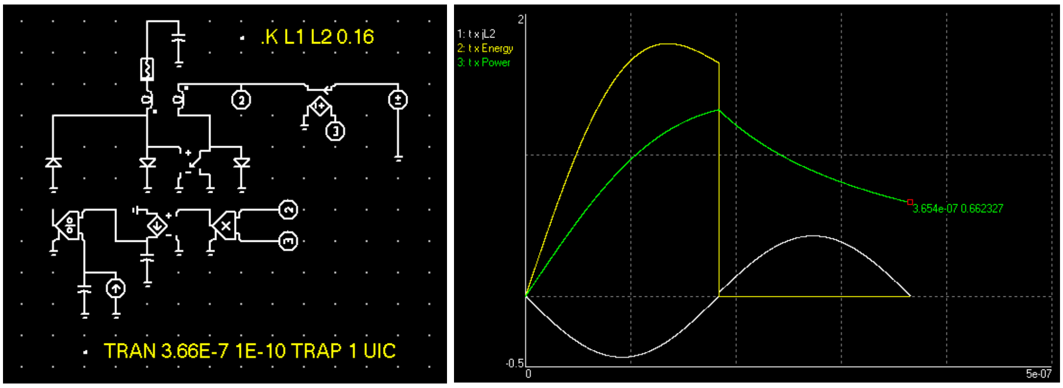

Fig. 10. Simulation of the power in v_{cc}.

The efficiency of the oscillator can be obtained as the ratio between output and input powers:

$$\text{Efficiency} = \frac{P_{v_R}}{P_{v_{cc}}} = \frac{8v_R(L_1(v_{be}-v_{omax}+v_R)-k(v_{cc}-v_{cesat})\sqrt{L_1L_2})}{v_{cc}(4k(v_{be}-v_{omax}+v_R)\sqrt{L_1L_2}+L_2(v_{cc}-v_{cesat})(k^2(\pi^2-4)-\pi^2))}$$

The expression evaluates to 0.3653. The formula can be used to evaluate the efficiency, but is not useful for dimensioning, since v_{omax} and v_R are associated.

As an evaluation of how real the model used is, Fig. 11 shows the waveforms obtained with the switch and v_{be} diode replaced by a transistor modeled with the Ebers-Moll model, and with capacitances (linear) added (C_{be} = 100 pF, C_{bc} = 5 pF, C_{ce} = 5 pF).

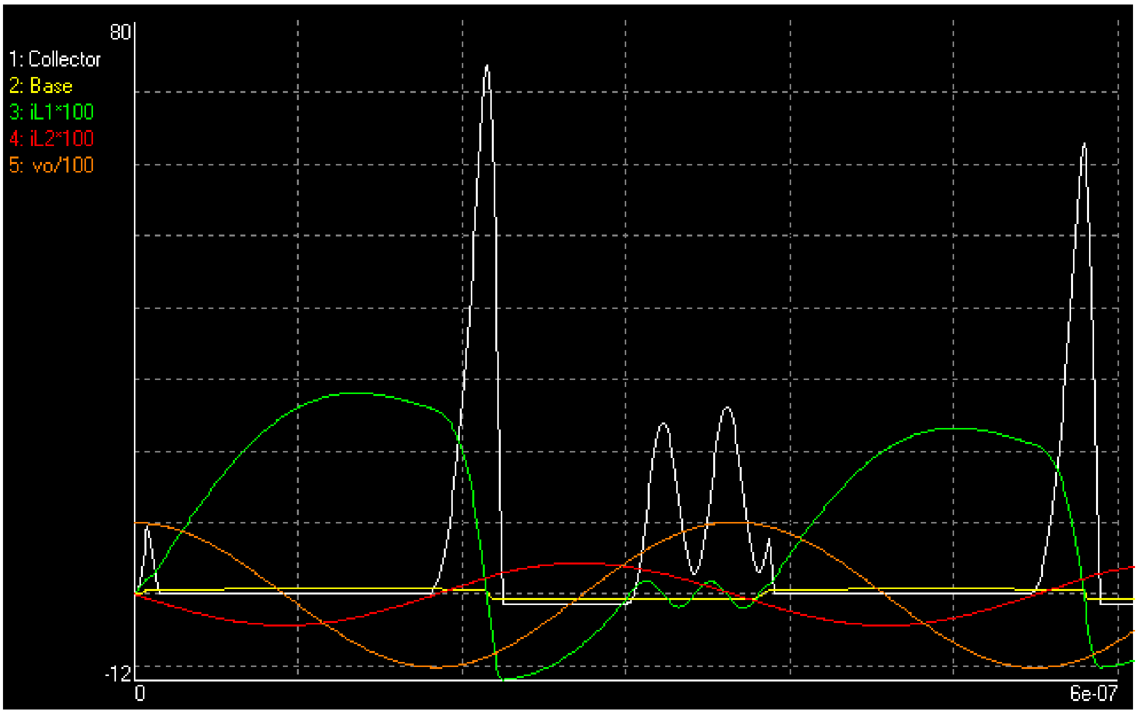

Figure 11. More realistic simulation, with a transistor model and capacitances.

Remarkable differences are the stretching of the impulse, reversal of i_{L1} after the pulse, returning some energy to the power supply (the first cycle is different, because the current starts from zero), oscillations when the transistor ceases to conduct, not affecting the output, and delayed turn on and turn-off of the transistor (in the model used, due to the capacitances only). After several cycles, the average input power evaluates as 491.1 mW, the output power as 252.9 mW (using the same v_R), with the output voltage growing to 1231 V. The efficiency is better, 0.5150.

Wireless energy transfer

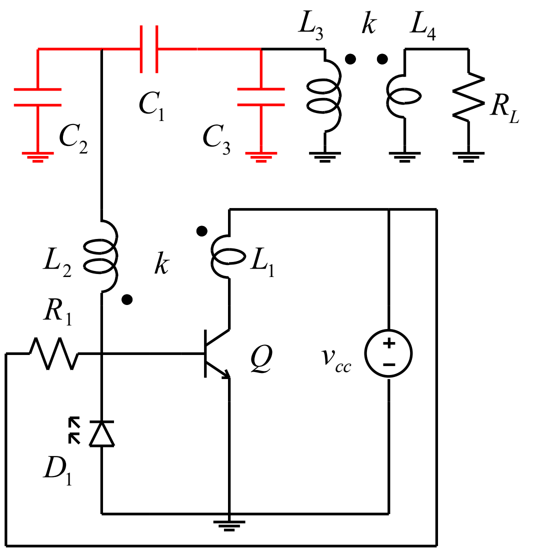

Figure 12. Model for the wireless energy transfer.

Fig. 12 shows an idealized model for the wireless energy transfer. C_2 is the distributed capacitance associated with the transmitter, C_3 the same for the receiver, and C_1 a small capacitance between the two coils. There are no losses included, so the system shall be capable of perfect energy transfer to R_L with proper tuning.

The system will be calculated with C_3 and R_L adjusted so the transmitter operates as in the previous analysis when at steady state. The procedure will start with the calculation of the impedance seen from the top of L_2. Then C_2, that was in series with the load in the basic transmitter, will be discounted from the impedance, so what remains shall be a pure resistance, equal to the resistance of $R_0 = 258.5\ \Omega$ derived previously. R_L will be then calculated, but a complex value will be obtained. One of the elements, in the case C_3, will be adjusted to make the impedance of R_L purely real, and then the final value for R_L will be obtained. Two solutions exist.

The impedance seen from the top of L_2 is found as, in Laplace transform (the negative signs are because k<1):

$$Z_{in}(s) = \frac{L_3L_4(C_1+C_3)(k^2-1)s^3 - L_3R_L(C_1+C_3)s^2 - L_4s - R_L}{\begin{array}{l} s(L_3L_4(k^2-1)(C_1(C_2+C_3)+C_2C_3)s^3 - L_3R_L(C_1(C_2+C_3)+C_2C_3)s^2 - \\ -L_4(C_1+C_2)s - R_L(C_1+C_2)) \end{array}}$$

The impedance to consider is $Z_{in}(s) - 1/(sC_2)$. Making it equal to R_0, R_L can be calculated (It is strange to have a resistance expressed in Laplace transform, but the actual calculation is in the sinusoidal steady state, with $s = j\omega$, where $j = \sqrt{-1}$ as usual, so the resistance is a complex value):

$$R_L(s) = \frac{sL_4(C_2L_3R_0(k^2-1)(C_1(C_2+C_3)+C_2C_3)s^3 + C_1C_3L_3(k^2-1)s^2 - C_2R_0(C_1+C_2)s - C_1)}{C_2L_3R_0(C_1(C_2+C_3)+C_2C_3)s^3 + C_1C_3L_3s^2 + C_2R_0(C_1+C_2)s + C_1}$$

Transforming the expression to the sinusoidal steady state, the correct frequency must be considered, by making $s = j2\pi/(T_1+T_2)$. The resulting expression is complex and too complicated to be copied here but can be evaluated numerically. The values used are:

$C_1 = 0.1$ pF; $C_2 = 2.5$ pF; $k = 0.16$; $L_4 = 4$ µH; $L_3 = 1.4$ mH; $T_1+T_2 = 0.3661$ µs; $R_0 = 258.5\ \Omega$

Solving $\mathrm{Im}(R_L) = 0$ as function of C_3, results in two possible values for it:

$C_3 = 2.430$ pF; $C_3 = 2.468$ pF

Replacing C_3 in the complex expression for R_L results in two purely real values:

$R_L = 132.7\ \Omega$; $R_L = 33.72\ \Omega$

The impedances seen for the two cases are almost identical, as seen in Fig. 13. Note that 20log(258.5) = 48.25 dB, confirming that the calculation is correct. Figs. 14 and 15 show results of a simulation of the complete circuit, considering the two cases, that are almost identical. C_1 and C_2 are initialized with 1000 V, and it's seen that this continues to be the peak voltage on them after an initial transient, as expected. It's curious to see that the voltage over C_3 almost does not change even with different C_3 and R_L. The power over R_L is identical in both cases, 241.5 mW, as calculated to be.

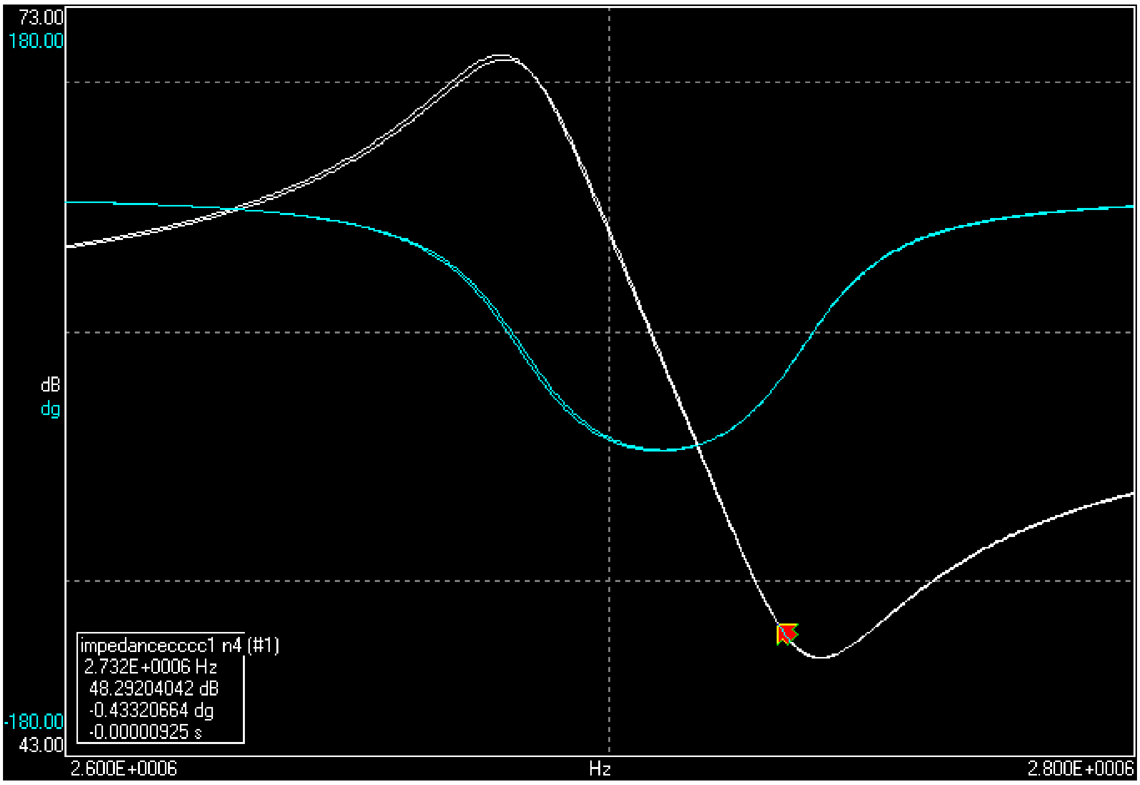

Figure 13. Impedances at the top of L_2 with C_2 in series removed, magnitude (dB) and phase (degrees), for the two cases of C_3 and R_L, giving almost identical curves. IFFT program used.

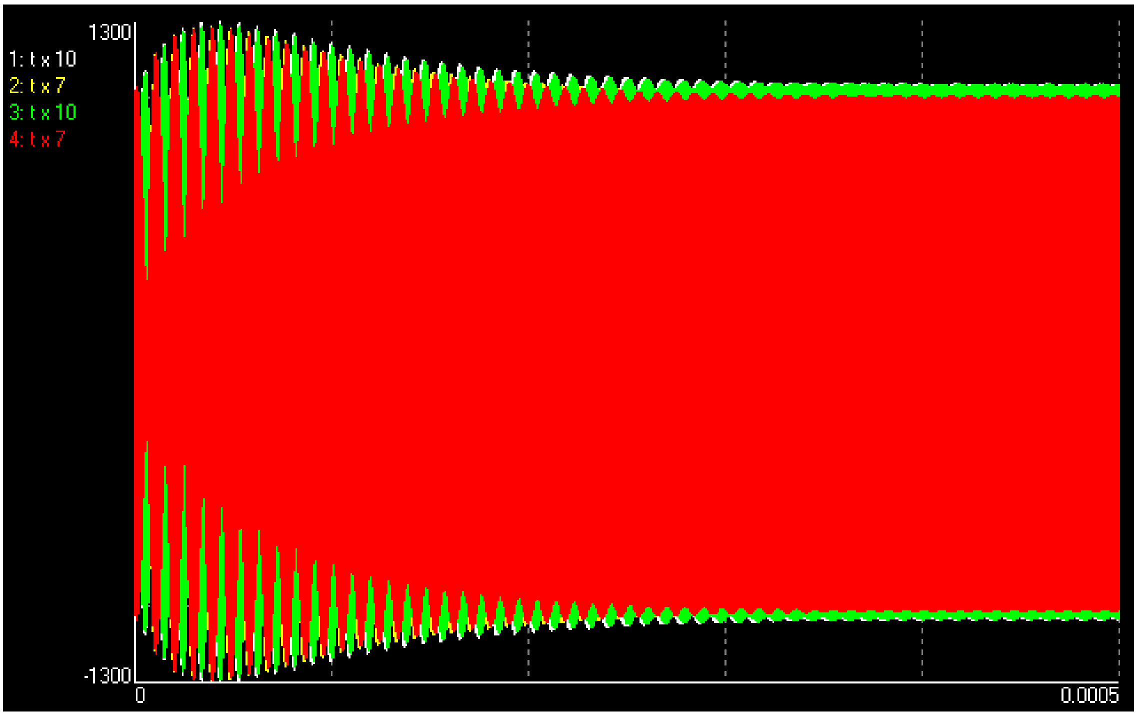

Figure 14. Simulation of the voltages over C_2 (node 7) and C_3 (node 10).

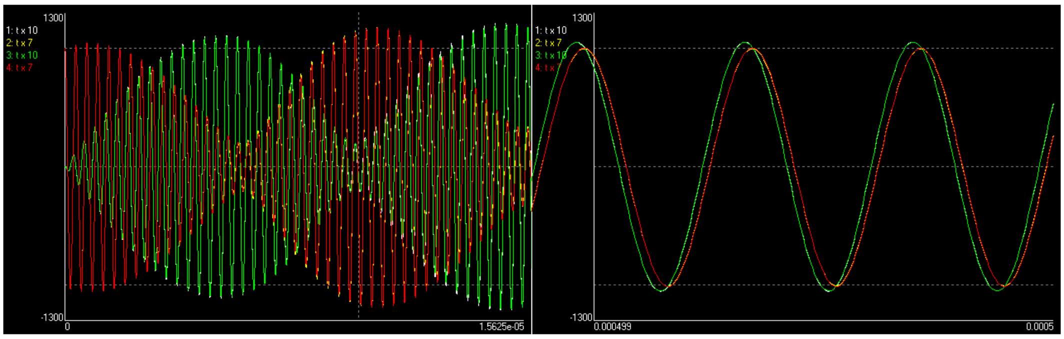

Figure 15. Expansions of the simulation at the beginning (left) and at the end (right).

The same idea can be applied to force the value of any of the elements, maybe with restrictions on range or problems due to the nonlinearity of the transmitter. For example, R_L can be specified instead of calculated if the two top loads C_2 and C_3 are used for tuning. The procedure starts with the impedance seen at the top of L_2, discounted by the impedance of C_2, with the value used in the calculations of the transmitter (C_{20} = 2.5 pF). A new C_2 is then calculated as a complex value by replacing s as in the previous calculation. C_3 is then calculated so the imaginary part of C_2 is zero, what results in two possible values, and finally two possible values of C_2 are obtained.

Departing from $Z_{in}(s)-1/(sC_{20})=R_0$, C_2 is found as:

$$C_2(s)=-\frac{\begin{array}{c}C_1C_{20}C_3L_3L_4R_0(k^2-1)s^4-L_3(C_1(C_{20}(C_3R_0R_L+L_4(k^2-1)+C_3L_4(1-k^2))+C_{20}C_3L_4(k^2-1))s^3+\\+(C_1(C_{20}(L_3R_L-L_4R_0)-C_3L_3R_L)+C_{20}C_3L_3R_L)s^2+(C_{20}L_4-C_1(C_{20}R_0R_L+L_4))s-R_L(C_1-C_{20})\end{array}}{(C_{20}R_0s+1)(L_3L_4(C_1+C_3)(k^2-1)s^3-L_3R_L(C_1+C_3)s^2-L_4s-R_L)}$$

Replacing $s=j2\pi/(T_1+T_2)$, making R_L = 200 Ω, and using the same elements of the last example, proceeding numerically due to the complexity of the expressions, the imaginary part of C_2 is a function of C_3. Solving it to zero results in two values for C_3:

C_3 = 2.250 pf; C_3 = 2.412 pF

Corresponding to two values for C_2:

C_2 = 2.283 pF; C_2 = 2.517 pF

Solutions exist for any R_L, but simulations of the resulting network show that the system does not stay operating at the correct frequency, but shifts it to another value, ruining the impedance matching. Apparently, C_2 cannot be modified because the transmitter starts using it (and C_1) as capacitive load and does not update it to the designed value.

Trying then to adjust C_3 and L_4 by a similar procedure, supposing that k does not change, also with R_L = 200 Ω specified, departing from $Z_{in}(s)-1/(sC_2)=R_0$, C_3 is found as:

$$C_3(s)=\frac{\begin{array}{l}C_1C_2{}^2L_3L_4R_0(k^2-1)s^4-C_1C_2{}^2L_3R_0R_Ls^3-C_2L_4R_0(C_1+C_2)s^2-\\-(C_1(C_2R_0R_L+L_4)+C_2{}^2R_0R_L)s-C_1R_L\end{array}}{L_3s^2(R_L-L_4(k^2-1)s)(C_2R_0(C_1+C_2)s+C_1)}$$

Again, replacing $s=j2\pi/(T_1+T_2)$ and making R_L = 200 Ω, the imaginary part of C_3 is function of L_4. Making it zero results in two values for L_4:

L_4 = 6.027 µH; L_4 = 0.2372 µH

Replacing these values in the expression for C_3, two corresponding values are found:

C_3 = 2.450 pF; C_3 = 2.468 pF

In this case the circuit works correctly, with results similar to the first example.

Note that the required adjustments are small, and high precision is required. With increased distance between transmitter and receiver, smaller C_1, the sensitivity of the system to small changes increases. All the losses were ignored, as resistances of the coils, what is not realistic. Also ignored were the delay of the signal propagation between transmitter and receiver, the energy dispersion due to radiation, and parasitic receivers, all contributing to the impossibility to make the system work at large distances.

Improving the efficiency of the transmitter

It is well known that the transmitter used is inefficient, fact verified in the analysis made. The main loss is caused by the impulse over the transistor at the start of the second semicycle. The energy of the impulse can be in great part returned to the power supply if a proper regenerative snubber circuit is added. Figure 15 shows a possibility. When the transistor conducts, current flows through L_1, as previously calculated and energy is transferred to the secondary circuit. When the transistor ceases to conduct, a pulse appears over it, but now C_x gets charged through D_2 and L_1 and limits its amplitude. The pulse ends when the resonance of C_x with L_1 tries to reverse the current. D_3 then conducts, pushing current back to the power supply through D_3, C_x, and L_1. The current continues through D_2 when C_x is discharged due to the energy stored in L_x. Figure 16 shows some of the waveforms. The model used a transistor with Ebers-Moll model, without capacitances to allow easier interpretation of the results. C_x was chosen (by simulation) to keep v_{omax} around 1000 V (in the simulation larger C_x increases it, but increases also the consumed power) as 70 pF. L_x was chosen to keep current flowing for the entire cycle, as 15 μH (the current through it is shown negative.). The first cycle is different, because there is no current in L_3.

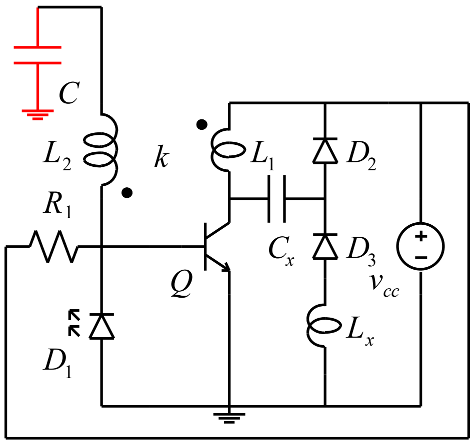

Figure 16. Regenerative snubber.

The power consumed by the load (R_0 = 258.5 Ω in series with L_2, as previously assumed) was simulated as 233.0 mW, with the power coming from v_{cc} as 327.0 mW, approximately. The simulated efficiency is then 0.7125, and probably can be better.

Analysis of the regenerative snubber

A complete analysis seems overly complicated, but the initial segments can be used to find how to dimension C_x and L_x approximately.

Consider the charging of C_x when the transistor turns off. The coupling between L_1 and L_2 is ignored, because the current in L_2 is almost unaffected, as verified by simulation. The mesh equation of the circuit L_1, C_x, D_2 is, where i_y is the mesh current:

$$-i_{1final}L_1 + sL_1 I_y(s) + \frac{1}{sC_x} I_y(s) + \frac{v_{D2}}{s} = 0$$

The current i_y is then found as:

$$I_y(s) = \frac{i_{1final}s}{s^2 + \frac{1}{L_1C_x}} - \frac{\frac{v_{D2}}{L_1}}{s^2 + \frac{1}{L_1C_x}}$$

$$i_y(t) = i_{1final} \cos\sqrt{\frac{1}{L_1C_x}}t - \frac{v_{D2}}{L_1}\sqrt{L_1C_x} \sin\sqrt{\frac{1}{L_1C_x}}t$$

Finding when $i_y(t) = 0$ leads to:

$$t = \sqrt{L_1C_x} \operatorname{atan}\frac{i_{1final}\sqrt{L_1C_x}}{C_x v_{D2}}$$

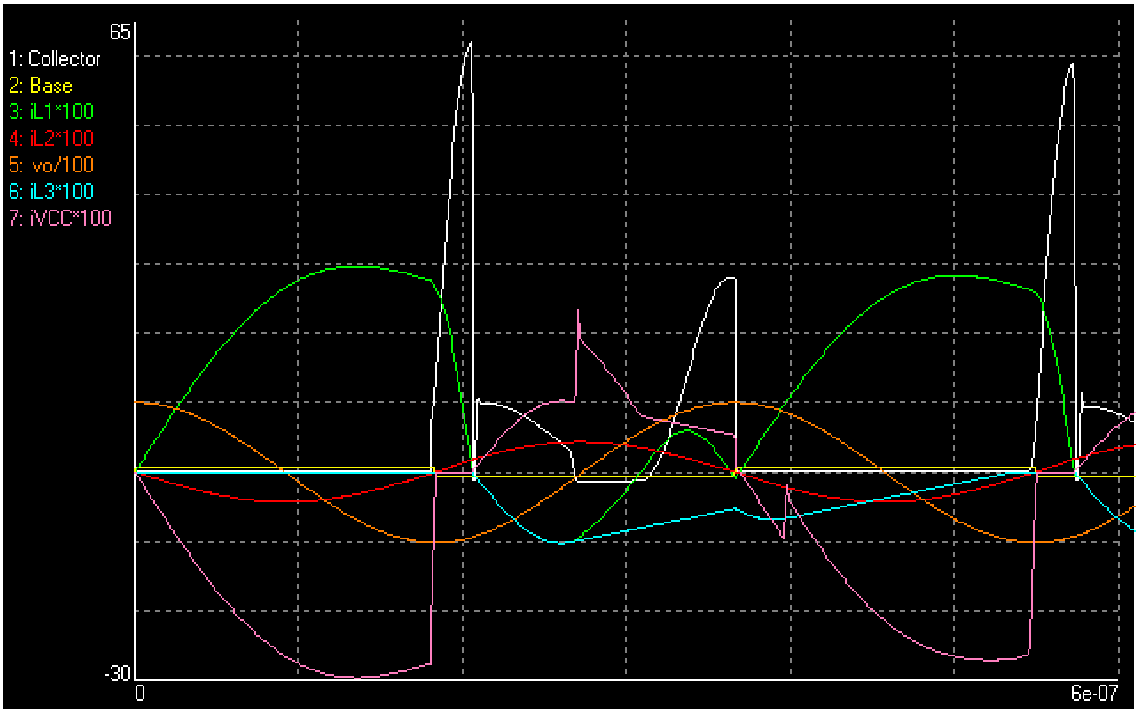

Figure 17. Waveforms with a regenerative snubber.

Finding v_{ce}:

$$V_{ce}(s) = \frac{v_{cc}}{s} + i_{1final}L_1 - sL_1I_y(s) = \frac{v_{cc}}{s} + \frac{v_{D2s}}{s^2 + \frac{1}{L_1C_x}} + \frac{\frac{i_{1final}}{C_x}}{s^2 + \frac{1}{L_1C_x}}$$

$$v_{ce}(t) = v_{cc} + v_{D2} \cos\sqrt{\frac{1}{L_1C_x}}t + \frac{i_{1final}L_1}{\sqrt{L_1C_x}} \sin\sqrt{\frac{1}{L_1C_x}}t$$

Substituting the calculated t when $i_1(t) = 0$:

$$v_{cemax} = v_{cc} + \sqrt{\frac{C_x v_{D2}{}^2 + i_{1final}{}^2 L_1}{C_x}}$$

Solving for C_x:

$$C_x = \frac{{i_{1final}}^2 L_1}{(v_{cc} - v_{cemax})^2 - {v_{D2}}^2}$$

Substituting i_{1final}:

$$C_x = \frac{\pi^2 C L_2 (1-k^2)(v_{cc} - v_{cesat})^2}{L_1((v_{cc} - v_{cemax})^2 - {v_{D2}}^2)}$$

Using the example values and v_{cemax} = 70 V, the formula gives C_x = 73.96 pF. Close to the values in Fig. 17. It would be more consistent to use k = 0, since i_{L2} was ignored. This results in C_x = 75.91 pF.

To calculate L_x, first the voltage over C_x , v_x, during the pulse in v_{ce} is calculated:

$$V_x(s) = \frac{1}{sC_x} I_y(s) = \frac{v_{D2} s}{s^2 + \frac{1}{L_1 C_x}} + \frac{\frac{i_{1final}}{C_x}}{s^2 + \frac{1}{L_1 C_x}} - \frac{v_{D2}}{s}$$

$$v_x(t) = V_{D2} \cos\sqrt{\frac{1}{L_1 C_x}} t + \frac{i_{1final}}{C_x}\sqrt{L_1 C_x} \sin\sqrt{\frac{1}{L_1 C_x}} t - v_{D2}$$

In the time when $i_y(t)$ = 0 C_x is fully charged with v_{xfinal}:

$$v_{xfinal} = \sqrt{\frac{C_x {v_{D2}}^2 + {i_{1final}}^2 L_1}{C_x}} - v_{D2} = v_{cemax} - v_{cc} - v_{D2}$$

Following then the mesh including v_{cc}, L_x, v_{D3}, charged C_x, and L_1 to find the current returning to v_{cc} through L_3, i_x:

$$\frac{v_{cc}}{s} + sL_x I_x + \frac{v_{D3}}{s} - \frac{v_{xfinal}}{s} + \frac{1}{sCx} I_x + sL_1 I_x = 0$$

Finding i_x:

$$I_x(s) = \frac{\frac{v_{xfinal} - v_{cc} - v_{D3}}{L_1 + L_x}}{s^2 + \frac{1}{(L_1 + L_x)C_x}}$$

$$i_x(t) = \frac{v_{xfinal} - v_{cc} - v_{D3}}{L_1 + L_x}\sqrt{(L_1 + L_x)C_x} \sin\sqrt{\frac{1}{(L_1 + L_x)C_x}} t$$

The maximum value of $i_x(t)$ is the maximum current returning to the power supply, i_{xmax}:

$$i_{xmax=} \frac{v_{xfinal} - v_{cc} - v_{D3}}{L_1 + L_x}\sqrt{(L_1 + L_x)C_x}$$

Solving for L_x:

$$L_x = \frac{C_x(v_{xfinal} - v_{cc} - v_{D3})^2}{i_{xmax}{}^2} - L_1 = \frac{C_x(2v_{cc} - v_{cemax} + v_{D2} + v_{D3})^2}{i_{xmax}{}^2} - L_1$$

Substituting C_x:

$$L_x = \frac{\pi^2 C L_2 (1-k^2)(v_{cc} - v_{cesat})^2 (2v_{cc} - v_{cemax} + v_{D2} + v_{D3})^2}{i_{xmax}{}^2 L_1((v_{cc} - v_{cemax})^2 - v_{D2}{}^2)} - L_1$$

Evaluating this expression for v_{cemax} = 70 V and i_{xmax} = 0.1135 A (simulated), with k = 0, gives the expected L_x = 15 µH. Note in Fig. 17 that after the returned current reaches i_{xmax}, the current through L_1 adds to it passing in D_2, practically doubling its value. This current flows through the transistor operating in inverse saturation, and must be at a safe level. Another diode can be added across the transistor to divert it.

Experimental results

A question is if this snubber is effective for the intended circuit. As k is not zero there is some interaction between L_1 and L_2, that can steal energy from L_2 when the snubber acts. The slow turn-off of the transistor can change the assumed conditions, by reducing i_{1final}. Circuit capacitances can also cause big changes, starting with limiting v_{cemax} in a high-frequency circuit. The final effect may be a reduction in power consumption, but a reduction of the output power too. The three circuits in Figs. 7, 8 and 9 will be evaluated for efficiency and effect of the snubber, considering the input power and the received power over a fixed resistance, with the receiver at a fixed distance and tuned for maximum power, using the top load capacitances C_2 and C_3, made adjustable by two small telescopic antennas, as in Fig. 9. Frequency jumping during the tuning is expected, as happened in the simulation of using these capacitances for tuning.

The setup places both coils at 27 cm of distance between centers. The primary coils are the ones shown in Fig. 8 and Fig. 9, used also with the smaller coils in Fig. 7. The transistor was a BC337, diodes 1N4148, red LED, startup resistor R_1 = 47 kΩ. A 47 nF capacitor was put in series with R_1, to eliminate unneeded power waste. The circuit continues to start normally with it. Tables 1 and 2 summarize the three setups. The numbers of turns for the secondary coils were not counted, but calculated with the Inca program. The numbers vary a bit between the pairs, but this was ignored. The snubber elements C_x and L_x were initially calculated, but then adjusted experimentally, trying to do not reduce much the output power.

Coil	Turns	Height	Radius	Base height
L_1, L_4	8	1.5 cm	2.51 cm	1.8 cm
L_2, L_3, Fig. 7	491	14 cm	1.5 cm	1.3 cm
L_2, L_3, Fig. 8	621	17.2 cm	2 cm	1.5 cm
L_2, L_3, Fig. 9	907	16.5 cm	2 cm	1.8 cm

Table 1. Coil dimensions.

Coils	L_1, L_4	L_2, L_3	k	Frequency	C_1, C_2	C_x	L_x
Fig. 7	4.3 µH	1.4 mH	0.23	2.3 mHz	3.3 pF	69 pF	0.91 µH
Fig. 8	4.6 µH	3.2 mH	0.28	1.4 MHz	4.0 pF	470 pF	0.91 µH
Fig. 9	4.6 µH	7.1 mH	0.27	870 kHz	4.7 pF	2.2 nF	4.3 µH

Table 2. Experimental values.

Tables 2 and 3 list the obtained results. The effect of the snubber is not conclusive, but it appears to help in some cases, particularly in the first. In the second case the circuit without snubber was quite efficient, so the snubber didn't help.

Coils	v_{cc}	i_{in}	v_{out}	p_{in}	p_{out}	Efficiency
Fig. 7	6 V	220 mA	36.4 V	1.32 W	752.8 mW	0.5703
Fig. 8	6 V	160 mA	34.4 V	960 mW	672.3 mW	0.7004
Fig. 9	6 V	240 mA	33.2 V	1.44 W	626.3 mW	0.4349

Table 3. Results without snubber.

Coils	v_{cc}	i_{in}	v_{outpp}	p_{in}	p_{out}	Efficiency
Fig. 7	6 V	100 mA	27.2 V	600 mW	420.4 mW	0.7006
Fig. 8	6 V	130 mA	30.8 V	780 mW	539.0 mW	0.6910
Fig. 9	6 V	200 mA	31.2 V	1.2 W	553.1 mW	0.4609

Table 4. Results with snubber.

The following oscilloscope screens show the voltage at the collector (yellow) and the output voltage at the receiver (blue). The load resistance used was of 220 Ω.

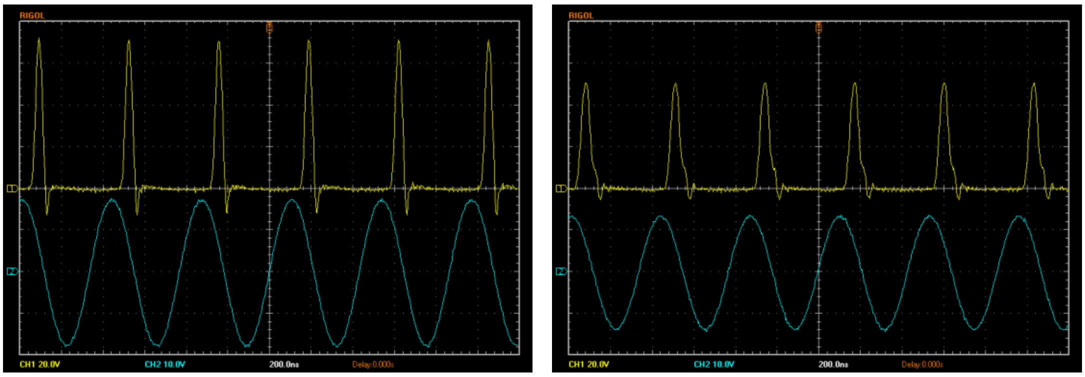

Figure 18. Circuit in Fig. 7, without (left) and with (right) snubber.

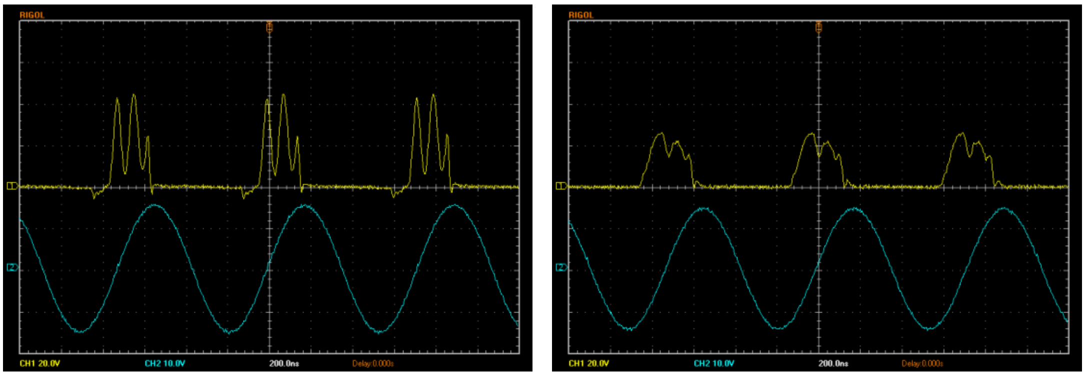

Figure 19. Circuit in Fig. 8, without (left) and with (right) snubber.

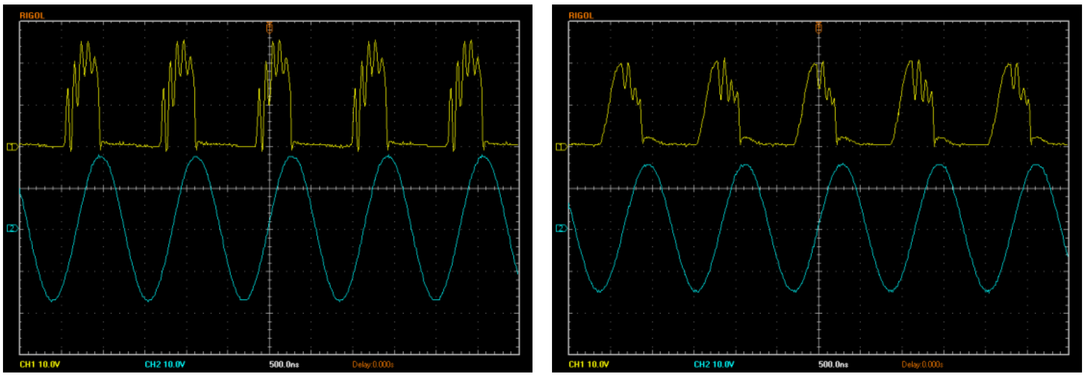

Figure 20. Circuit in Fig. 9, without (left) and with (right) snubber.

The most efficient system, the one in Fig. 8, was also tested with a load resistance of 100 Ω. The results were similar: Without snubber, i_{in} = 150 mA, v_{out} = 22.8 V, efficiency = 0.7220. With snubber, i_{in} = 110 mA, v_{out} = 18.2 V, efficiency = 0.6273.

Another test was to vary the separation, using the circuit in Fig. 8 with 100 Ω load and 6 V input, without snubber. Table 5 shows the results. The circuit was not retuned for each distance, and the antennas used as top loads were fully extended. The measuments were not so easy, because the system tended to show frequency jumps when close to the maximum power transfer condition, particularly at small distances. This caused the waveforms to not repeat at every cycle, resulting in an apparent increase in efficiency followed by a decrease.

Separation	p_{in}	p_{out}	Efficiency
21 cm	960 mW	672.8 mW	0.6540
23 cm	1.240 W	781.3 mW	0.6854
25 cm	900 mW	672.8 mW	0.7476
27 cm	840 mW	649.8 mW	0.7736
30 cm	900 mW	649.8 mW	0.7220
33 cm	840 mW	540.8 mW	0.6438
36 cm	960 mW	500.0 mW	0.5208
39 cm	960 mW	480.2 mW	0.5002
42 cm	1.080 W	336.2 mW	0.3313

Table 5. Varying the separation.

Energy transfer as an impedance matching problem

It is interesting to see how the load R_L is converted into the resistance calculated as load in the transmitter R_0 by the LCM network connecting both. To plot in a Smith chart the steps of the impedance conversion, consider the model in Fig. 21, where the output transformer is modeled as a T network of inductors. This is the model used in the calculation using C_2 and C_3 to perform the matching at 2.732 MHz.

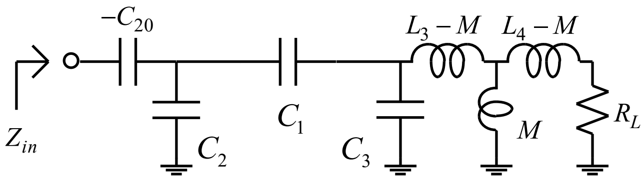

Figure 21. Model for calculating Z_{in}.

The path showing the effect of each element on the impedance is shown in Fig. 22. Starting from R_L = 200 Ω, the elements transform the impedance into R_0 = 258.5 Ω, for the two possible cases.

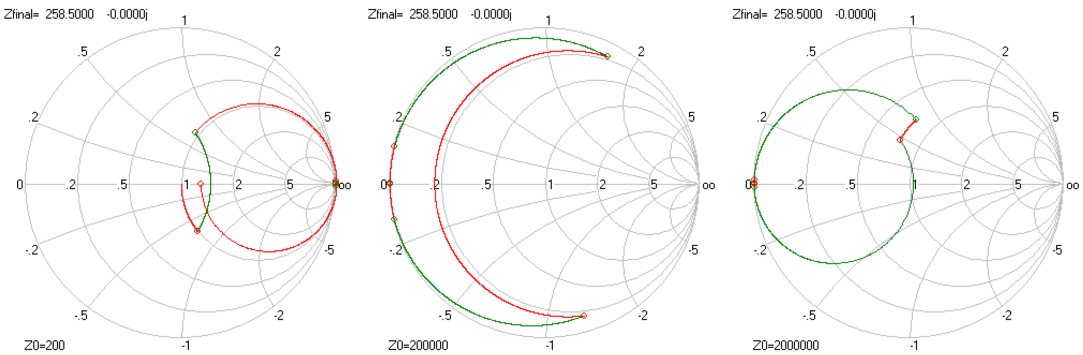

Figure 22. Paths in the Smith chart. Left: starting from 200 Ω at the center and reaching 258.5 Ω. Center: expansion of the high-impedance section in the first solution. Right: expansion for the second solution.

The three fist segments starting from the center (left) are from the transformer, changing the impedance level to a high value. The negative capacitor completes the return to R_0. The two solutions use C_1 (central red segment at the expansions) at different impedance levels. There are just two solutions where the arcs defined by C_2 and C_3 end in the same circle of constant resistance and are joined by the arc of the fixed C_1.

Conclusions

A study was made about Tesla´s wireless energy transfer, using a "Slayer exciter" blocking oscillator as a convenient transmitter. All the waveforms in the transmitter were calculated from an idealized model and verified by simulation. It was observed that the oscillator is quite inefficient but can be improved by a variable amount with the use of a regenerative snubber. It was shown that tuning for maximum power transfer can be achieved by tuning two elements in the system, in the experimental case the two top load capacitances. The simulation of this tuning was not successful, because the nonlinear system changed the oscillation frequency. In the experiments this could also be observed as frequency jumps, but it was possible to achieve good power transmission by guiding the system towards maximum output by testing if the output voltage increases or decreases with a hand close to the antennas and adjusting the antennas to have the same effect. At this condition, any change in the capacitances, by approaching a hand to the transmitter or the receiver antenna, for example, was observed to change the oscillation frequency and reduce the efficiency. The best condition could be restored by approaching a hand to the other antenna. The energy transmission is quite efficient at a distance similar to the length of the coils, considering that the oscillator has significant losses. This may explain the long-lasting hope that the method could be perfected for transmission at large distances, although a bit of reasoning shows that this is not possible.

Most of the algebraic calculations were done with the Derive 6.10 program ("abandonware") and verified with precise simulations with the programs MNAE (time domain) and IFFT (frequency domain). The coils were analyzed with the Inca program. The last programs are available at http://www.coe.ufrj.br/~acmq/programs.

This is not a peer-reviewed paper.

First version: 17/5/2020. Last update: 1/7/2020.