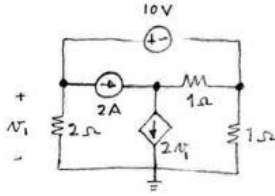
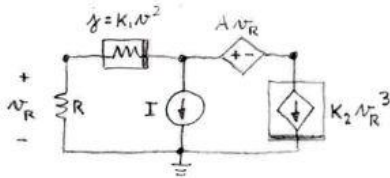


CIRCUITOS ELÉTRICOS II - 2º SEMESTRE DE 2000 1ª PROVA

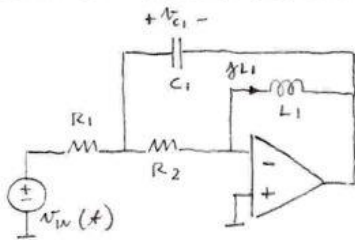
- ① PARA O CIRCUITO ABAIXO, ESCRIBA O SISTEMA NODAL, DIRETAMENTE EM FORMA MATRICIAL DESLOCUE FONTES DE TENSÃO. RESOLVA PARA AS TENSÕES NOS TRÊS NÓS ORIGINAIS.

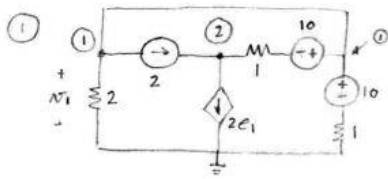


- ② PARA O CIRCUITO ABAIXO, MONTE O SISTEMA NODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO PARA A SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON



- ③ PARA O CIRCUITO ABAIXO, MONTE O SISTEMA NODAL QUE CALCULA A SOLUÇÃO EM $t = t_0 + \Delta t$, CONHECIDA A SOLUÇÃO EM $t = t_0$, USANDO O MÉTODO DE EULER "BACKWARD". TRATE O AMP. OPERACIONAL EM UMA FORMA QUE REDUZA O TAMAHO DO SISTEMA A RESOLVER.





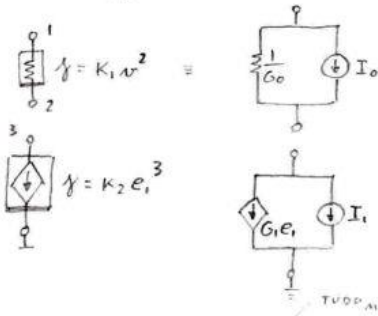
$$\begin{bmatrix} \frac{1}{2} + 1 & -1 \\ -1 + 2 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} -2 + 10 + 10 \\ 2 - 10 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{5}{2} & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 18 \\ -8 \end{bmatrix}$$

$$e_1 = \frac{18 - 8}{\frac{5}{2} + 1} = \frac{10}{7/2} = +\frac{20}{7}$$

$$e_2 = \frac{-\frac{5}{2} \times 8 - 18}{7/2} = \frac{-40 - 36}{7} = -\frac{76}{7}$$

$$e_3 = e_1 - 10 = \frac{20}{7} - 10 = \frac{20 - 70}{7} = -\frac{50}{7}$$

2) MODELOS:

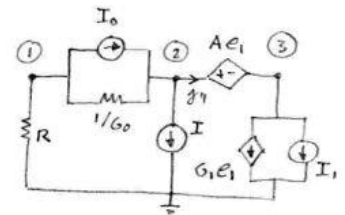


$$G_0 = 2K_1(e_1 - e_{2m})$$

$$I_0 = K_1(e_{2m} - e_{2n})^2 - G_0(e_{1m} - e_{2m})$$

$$G_1 = 3K_2 e_{1m}^2$$

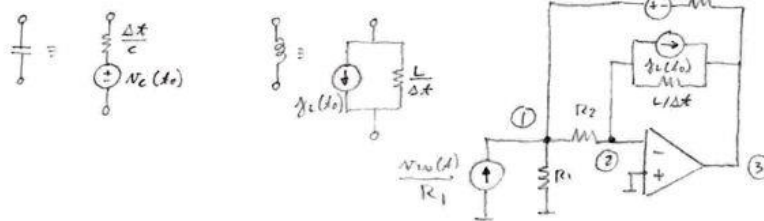
$$I_1 = K_2 e_{1m}^3 - G_1 e_{1m}$$



$$e_2 - e_3 - Ae_1 = 0$$

$$\begin{bmatrix} \frac{1}{R} + G_0 & -G_0 & 0 & 0 \\ -G_0 & G_0 & 0 & +1 \\ +G_1 & 0 & 0 & -1 \\ -A & +1 & -1 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \psi_4 \end{bmatrix} = \begin{bmatrix} -I_0 \\ +I_0 - I \\ -I_1 \\ 0 \end{bmatrix}$$

3) MODELOS:



SEM O AMP. OP.:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{C}{\Delta t} & -\frac{1}{R_2} & -\frac{C}{\Delta t} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{\Delta t}{L} & -\frac{\Delta t}{L} \\ -\frac{C}{\Delta t} & -\frac{\Delta t}{L} & \frac{C}{\Delta t} + \frac{\Delta t}{L} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} N_w(t+\Delta t)/R_1 + N_c(t_0)C/\Delta t \\ -I_c(t_0) \\ +I_c(t_0) - N_c(t_0)C/\Delta t \end{bmatrix}$$

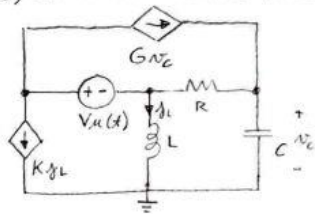
RESPOSTA:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{C}{\Delta t} & -\frac{C}{\Delta t} \\ -\frac{1}{R_2} & -\frac{\Delta t}{L} \end{bmatrix} \begin{bmatrix} e_1(t+\Delta t) \\ e_3(t+\Delta t) \end{bmatrix} = \begin{bmatrix} N_w(t+\Delta t)/R_1 + N_c(t_0)C/\Delta t \\ -I_c(t_0) \end{bmatrix}$$

O AMP. OP. FAZ $e_2 = 0 \rightarrow$ Elimina-se coluna 2
A B É EQUAÇÃO DE SUPLENÇÃO

① PARA O CIRCUITO ABAIXO, ESCREVA: (ANÁLISE EM TRANSFORMADA DE LAPLACE)

- a) UM SISTEMA MODAL MODIFICADO
- b) UM SISTEMA MODAL PURO

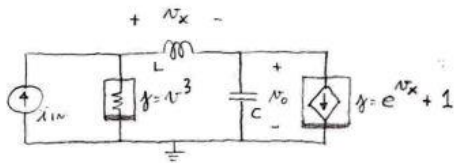


DADOS $\beta_L(\omega)$, $\beta_C(\omega)$

② PARA O CIRCUITO ABAIXO, DESCREVA, USANDO ANÁLISE MODAL:

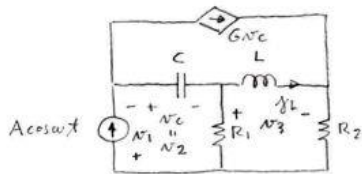
- a) COMO OBTER O PONTO DE OPERAÇÃO. ACHÉ O PONTO DE OPERAÇÃO (POR IMPEÇÃO).
- b) COMO OBTÉR A BRILHOITA EM FREQUÊNCIA DE $\frac{\omega_0}{\lambda_{11}}$, PARA PEQUENOS SINAIS EM TORNO DO P.O.

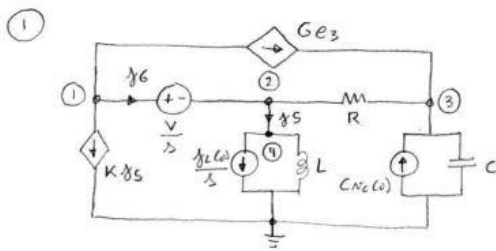
ESCREVA OS SISTEMAS MODAIS NECESSÁRIOS NAS DUAS ANÁLISES.



③ ESCREVA UM SISTEMA PARA A ANÁLISE NO TEMPO DO CIRCUITO ABAIXO, DADO

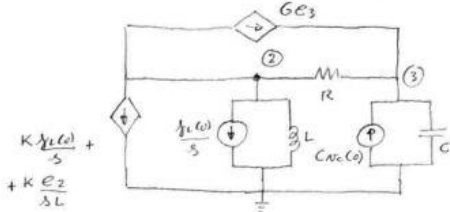
AS INDETERMINAÇÕES λ_1 , λ_2 E λ_3 . USE A OPERAÇÃO B.E., CALCULANDO A SOLUÇÃO EM $t = t_0 + \Delta t$, DADO $\beta_C(t_0)$ E $\beta_L(t_0)$





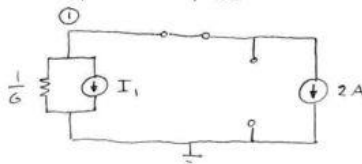
$$\begin{bmatrix} 0 & 0 & +G & 0 & +K & +1 \\ 0 & \frac{1}{R} & -\frac{1}{R} & 0 & +1 & -1 \\ 0 & -\frac{1}{R} & \cancel{G} + \frac{1}{R} - G & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{sL} & -1 & 0 \\ 0 & +1 & 0 & -1 & 0 & 0 \\ +1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ i_{gs} \\ i_g \end{bmatrix} = \begin{bmatrix} -Ge_3 \\ 0 \\ Cvc(\omega) + Gv \\ -\frac{1}{s} \\ 0 \\ \frac{v}{s} \end{bmatrix}$$

DESLOCANDO A FONTE $\frac{v}{s}$ NA DIREÇÃO DAS FONTES DE CORRENTE
EJA DESAPARECE, B A N C TAMBÉM



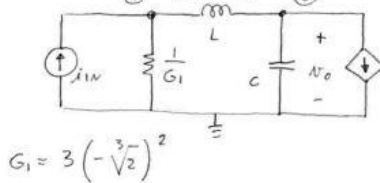
$$\begin{bmatrix} \frac{1}{sL} + \frac{1}{R} + \frac{K}{sL} & -\frac{1}{R} + G \\ -\frac{1}{R} & \frac{1}{R} + sC - G \end{bmatrix} \begin{bmatrix} e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} -K \frac{1}{s} - K \frac{e_2}{sL} - Ge_3 - \frac{1}{s} \\ Ge_3 + Cvc(\omega) \end{bmatrix}$$

2) PARA ACHAR O P.O., FAZER $i_{in} = 0$ E TIRAR: $[G] e_{in+1} = [-I_1 - 2]$
com: $\begin{cases} G_1 = 3e_{in}^2 \\ I_1 = e_{in}^3 - G_1 e_{in} \end{cases}$ (A FONTE CONTROLADA É CONSTANTE, POIS $N_x = 0 \Rightarrow i = 2A$)



A SOLUÇÃO FINAL $e_1 - 2 = e_1^3 \Rightarrow e_1 = -\sqrt[3]{2}$

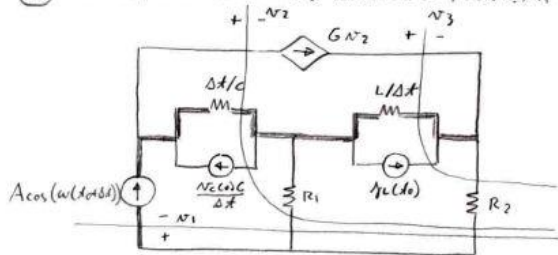
PARA ACHAR A RESPOSTA EM FREQUÊNCIA BASTA RESOLVER O CIRCUITO EM fw , PARA A FAIXA DE ω DESEJADA:



A TRANSCONDUTÂNCIA É $C \frac{N_x}{N_0}$ COM $N_x = 0$, OU 1Ω

$$\begin{bmatrix} G_1 + \frac{1}{j\omega L} & -\frac{1}{j\omega C} \\ -\frac{1}{j\omega L} + 1 & \frac{1}{j\omega C} + j\omega C - 1 \end{bmatrix} \begin{bmatrix} e_1(j\omega) \\ e_2(j\omega) \end{bmatrix} = \begin{bmatrix} i_{in} \\ -(e_1 - e_2) \end{bmatrix} \quad N_0 = e_2$$

3) UM SISTEMA DE CORDES RESOLVE O PROBLEMA:

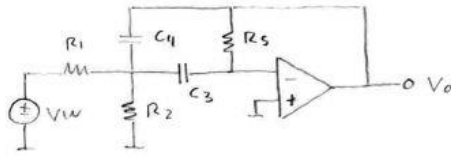


$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & \frac{1}{R_1} + \frac{1}{R_2} & \frac{1}{R_2} \\ \frac{1}{R_1} + \frac{1}{R_2} & \frac{C}{\Delta t} + \frac{1}{R_1} + \frac{1}{R_2} + G & \frac{1}{R_2} \\ \frac{1}{R_2} & \frac{1}{R_2} + G & \frac{\Delta t}{L} + \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} =$$

$$= \begin{bmatrix} -A \cos(\omega(t_0 + \Delta t)) \\ + \frac{N_c(t_0)C}{\Delta t} & -GN_2 \\ -j_c(t_0) & -GN_2 \end{bmatrix}$$

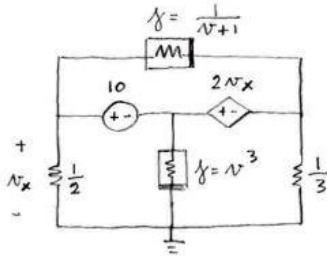
CIRCUITOS ELÉTRICOS II - 1ª PROVA - 1º SEMESTRE DE 2003

1



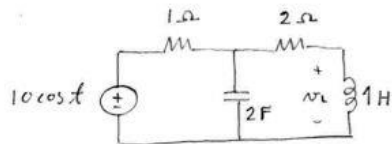
ACHE $\frac{V_o(s)}{V_w}$ USANDO UMA ANÁLISE NODAL

2



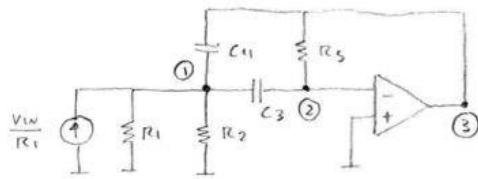
ESCREVA O SISTEMA NODAL MODIFICADO QUE ACHA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO DO CIRCUITO, SUPONDO A SOLUÇÃO ATUAL COM TODAS AS TRANSISTORES (CORRETO) ACERTAS

3



USE UM SISTEMA DE MALHAS PARA ACHAR A TENSÃO SOBRE O INDUTOR NO ESTADO PERMANENTE SENOIDAL

1



$$\text{SAI} \rightarrow \begin{bmatrix} sC_4 + sC_3 + \frac{1}{R_1} + \frac{1}{R_2} & -sC_3 & -sC_4 \\ -sC_3 & \frac{1}{R_5} + sC_3 & -\frac{1}{R_5} \\ -sC_4 & -\frac{1}{R_5} & \frac{1}{R_5} + sC_4 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 = 0 \\ E_3 \end{bmatrix} = \begin{bmatrix} \frac{V_w}{R_1} \\ 0 \\ 0 \end{bmatrix}$$

SAI ($E_2 = 0$)

$$\begin{bmatrix} sC_3 + sC_4 + \frac{1}{R_1} + \frac{1}{R_2} & -sC_4 \\ -sC_3 & -\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} E_1 \\ E_3 \end{bmatrix} = \begin{bmatrix} \frac{V_w}{R_1} \\ 0 \end{bmatrix}$$

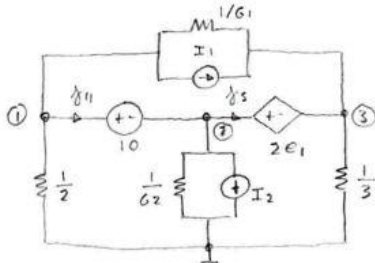
$$\frac{E_3}{V_w} = \frac{+sC_3}{-\frac{sC_3}{R_5} - \frac{sC_4}{R_5} - \frac{1}{R_1 R_3} - \frac{1}{R_2 R_3} - s^2 C_3 C_4}$$

÷ $C_3 C_4$ FACILMAIS ENGRAÇA

$$\frac{V_o}{V_w} = \frac{-\frac{1}{R_1 C_4}}{s^2 + s \left(\frac{1}{R_5 C_4} + \frac{1}{R_5 C_3} \right) + \frac{1}{C_3 C_4 R_5} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)}$$

FILTRO PASSA-Faixa DE 2ª ORDEM INVERSOR

2



$$G_1 = -\frac{1}{(N_1+1)^2} = -1$$

$$E_1 - E_2 = 10$$

$$E_2 - E_3 = 2E_1$$

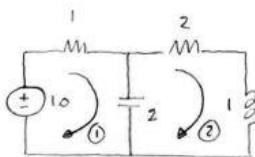
$$I_1 = \frac{1}{N_1+1} - G_1 N_1 = 1 + 1 \times 0 = 1$$

$$G_2 = 3N_2^2 = 0$$

$$I_2 = N_3^3 - G_2 N_3 = 0 - 0 \times 0 = 0$$

$$\begin{bmatrix} 2-1 & 0 & +1 & +1 & 0 \\ 0 & 0 & 0 & -1 & +1 \\ +1 & 0 & 3-1 & 0 & -1 \\ +1 & -1 & 0 & 0 & 0 \\ -2 & +1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{1,n+1} \\ E_{2,n+1} \\ E_{3,n+1} \\ I_{4,n+1} \\ I_{5,n+1} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ +1 \\ 10 \\ 0 \end{bmatrix}$$

3



MAS FACIL FAZER E.A.S (?)

$$\begin{bmatrix} 1 + \frac{1}{2s} & -\frac{1}{2s} \\ -\frac{1}{2s} & \frac{1}{2s} + 2 + s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$I_2 = \frac{+\frac{5}{s}}{\frac{1}{2s} + 2 + s + \frac{1}{4s^2} + \frac{1}{s} + \frac{1}{2} - \frac{1}{4s^2}} = \frac{+\frac{5}{s}}{\frac{3}{2}s + s + \frac{5}{2}} = \frac{+10}{3 + 2s^2 + 5s} \quad \therefore V_L = \frac{+10s}{2s^2 + 5s + 3}$$

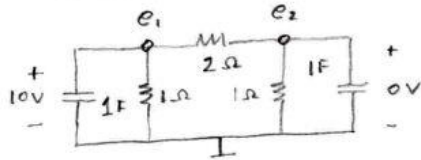
com $s = j$:

$$V_L = \frac{+10j}{-2 + 5j + 3} = \frac{+10j}{1 + 5j} = \frac{+10j(1-5j)}{26} = \frac{+50}{26} + \frac{10j}{26}$$

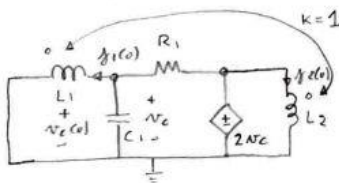
$$V_L(t) = +\frac{50}{26} \cos t - \frac{10}{26} \sin t$$

$$V_L = \frac{10}{\sqrt{26}} \cos \left(t + \tan^{-1} \frac{1}{5} \right)$$

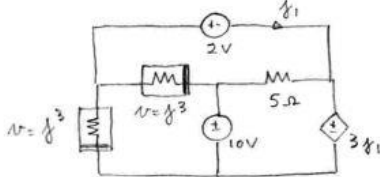
- ① PARA O CIRCUITO ABAIXO, CALCULE $e_1(t)$ E $e_2(t)$, A PARTIR DE UM SISTEMA MODAL EM TRANSFORMADA DE LAPLACE, DADO O ESTADO EM $t=0$:



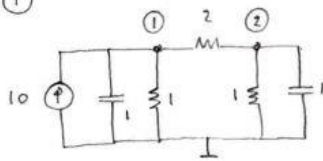
- ② PARA O CIRCUITO ABAIXO, ESCRVA O SISTEMA MODAL MODIFICADO QUE CALCULA $\vec{e}(t_0 + \Delta t)$, DADO O ESTADO DO CIRCUITO EM $t = t_0$, USANDO A APROXIMAÇÃO "BACKWARD" DE EULER



- ③ ESCRVA UM SISTEMA DE MALHAS QUE CALCULE A PRÓXIMA APROXIMAÇÃO DE \vec{x} PELO MÉTODO DE NEWTON-RAPHSON



1



$$\begin{bmatrix} s+1+0.5 & -0.5 \\ -0.5 & s+1+0.5 \end{bmatrix} \begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$E_1(s) = \frac{10(s+1.5)}{(s^2+3s+1.5^2)-0.5^2} = \frac{10(s+1.5)}{s^2+3s+2} = \frac{10s+15}{(s+1)(s+2)}$$

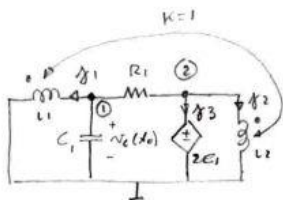
$$= \frac{A_1}{s+1} + \frac{A_2}{s+2} \quad A_1 = \frac{10s+15}{s+2} \Big|_{s=-1} = \frac{-10+15}{-1+2} = 5 \quad A_2 = \frac{10s+15}{s+1} \Big|_{s=-2} = \frac{-20+15}{-2+1} = 5$$

$$e_1(t) = 5e^{-t} + 5e^{-2t} \quad \text{COMEÇA DE 10, ACABA EM 0}$$

$$E_2(s) = \frac{+5}{(s+1)(s+2)} = \frac{A_1}{s+1} + \frac{A_2}{s+2} \quad A_1 = \frac{5}{s+2} \Big|_{s=-1} = 5 \quad A_2 = \frac{5}{s+1} \Big|_{s=-2} = -5$$

$$e_2(t) = 5e^{-t} - 5e^{-2t} \quad \text{COMEÇA E ACABA EM 0}$$

2

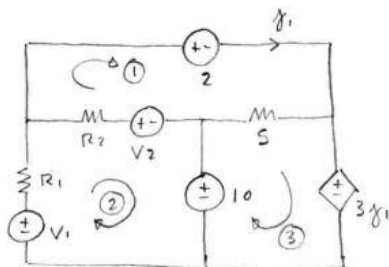


$$\begin{bmatrix} \frac{C_1+1/R_1}{\Delta t} & -1/R_1 & +1 & 0 & 0 \\ -1/R_1 & 1/R_1 & 0 & +1 & +1 \\ -1 & 0 & \frac{L_1}{\Delta t} & -\frac{M}{\Delta t} & 0 \\ 0 & -1 & \frac{M}{\Delta t} & \frac{L_2}{\Delta t} & 0 \\ -2 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1(t_0+\Delta t) \\ e_2(t_0+\Delta t) \\ i_1(t_0+\Delta t) \\ i_2(t_0+\Delta t) \\ i_3(t_0+\Delta t) \end{bmatrix} = \begin{bmatrix} \frac{C_1}{\Delta t} i_2(t_0) \\ 0 \\ \frac{C_1}{\Delta t} i_1(t_0) - \frac{M}{\Delta t} i_2(t_0) \\ -\frac{M}{\Delta t} i_1(t_0) + \frac{L_2}{\Delta t} i_2(t_0) \\ 0 \end{bmatrix}$$

TRANSFORMADOR:

$$\vec{i}(t_0+\Delta t) = \vec{i}(t_0) + [r] \Delta t \vec{i}(t_0+\Delta t) \Rightarrow -\vec{i}(t_0+\Delta t) + \frac{1}{\Delta t} [L] \vec{i}(t_0+\Delta t) = \frac{1}{\Delta t} [L] \vec{i}(t_0) \quad M = K\sqrt{L_1 L_2} = \sqrt{L_1 L_2}$$

3

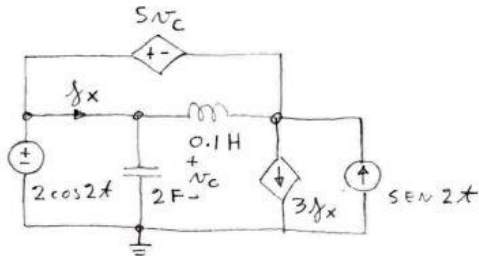


$$\begin{aligned} R_1 &= 3(-i_{2m})^2 \\ V_1 &= (-i_{2m})^3 - R_1(-i_{2m}) \\ R_2 &= 3(i_{2m} - i_{1m})^2 \\ V_2 &= (i_{2m} - i_{1m})^3 - R_2(i_{2m} - i_{1m}) \end{aligned}$$

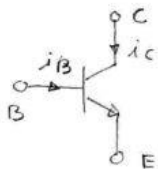
$$\begin{bmatrix} R_2+S & -R_2 & -S \\ -R_2 & R_1+R_2 & 0 \\ -S+3 & 0 & S \end{bmatrix} \begin{bmatrix} i_{1m+1} \\ i_{2m+1} \\ i_{3m+1} \end{bmatrix} = \begin{bmatrix} V_2-2 \\ V_1-V_2-10 \\ 10-3i_1 \end{bmatrix}$$

CIRCUITOS ELÉTRICOS II - 1º SEMESTRE DE 2005 - 1ª PROVA 29/4/2005

- ① ESCREVA O SISTEMA NODAL MODIFICADO PARA ANÁLISE DO CIRCUITO, NO ESTADO PERMANENTE SENOIDAL



- ② MOSTRE COMO FICA A ESTAMPA EM UM SISTEMA DE EQUAÇÕES NODAIS DE UM TRANSISTOR MODELAO PECO MODELO HÍBRIDO h PARA DEQUENOS SIMILIS

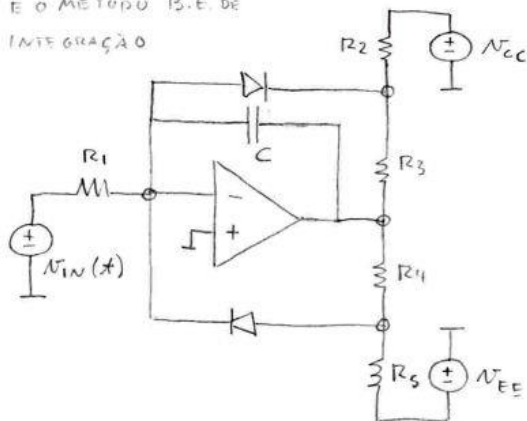


$$v_{BE} = h_{ie} i_B + h_{re} v_{CE}$$

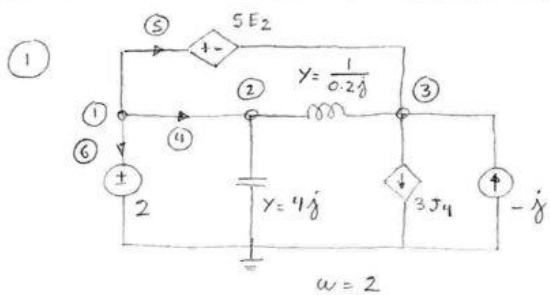
$$i_c = h_{fe} i_B + h_{oe} v_{CE}$$

O MODELO DEVE TER 3 NÓS, E APENAS ELEMENTOS PERMITIDOS NA ANÁLISE NODAL

- ③ ESCREVA O SISTEMA NODAL QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO DO CIRCUITO EM $t = t_0 + \Delta t$ PELO MÉTODO DE NEWTON-RAPHSON USANDO REDUÇÃO DO TAMANHO DO SISTEMA DEVIDA AO AMP. OP. IDEAL E O MÉTODO B.E. DE INTEGRAÇÃO



$$\text{DIODOS: } i = I_s (e^{v/V_T} - 1)$$



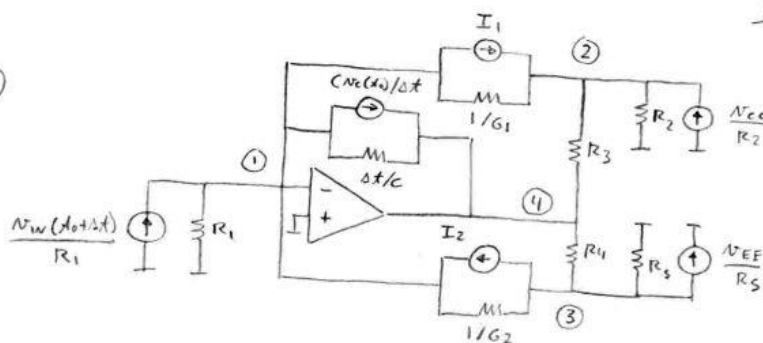
$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & +1 & +1 & +1 \\ 2 & 0 & 4j-5j & +5j & -1 & 0 & 0 \\ 3 & 0 & +5j & -5j & +3 & -1 & 0 \\ 4 & -1 & +1 & 0 & 0 & 0 & 0 \\ 5 & -1 & +5 & +1 & 0 & 0 & 0 \\ 6 & +1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -j \\ 0 \\ 0 \\ +2 \end{bmatrix}$$

2) $V_{BE} = h_{ie} i_B + h_{re} V_{CE} \Rightarrow i_B = \frac{V_{BE}}{h_{ie}} - \frac{h_{re}}{h_{ie}} V_{CE}$

$i_C = \frac{h_{fe}}{h_{ie}} V_{BE} - \frac{h_{fe} h_{re}}{h_{ie}} V_{CE} + h_{oe} V_{CE} = \frac{h_{fe}}{h_{ie}} V_{BE} + \left(h_{oe} - \frac{h_{fe} h_{re}}{h_{ie}} \right) V_{CE}$

$$\begin{matrix} C \\ B \\ E \end{matrix} \begin{bmatrix} C & B & E \\ +G & \frac{h_{fe}}{h_{ie}} & -\frac{h_{ie}}{h_{ie}} - G \\ -\frac{h_{re}}{h_{ie}} & \frac{1}{h_{ie}} & -\frac{1}{h_{ie}} + \frac{h_{re}}{h_{ie}} \\ +\frac{h_{re}}{h_{ie}} - G & -\frac{1}{h_{ie}} \cdot \frac{h_{fe}}{h_{ie}} & +\frac{1}{h_{ie}} + \frac{h_{fe}}{h_{ie}} + G - \frac{h_{re}}{h_{ie}} \end{bmatrix} \begin{bmatrix} e_C \\ e_B \\ e_E \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

3)



usando $\vec{e}_m(t+\Delta t)$:

$G_1 = \frac{I_3}{V_A} e^{\frac{e_{1m} - e_{2m}}{V_A}}$

$I_1 = I_5 \left(e^{\frac{e_{1m} - e_{2m}}{V_A}} - 1 \right) - G_1 (e_{1m} - e_{2m})$

$G_2 = \frac{I_5}{V_A} e^{\frac{e_{3m} - e_{1m}}{V_A}}$

$I_2 = I_5 \left(e^{\frac{e_{3m} - e_{1m}}{V_A}} - 1 \right) - G_2 (e_{3m} - e_{1m})$

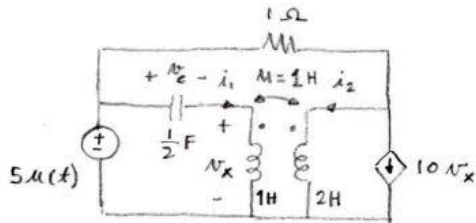
$$\begin{matrix} SA1 \\ \rightarrow \end{matrix} \begin{bmatrix} \frac{1}{R_1} + \frac{C}{\Delta t} + G_1 + G_2 & -G_1 & -G_2 & -\frac{C}{\Delta t} \\ -G_1 & G_1 + \frac{1}{R_3} + \frac{1}{R_2} & 0 & -\frac{1}{R_3} \\ -G_2 & 0 & G_2 + \frac{1}{R_4} + \frac{1}{R_5} & -\frac{1}{R_4} \\ -\frac{C}{\Delta t} & -\frac{1}{R_3} & -\frac{1}{R_4} & \frac{C}{\Delta t} + \frac{1}{R_6} + \frac{1}{R_4} \end{bmatrix} \begin{bmatrix} e_{1m+1}^o(t+\Delta t) \\ e_{2m+1}^o(t+\Delta t) \\ e_{3m+1}^o(t+\Delta t) \\ e_{4m+1}^o(t+\Delta t) \end{bmatrix} = \begin{bmatrix} \frac{V_w(t+\Delta t)}{R_1} - \frac{C V_e(t)}{\Delta t} \\ -I_1 + I_2 \\ I_1 + \frac{V_{CC}}{R_2} \\ -I_2 - \frac{V_{EE}}{R_5} \\ \frac{C V_e(t)}{\Delta t} \end{bmatrix}$$

RESTA:

$$\begin{bmatrix} -G_1 & -G_2 & -\frac{C}{\Delta t} \\ G_1 + \frac{1}{R_3} + \frac{1}{R_2} & 0 & -\frac{1}{R_3} \\ G & G_2 + \frac{1}{R_4} + \frac{1}{R_5} & -\frac{1}{R_4} \end{bmatrix} \begin{bmatrix} e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} \frac{V_w(t+\Delta t)}{R_1} - \frac{C V_e(t)}{\Delta t} \\ I_1 + \frac{V_{CC}}{R_2} \\ -I_2 - \frac{V_{EE}}{R_5} \end{bmatrix}$$

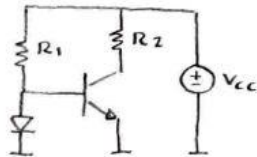
CIRCUITOS ELÉTRICOS II - 2º SEMESTRE DE 2006 - 1ª PROVA

- 1) ESCREVA O SISTEMA MODAL MODIFICADO QUE RESOLVE O CIRCUITO USANDO TRANSFORMADA DE LAPLACE.



DADOS: $u_x(0)$, $i_1(0)$ E $i_2(0)$

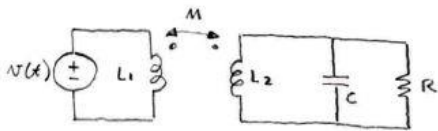
- 2) MOSTRE COMO RESOLVER O CIRCUITO USANDO ANÁLISE MODAL E O MÊTODO DE NEWTON-RAPHSON. ESCREVA O SISTEMA DE EQUAÇÕES USADO NO MÊTODO.



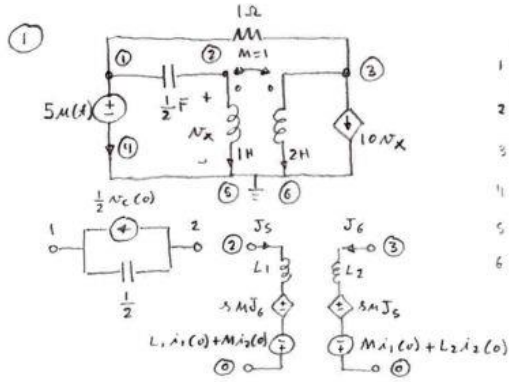
DIODO: $j = I_s (e^{u/V_T} - 1)$

TRANSISTOR: MODELO DE EBERS-MOLL

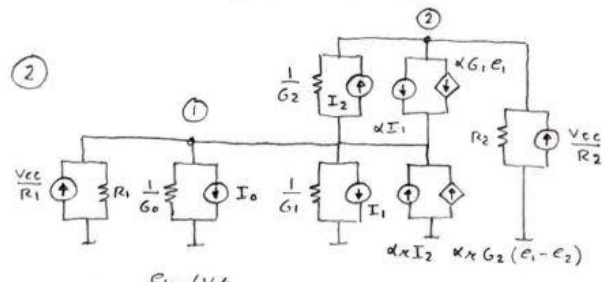
- 3) ESCREVA O SISTEMA DE MALHAS QUE CALCULA A SOLUÇÃO DO CIRCUITO EM $t = t_0 + \Delta t$, USANDO INTEGRAÇÃO BACKWARD DE EULER.



ENCONTRE UMA FORMA DE COMBINAR C E R, DE FORMA A GERAR UM SISTEMA COM APENAS DUAS MALHAS (EQUIVALENTE THÉVENIN)



$$\begin{bmatrix}
 1 & 2 & 3 & 4 & 5 & 6 \\
 \frac{1}{2}S + 1 & -\frac{1}{2}S & -1 & +1 & 0 & 0 \\
 -\frac{1}{2}S & \frac{1}{2}S & 0 & 0 & +1 & 0 \\
 -1 & 1 & 0 & 0 & 0 & +1 \\
 -1 & 0 & 0 & 0 & 0 & 0 \\
 0 & -1 & 0 & 0 & -5M & 5M \\
 0 & 0 & -1 & 0 & 5M & -5M
 \end{bmatrix}
 \begin{bmatrix}
 E_1 \\
 E_2 \\
 E_3 \\
 J_4 \\
 J_5 \\
 J_6
 \end{bmatrix}
 =
 \begin{bmatrix}
 \frac{1}{2}N_c(\omega) \\
 -\frac{1}{2}N_c(\omega) \\
 0 \\
 -\frac{5}{S} \\
 L_1 i_1(\omega) + M i_2(\omega) \\
 M i_1(\omega) + L_2 i_2(\omega)
 \end{bmatrix}$$



$$\begin{bmatrix}
 \frac{1}{R_1} + G_0 + G_1 + G_2 - \alpha G_1 - \alpha \alpha G_2 & -G_2 + \alpha \alpha G_2 \\
 -G_2 + \alpha G_1 & \frac{1}{R_2} + G_2
 \end{bmatrix}
 \begin{bmatrix}
 e_{1, m+1} \\
 e_{2, m+1}
 \end{bmatrix}$$

$$\begin{bmatrix}
 \frac{V_{CC}}{R_1} - I_0 - I_1 + \alpha \alpha I_2 - I_2 + \alpha I_1 + \alpha \alpha G_2 (e_{1, m+1} - e_{2, m+1}) + \alpha G_1 e_{1, m+1} \\
 \frac{V_{CC}}{R_2} + I_2 - \alpha I_1 - \alpha G_1 e_{1, m+1}
 \end{bmatrix}$$

$$G_0 = \frac{I_5}{V_A} e^{e_{1, m} / V_A}$$

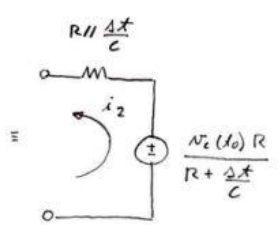
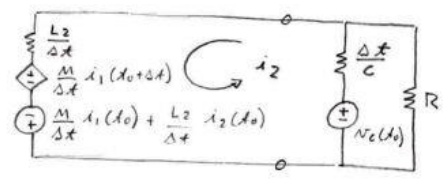
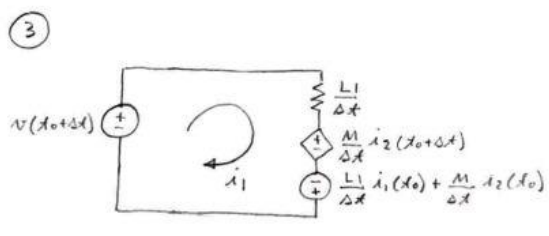
$$I_0 = I_5 \left(e^{e_{1, m} / V_A} - 1 \right) - G_0 e_{1, m}$$

$$G_1 = G_0 \quad I_1 = I_0$$

$$G_2 = \frac{I_3}{V_A} e^{(e_{1, m} - e_{2, m}) / V_A}$$

$$I_2 = I_3 \left(e^{(e_{1, m} - e_{2, m}) / V_A} - 1 \right) - G_2 (e_{1, m} - e_{2, m})$$

CALCULA-SE OS VALORES À ESQUERDA, E RESOLVE-SE O SISTEMA. NA PRIMEIRA VEZ, \vec{e}_m É UMA APROXIMAÇÃO INICIAL. NAS VEZS SEQUENTES É A ÚLTIMA SOLUÇÃO. ITERA-SE ATÉ $\vec{e}_{m+1} \approx \vec{e}_m$.

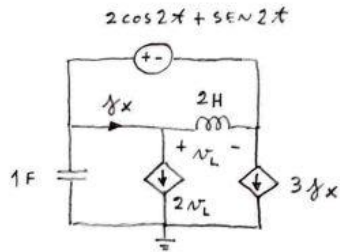


$$\begin{bmatrix}
 \frac{L_1}{\Delta x} & \frac{M}{\Delta x} \\
 \frac{M}{\Delta x} & \frac{L_2}{\Delta x} + R // \frac{\Delta x}{C}
 \end{bmatrix}
 \begin{bmatrix}
 i_1(x_0 + \Delta x) \\
 i_2(x_0 + \Delta x)
 \end{bmatrix}
 =
 \begin{bmatrix}
 V(x_0 + \Delta x) + \frac{L_1}{\Delta x} i_1(x_0) + \frac{M}{\Delta x} i_2(x_0) \\
 \frac{M}{\Delta x} i_1(x_0) + \frac{L_2}{\Delta x} i_2(x_0) + \frac{N_c(x_0) R}{R + \frac{\Delta x}{C}}
 \end{bmatrix}$$

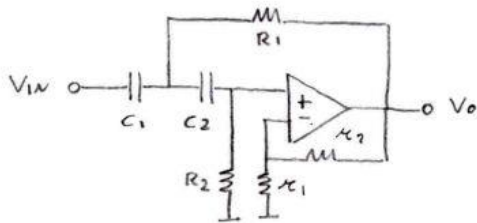
$N_c(x_0 + \Delta x)$ É CALCULADA COM $N_c(x_0 + \Delta x) = N_c(x_0) \frac{R}{R + \frac{\Delta x}{C}} = R // \frac{\Delta x}{C} i_2(x_0 + \Delta x)$

CIRCUITOS ELÉTRICOS II - 1º SEMESTRE DE 2007 - 1ª PROVA

- ① ESCREVA O SISTEMA NODAL MODIFICADO QUE RESOLVE O CIRCUITO, NO ESTADO PERMANENTE SENOIDAL



- ② ACITE A FUNÇÃO DE TRANSFERÊNCIA $\frac{V_o(s)}{V_{in}(s)}$ USANDO UMA ANÁLISE NODAL COM ELIMINAÇÃO DO AMP. OP.

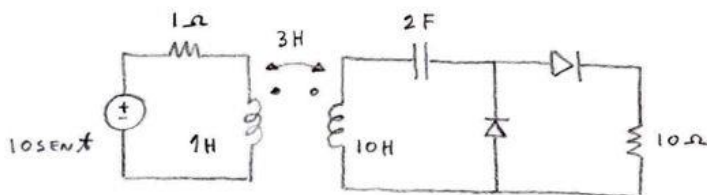
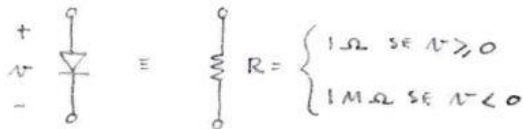


FAÇA $1 + \frac{K_2}{K_1} = K$

O QUE FAZ ESSE CIRCUITO?

- ③ ESCREVA UM SISTEMA DE MALHAS QUE CALCULE A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO DO CIRCUITO EM $t = t_0 + \Delta t$, USANDO O MÉTODO "BACKWARD" DE EULER E O MÉTODO DE NEWTON-RAPHSON.

OS DIODOS SÃO MODELADOS COMO:



①

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} j2 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{j4} + 2 & -\frac{1}{j4} - 2 & 0 & 0 \\ 0 & -\frac{1}{j4} & \frac{1}{j4} & 0 & 0 \\ +1 & 0 & -1 & 0 & 0 \\ +1 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ J_4 \\ J_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2-j \\ 0 \end{bmatrix}$$

O INDIADOR FOI TRATADO COMO ADMITÂNCIA

②

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} sC_1 + sC_2 + \frac{1}{R_1} & -sC_2 & 0 & -\frac{1}{R_1} \\ -sC_2 & sC_2 + \frac{1}{R_2} & 0 & 0 \\ 0 & 0 & \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_1} & 0 & -\frac{1}{R_2} & \frac{1}{R_1} + \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} sC_1 VIN \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

RESTA:

$$\begin{bmatrix} sC_1 + sC_2 + \frac{1}{R_1} & -sC_2 & -\frac{1}{R_1} \\ -sC_2 & sC_2 + \frac{1}{R_2} & 0 \\ 0 & \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2,3 \\ E_4 \end{bmatrix} = \begin{bmatrix} sC_1 VIN \\ 0 \\ 0 \end{bmatrix}$$

$K = 1 + \frac{R_2}{R_1}$
 $\frac{R_2}{R_1} = K - 1$
 $1 - \frac{R_2}{R_1} = 1 - K + 1 = -K$

$$\frac{E_4}{VIN} = \frac{-s^2 C_1 C_2 \left(\frac{1}{R_1} + \frac{1}{R_2}\right)}{\left(s^2 C_1 C_2 + \frac{sC_1}{R_2} + \frac{sC_2}{R_2} + \frac{sR_2}{R_1} + \frac{1}{R_1 R_2}\right) \left(-\frac{1}{R_2}\right) + \frac{sC_2}{R_1} \left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = \frac{\left(1 + \frac{R_2}{R_1}\right) s^2}{s^2 + s \left(\frac{1}{C_2 R_2} + \frac{1}{C_1 R_2} - \frac{K-1}{C_1 R_1}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

É UM RESSONADOR PASSA-ALTAS DE 2º ORDEM (OU FICHA)

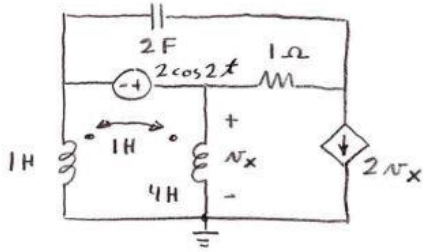
③

$R_1 = \begin{cases} 1 \text{ SE } i_{3M} - i_{2M} \geq 0 \\ 10^6 \text{ SE NÃO} \end{cases}$
 $R_2 = \begin{cases} 1 \text{ SE } i_{3M} \geq 0 \\ 10^6 \text{ SE NÃO} \end{cases}$
 $V_1 = V_2 = 0!$
 $N_c(t_0) = N_c(t_0 - \Delta t) + \frac{\Delta t}{2} i_2(t_0)$ ou $N_c(t_0)$

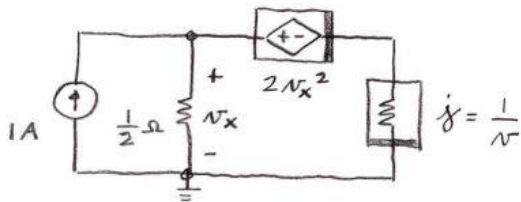
$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 + \frac{1}{\Delta t} & -\frac{3}{\Delta t} & 0 \\ -\frac{3}{\Delta t} & \frac{10}{\Delta t} + \frac{\Delta t}{2} + R_1 & -R_1 \\ 0 & -R_1 & R_1 + R_2 + 10 \end{bmatrix} \begin{bmatrix} i_{1,t+1}(t_0 + \Delta t) \\ i_{2,t+1}(t_0 + \Delta t) \\ i_{3,t+1}(t_0 + \Delta t) \end{bmatrix} = \begin{bmatrix} 10 \text{sen}(t_0 + \Delta t) + \frac{1}{\Delta t} (i_1(t_0) - 3i_2(t_0)) \\ -\frac{1}{\Delta t} (3i_1(t_0) - 10i_2(t_0)) - N_c(t_0) \\ 0 \end{bmatrix}$$

CIRCUITOS ELÉTRICOS II - 2º SEMESTRE DE 2007 - 1ª PROVA

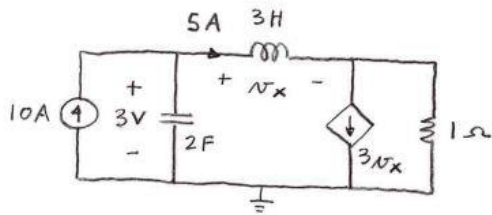
- ① PARA O CIRCUITO ABAIXO, ESCREVA O SISTEMA NODAL COM TAMANHO MÍNIMO E O SISTEMA NODAL MODIFICADO, PARA ANÁLISE NO ESTADO PERMANENTE SENOIDAL.

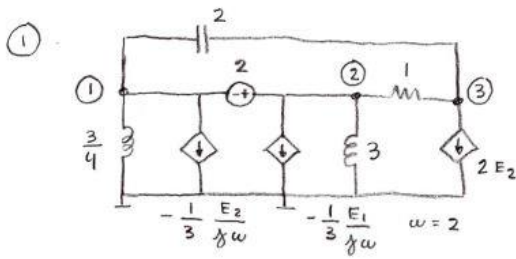


- ② PARA O CIRCUITO NÃO LINEAR ABAIXO, ESCREVA AS EQUAÇÕES NODAIS MODIFICADAS NÃO LINEARES, E O SISTEMA QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON.



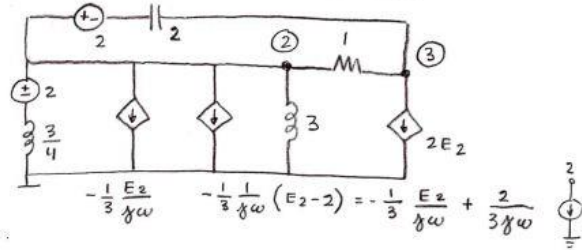
- ③ PARA O CIRCUITO ABAIXO, ESCREVA O SISTEMA NODAL QUE CALCULA A SOLUÇÃO 0.1 SEGUNDOS ALÉM DO TEMPO EM QUE O CIRCUITO É MOSTRADO, USANDO O MÉTODO "BACKWARD" DE EULER.



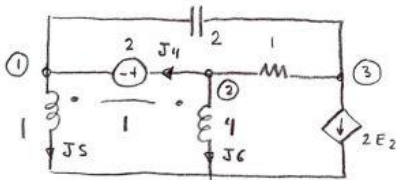


$$[L] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow [r] = \frac{1}{3} \begin{bmatrix} 4 & -1 \\ -1 & 1 \end{bmatrix}$$

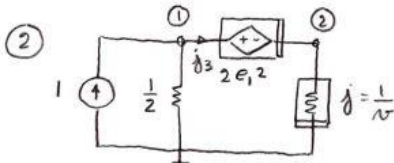
DESCOLOCANDO A FONTE PARA A ESQUERDA



$$\begin{bmatrix} \frac{4}{3j2} + 2j2 - \frac{2}{3j2} + \frac{1}{3j2} + 1 & -1 - 2j2 \\ -1 - 2j2 + 2 & 1 + 2j2 \end{bmatrix} \begin{bmatrix} E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{3j2} + 2j2 \cdot 2 - \frac{2}{3j2} \\ -2j2 \cdot 2 \end{bmatrix}$$

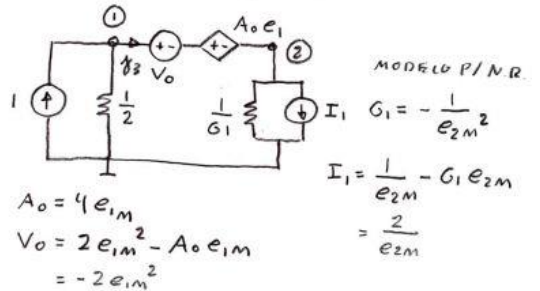


$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & j2 \cdot 2 & 0 & -j2 \cdot 2 & -1 & +1 & 0 \\ 2 & 0 & 1 & -1 & +1 & 0 & +1 \\ 3 & -j2 \cdot 2 & -1 + 2 & 1 + j2 \cdot 2 & 0 & 0 & 0 \\ 4 & +1 & -1 & 0 & 0 & 0 & 0 \\ 5 & -1 & 0 & 0 & 0 & j2 & j2 \\ 6 & 0 & -1 & 0 & 0 & j2 & j2 \cdot 4 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$



MNA NÃO LINEAR

$$\begin{aligned} \textcircled{1} \quad 2e_1 + j3 &= 1 \\ \textcircled{2} \quad -j3 + \frac{1}{e_2} &= 0 \\ \textcircled{3} \quad e_1 - e_2 - 2e_1^2 &= 0 \end{aligned}$$



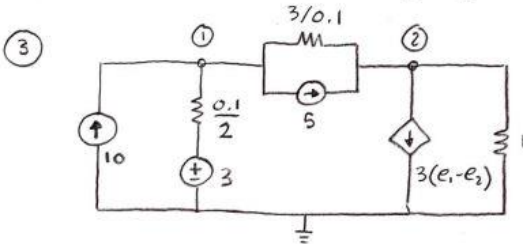
MODELO P/N.R.

$$\begin{aligned} G_1 &= -\frac{1}{e_{2M}^2} \\ I_1 &= \frac{1}{e_{2M}} - G_1 e_{2M} \\ &= \frac{2}{e_{2M}} \end{aligned}$$

SISTEMA MNA PARA N. R.:

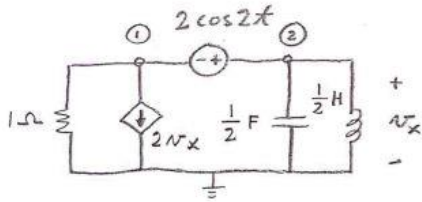
$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 0 & +1 \\ 2 & 0 & G_1 & -1 \\ 3 & -1 + A_0 & +1 & 0 \end{bmatrix} \begin{bmatrix} e_{1M+1} \\ e_{2M+1} \\ j_{3M+1} \end{bmatrix} = \begin{bmatrix} 1 \\ -I_1 \\ -V_0 \end{bmatrix}$$

$$e_1 - e_2 = V_0 + A_0 e_1$$

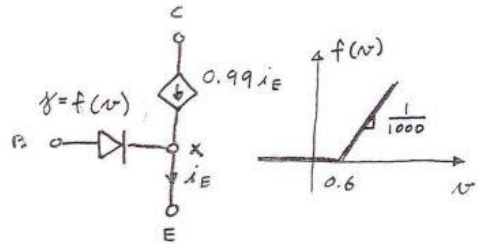
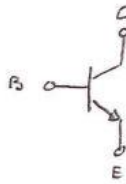
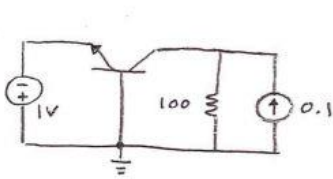


$$\begin{bmatrix} \frac{2}{0.1} + \frac{0.1}{3} & -\frac{0.1}{3} \\ -\frac{0.1}{3} + 3 & \frac{0.1}{3} + 1 - 3 \end{bmatrix} \begin{bmatrix} e_1(x_0 + 0.1) \\ e_2(x_0 + 0.1) \end{bmatrix} = \begin{bmatrix} \frac{2}{0.1} \cdot 3 - 5 \\ +10 \\ 5 \end{bmatrix}$$

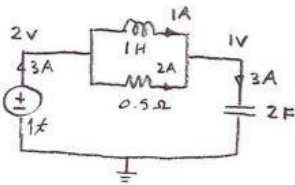
- 1) CALCULE $e_1(x)$ E $e_2(x)$ NO ESTADO PERMANENTE, USANDO UMA ANÁLISE MODAL. TRANSFORME O CIRCUITO QUANDO NECESSÁRIO.



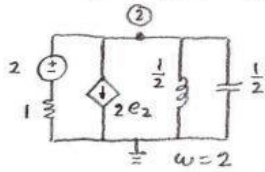
- 2) PARA O CIRCUITO ABAIXO, ESCREVA O SISTEMA MODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON. O TRANSISTOR B É MODELCADO COMO MOSTRADO.



- 3) PARA O CIRCUITO ABAIXO, DESENHE O MODELO E ESCREVA O SISTEMA MODAL MODIFICADO QUE CALCULA A SOLUÇÃO EM $t = 2.01$, DADO O ESTADO DO CIRCUITO EM $t = 2$, USANDO O MÉTODO DOS TRAPÉZIOS.



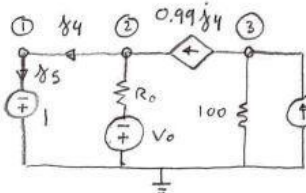
① DESCOLOCANDO A FONTE DE TENSÃO SÓ RESTA UM NÓ



$$\left[1 + 2 + \frac{1}{j2\frac{1}{2}} + j2\frac{1}{2} \right] E_2 = [2] \quad \therefore 3E_2 = 2 \quad \therefore E_2 = \frac{2}{3}$$

$$E_1 = E_2 - 2 = \frac{2}{3} - 2 = -\frac{4}{3} \quad e_1(t) = -\frac{4}{3} \cos 2t \quad e_2(t) = \frac{2}{3} \cos 2t$$

②



USANDO 5 INCÓGNITAS (PODIA FAZER $j5 = j4$)

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & -1 & +1 \\ 0 & \frac{1}{R_0} & 0 & +0.01 & 0 \\ 0 & 0 & \frac{1}{100} & +0.99 & 0 \\ -1 & +1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ j4 \\ j5 \end{bmatrix} & = & \begin{bmatrix} 0 \\ -V_0/R_0^* \\ 0.1 \\ 0 \\ +1 \end{bmatrix} \end{matrix}$$

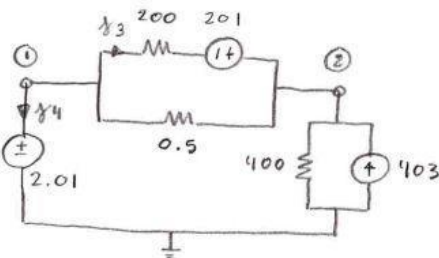
$$V_0 = 0.6$$

$$R_0 = 1000$$

NO CASO, A SOLUÇÃO É ACHADA IMEDIATAMENTE, POIS O PRÓXIMO SISTEMA SERÁ O MESMO

* COLOCAR ESTES TERMOS SE $-e_{2m} < 0.6V$, SENÃO COLOCAR 0

③



INDUTOR NO MÉTODO DOS TRIÂNGULOS:

$$j(\lambda_0 + \Delta\lambda) \approx j(\lambda_0) + \frac{\Delta\lambda}{2L} (N(\lambda_0) + N(\lambda_0 + \Delta\lambda))$$

$$N(\lambda_0 + \Delta\lambda) \approx \frac{2L}{\Delta\lambda} (j(\lambda_0 + \Delta\lambda) - j(\lambda_0)) - N(\lambda_0)$$

CAPACITOR:

$$j(\lambda_0 + \Delta\lambda) \approx \frac{2C}{\Delta\lambda} (N(\lambda_0 + \Delta\lambda) - N(\lambda_0)) - j(\lambda_0)$$

$$\frac{2L}{\Delta\lambda} = \frac{2}{0.01} = 200 \quad \frac{\Delta\lambda}{2C} = \frac{0.01}{4} = 0.0025$$

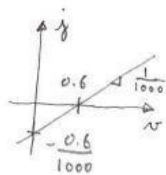
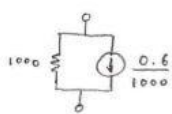
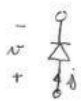
$$\frac{2C}{\Delta\lambda} = \frac{4}{0.01} = 400$$

$$\frac{2L}{\Delta\lambda} j_L(\lambda_0) + N_L(\lambda_0) = 200 \times 1 + 1 = 201$$

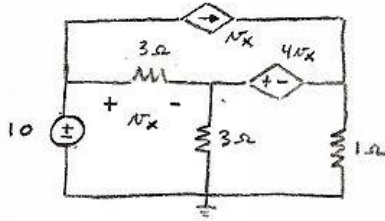
$$\frac{2C}{\Delta\lambda} N_C(\lambda_0) + j_C(\lambda_0) = 400 \times 1 + 3 = 403$$

SISTEMA:

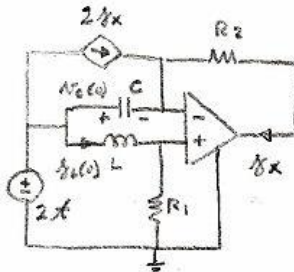
$$\begin{matrix} 1 & 2 & 3 & 4 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 2 & -2 & +1 & +1 \\ -2 & 2+400 & -1 & 0 \\ -1 & +1 & 200 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} e_1(2.01) \\ e_2(2.01) \\ j_3(2.01) \\ j_4(2.01) \end{bmatrix} & = & \begin{bmatrix} 0 \\ 403 \\ 201 \\ -2.01 \end{bmatrix} \end{matrix}$$



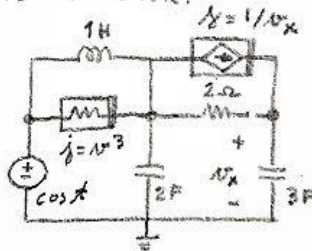
- 1) ENCONTRE A SOLUÇÃO DO CIRCUITO USANDO UMA ANÁLISE MODAL, APÓS TRANSFORMAR O CIRCUITO ONDE NECESSÁRIO



- 2) ESCREVA O SISTEMA MODAL MODIFICADO EM TRANSFORMADA DE LAPLACE, COM CONDIÇÕES INICIAIS

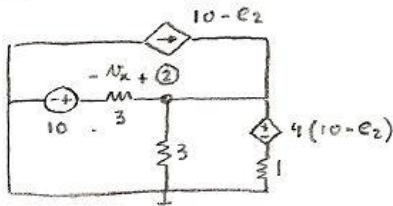


- 3) ESCREVA O SISTEMA MODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO DO CIRCUITO EM $t = 1.1$, CONHECIDA A SOLUÇÃO EM $t = 1$, $\vec{e}^3(1)$, E A ÚLTIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON, $\vec{e}_M(1.1)$. USE O MÉTODO "BACKWARD" DE FULLER.



CIRCUITOS ELÉTRICOS II - 1º SEMESTRE DE 2009 - 1ª PROVA - GABARITO

1) DESLOCANDO AS FONTES DE TENSÃO:



$V_x = 10 - e_2$

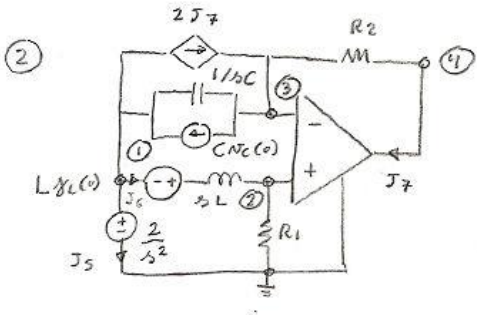
FAZENDO EQU. NORTON:

$$\left[\frac{1}{3} + \frac{1}{3} + 1 \right] e_2 = \left[\frac{10}{3} + 10 - e_2 + 4(10 - e_2) \right]$$

$$\left[\frac{2}{3} + 1 + 1 + 1 \right] e_2 = \left[\frac{10}{3} + 10 + 40 \right]$$

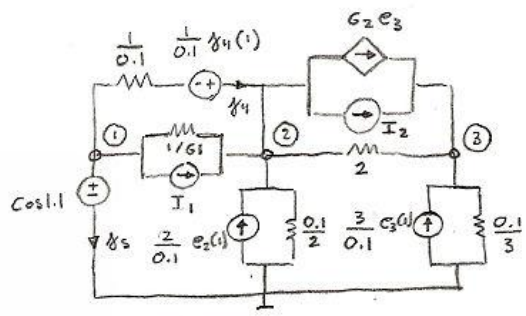
$20 e_2 = 160$

$e_2 = 8V \quad e_1 = 10V \quad e_3 = e_2 - 40 + 4e_2 = 8 - 40 + 32 = 0V$



$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & -sC & -sC & & +1 & +1 & +2 \\ 2 & & \frac{1}{R_1} & & & & \\ 3 & -sC & sC + \frac{1}{R_2} & -\frac{1}{R_2} & & & -2 \\ 4 & & -\frac{1}{R_2} & \frac{1}{R_2} & & & +1 \\ 5 & -1 & & & & & \\ 6 & -1 & & & & sL & \\ 7 & & -1 & +1 & & & \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ J_5 \\ J_6 \\ J_7 \end{bmatrix} = \begin{bmatrix} C N_2(\omega) \\ -C N_2(\omega) \\ -2/s^2 \\ L J_6(\omega) \end{bmatrix}$$

3)

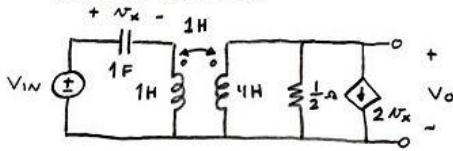


$G_1 = 3(e_{1m}(1.1) - e_{2m}(1.1))^2$
 $I_1 = (e_{1m}(1.1) - e_{2m}(1.1))^3 - G_1(e_{1m}(1.1) - e_{2m}(1.1))$
 $G_2 = -\frac{1}{e_{3m}(1.1)^2}$
 $I_2 = \frac{1}{e_{3m}(1.1)} - G_2 e_{3m}(1.1)$

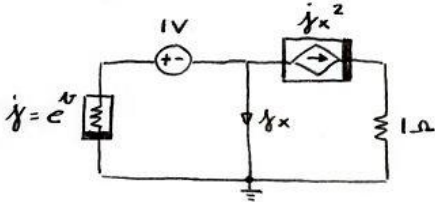
$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & G_1 & -G_1 & & +1 & +1 \\ 2 & -G_1 & G_1 + \frac{1}{2} + \frac{2}{0.1} & -\frac{1}{2} + G_2 & & -1 \\ 3 & & -\frac{1}{2} & \frac{1}{2} + \frac{3}{0.1} - G_2 & & \\ 4 & -1 & +1 & & \frac{1}{0.1} & \\ 5 & -1 & & & & \end{bmatrix} \begin{bmatrix} e_{1m}(1.1) \\ e_{2m}(1.1) \\ e_{3m}(1.1) \\ J_{4m}(1.1) \\ J_{5m}(1.1) \end{bmatrix} = \begin{bmatrix} -I_1 \\ +I_1 + \frac{2}{0.1} e_2(1) - I_2 \\ +I_2 + \frac{3}{0.1} e_3(1) \\ \frac{1}{0.1} J_4(1) \\ -\cos 1.1 \end{bmatrix}$$

CIRCUITOS ELÉTRICOS II - 2º SEMESTRE DE 2009 - 1ª PROVA

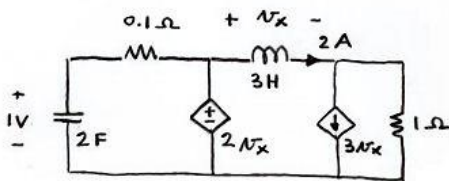
- ① PARA O CIRCUITO ABAIXO, OBTENHA $\frac{V_o}{V_{in}}(j\omega)$ USANDO UMA ANÁLISE NODAL SIMPLES.

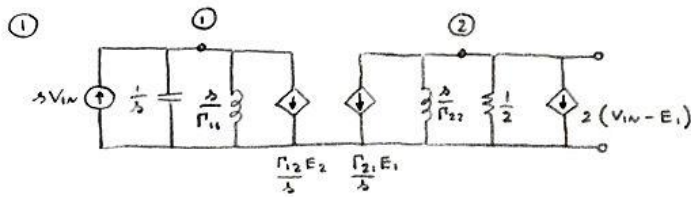


- ② PARA O CIRCUITO ABAIXO, ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON. QUAL A SOLUÇÃO EXATA DESTES CIRCUITO?



- ③ ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A SOLUÇÃO EM $t = t_0 + \Delta t$ PARA O CIRCUITO ABAIXO, USANDO O MÉTODO "BACKWARD" DE EULER.





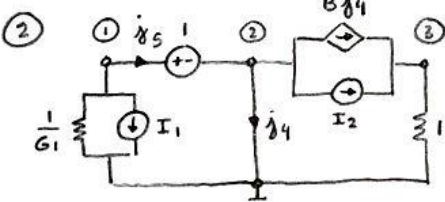
$$[L] = \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix} \quad [P] = \frac{1}{4-1} \begin{bmatrix} 4 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[P] = \begin{bmatrix} \frac{4}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} 5 + \frac{4}{3s} & -\frac{1}{3s} \\ -\frac{1}{3s} - 2 & 2 + \frac{1}{3s} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} 5V_{1W} \\ -2V_{1W} \end{bmatrix}$$

$$\frac{E_2}{V_{1W}} = \frac{-2/s - \frac{8}{3s} + \frac{1}{3} + 2/s}{2s + \frac{1}{3} + \frac{8}{3s} + \frac{4}{9s^2} - \frac{1}{9s^2} - \frac{2}{3s}} = \frac{\frac{1}{3} - \frac{8}{3s}}{2s + \frac{1}{3} + \frac{2}{s} + \frac{1}{3s^2}} \quad \times 3s^2 = \frac{s^2 - 8s}{6s^3 + s^2 + 6s + 1}$$

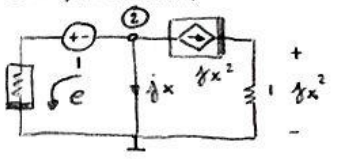
$$\frac{V_o}{V_{1W}}(j\omega) = \frac{-\omega^2 - 8j\omega}{-6j\omega^3 - \omega^2 + 6j\omega + 1}$$



$$G_1 = e^{e_{1m}} \quad I_1 = e^{e_{1m}} - G_1 e_{1m} \quad B = 2j4m \quad I_2 = j4m^2 - B j4m$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & G_1 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 1+B & -1 \\ 3 & 0 & 0 & 1 & -B & 0 \\ 4 & 0 & -1 & 0 & 0 & 0 \\ 5 & -1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ j4 \\ j8 \end{bmatrix} = \begin{bmatrix} -I_1 \\ -I_2 \\ I_2 \\ 0 \\ -1 \end{bmatrix}$$

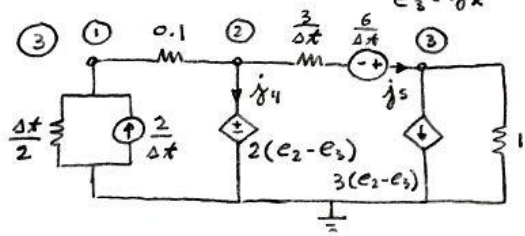
SOLUÇÃO EXATA:



$$e + jx + jx^2 = 0 \quad \therefore jx = \frac{-1 \pm \sqrt{1-4e}}{2}$$

É COMPLEXO
O CIRCUITO NÃO TEM
SOLUÇÃO REAL

ASSIM SÓ HAVERIA SOLUÇÃO SE A CORRENTE NA FONTE DE TENSÃO FOSSE MENOR QUE 1/4, OU SE A FONTE DE CORRENTE ESTIVESSE INVERTIDA



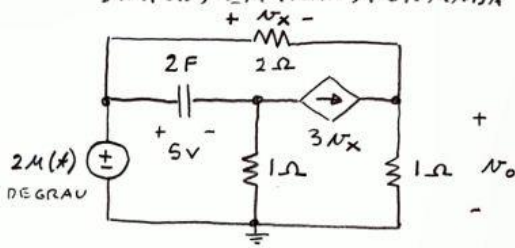
$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & \frac{2}{\Delta x} + 10 & -10 & 0 & 0 & 0 \\ 2 & -10 & 10 & 0 & 1 & 1 \\ 3 & 0 & 3 & 1-3 & 0 & -1 \\ 4 & 0 & 2-1 & -2 & 0 & 0 \\ 5 & 0 & -1 & 1 & 0 & \frac{3}{\Delta x} \end{bmatrix} \begin{bmatrix} e_1(x_0 + \Delta x) \\ e_2(x_0 + \Delta x) \\ e_3(x_0 + \Delta x) \\ j4(x_0 + \Delta x) \\ j5(x_0 + \Delta x) \end{bmatrix} = \begin{bmatrix} 2/\Delta x \\ 0 \\ 0 \\ 0 \\ \frac{6}{\Delta x} \end{bmatrix}$$

$$e_2 = 2(e_2 - e_3)$$

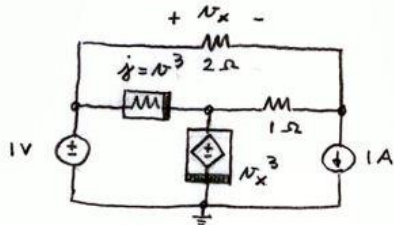
$$e_2 - e_3 = \frac{3}{\Delta x} j5 - \frac{6}{\Delta x}$$

CIRCUITOS ELÉTRICOS II - 1º SEMESTRE DE 2010 - 1ª PROVA

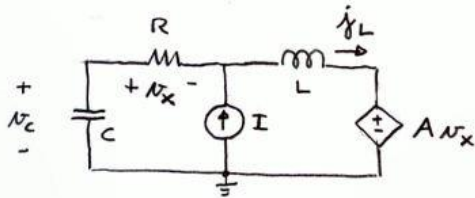
- ① PARA O CIRCUITO ABAIXO, ACHE $v_o(t)$ USANDO UMA ANÁLISE NODAL SIMPLES EM TRANSFORMADA DE LAPLACE



- ② PARA O CIRCUITO ABAIXO, ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON



- ③ ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A SOLUÇÃO EM $t = t_0 + \Delta t$ USANDO O MÉTODO DE GEAR DE 2ª ORDEM, CONHECIDAS AS SOLUÇÕES EM $t = t_0$ E $t = t_0 - \Delta t$ PARA v_C E i_L



$$y(t_0 + \Delta t) = y(t_0) + \int_{t_0}^{t_0 + \Delta t} x(t) dt$$

MÉTODO DE GEAR DE 2ª ORDEM:

$$y(t_0 + \Delta t) \approx \frac{4}{3} y(t_0) - \frac{1}{3} y(t_0 - \Delta t) + \frac{2}{3} \Delta t x(t_0 + \Delta t)$$

CIRCUITOS ELÉTRICOS II - 1º SEMESTRE DE 2010 - 1ª PROVA - GABARITO

1

$v_x = \frac{2}{5} - E_2$

$$\begin{bmatrix} 2S+1 & -3 \\ 0 & 3+\frac{3}{2} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} -6 - \frac{6}{5} \\ \frac{1}{5} + \frac{6}{5} \end{bmatrix} + 3E_2$$

$$V_0 = E_2 = \frac{(2S+1) \frac{7}{5}}{(2S+1) (\frac{9}{2})} = \frac{14}{9S} \quad v_0 = \frac{14}{9}$$

2

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & G_1+0.5 & -G_1 & -0.5 & 1 \\ 2 & -G_1 & G_1+1 & -1 & 0 \\ 3 & -0.5 & -1 & 1.5 & 0 \\ 4 & -1 & 0 & 0 & 1 \\ 5 & A_2 & -1 & -A_2 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} -I_1 \\ +I_1 \\ -1 \\ -1 \\ -V_2 \end{bmatrix}$$

$$e_2 = A_2(e_1 - e_3) + V_2$$

$$G_1 = 3(e_{1m} - e_{2m})^2$$

$$A_2 = 3(e_{1m} - e_{3m})^2$$

$$I_1 = (e_{1m} - e_{2m})^3 - G_1(e_{1m} - e_{2m})$$

$$V_2 = (e_{1m} - e_{3m})^3 - A_2(e_{1m} - e_{3m})$$

3

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & \frac{1}{R_1} + \frac{1}{R} & -\frac{1}{R} & 0 & 0 \\ 2 & -\frac{1}{R} & \frac{1}{R} & 0 & +1 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & -1 & +1 & R_2 \\ 5 & A & -A & -1 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} I_1 \\ I \\ 0 \\ V_2 \\ 0 \end{bmatrix}$$

CAPACITOR

$$v(t_0 + \Delta t) = \frac{4}{3} v(t_0) - \frac{1}{3} v(t_0 - \Delta t) + \frac{2}{3} \frac{\Delta t}{C} j(t_0 + \Delta t) \quad \therefore j(t_0 + \Delta t) = \frac{3C}{2\Delta t} v(t_0 + \Delta t) - \frac{2C}{\Delta t} v(t_0) + \frac{C}{2\Delta t} v(t_0 - \Delta t)$$

$$R_1 = \frac{2}{3} \frac{\Delta t}{C} \quad I_1 = \frac{2C}{\Delta t} v(t_0) - \frac{C}{2\Delta t} v(t_0 - \Delta t), \quad v = e_1$$

INDUTOR

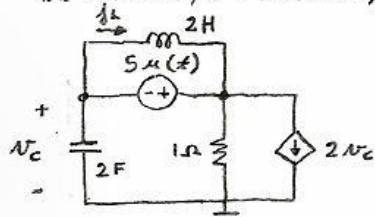
$$j(t_0 + \Delta t) = \frac{4}{3} j(t_0) - \frac{1}{3} j(t_0 - \Delta t) + \frac{2}{3} \frac{\Delta t}{L} v(t_0 + \Delta t)$$

$$v(t_0 + \Delta t) = \frac{3L}{2\Delta t} j(t_0 + \Delta t) - \frac{2L}{\Delta t} j(t_0) + \frac{L}{2\Delta t} j(t_0 - \Delta t)$$

$$R_2 = \frac{3L}{2\Delta t} \quad v_2 = \frac{2L}{\Delta t} j(t_0) - \frac{L}{2\Delta t} j(t_0 - \Delta t), \quad j = j_4$$

CIRCUITOS ELÉTRICOS I - 2º SEMESTRE DE 2010 - 1ª PROVA

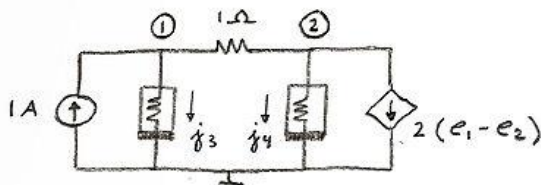
- ① PARA O CIRCUITO ABAIXO, ESCREVA UM SISTEMA NODAL MODIFICADO EM TRANSFORMADA DE LAPLACE, E O RESOLVA, ACHANDO A SOLUÇÃO NO TEMPO



$$v_c(0) = 1V$$

$$i_L(0) = 0$$

- ② NO CIRCUITO NÃO LINEAR ABAIXO, A ÚLTIMA SOLUÇÃO ENCONTRADA PELO MÉTODO DE NEWTON-RAPHSON FOI:



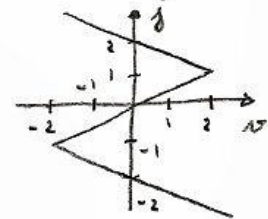
$$e_1 = 1V$$

$$e_2 = -2V$$

$$i_3 = \frac{1}{2}A$$

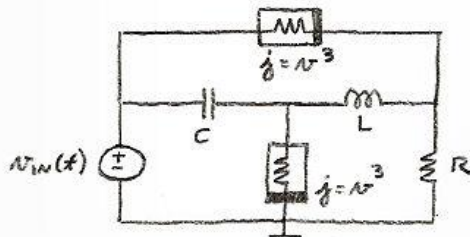
$$i_4 = 3A$$

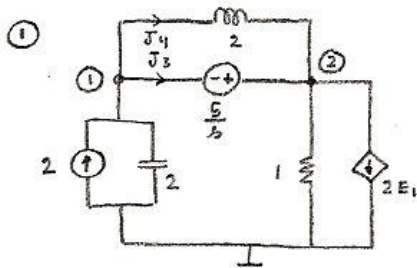
OS RESISTORES NÃO LINEARES SÃO ASSIM:



ESCREVA O SISTEMA QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO, POR QUE TEM QUE SER NODAL MODIFICADO?

- ③ PARA O CIRCUITO ABAIXO, ESCREVA UM SISTEMA NODAL SIMPLES QUE CALCULE A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO EM $t = t_0 + \Delta t$ PELO MÉTODO DE NEWTON-RAPHSON, USANDO O MÉTODO "BACKWARD" DE EULER





$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2\Omega & 0 & 1 & 1 \\ 2 & 2 & 1 & -1 & -1 \\ 3 & 1 & -1 & 0 & 0 \\ 4 & -1 & 1 & 0 & 2\Omega \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ J_3 \\ J_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -5/2 \\ 0 \end{bmatrix}$$

SOMANDO AS DUAS PRIMEIRAS EQUAÇÕES: $(2\Omega + 2)E_1 + E_2 = 2$ E TEM-SE, DA TERCEIRA

$$E_1 - E_2 = -\frac{5}{2} \therefore E_2 = E_1 + \frac{5}{2} \Rightarrow (2\Omega + 2)E_1 + E_1 + \frac{5}{2} = 2 \therefore E_1 = \frac{2 - \frac{5}{2}}{2\Omega + 3} = \frac{\Omega - \frac{5}{2}}{\Omega + \frac{3}{2}}$$

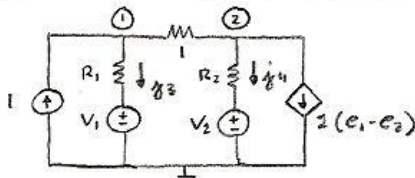
$$E_1 = \frac{-\frac{5}{2}/\frac{3}{2}}{\Omega + \frac{3}{2}} + \frac{-\frac{3}{2} - \frac{5}{2}}{-3/2} = \frac{-\frac{5}{3}}{\Omega + \frac{3}{2}} + \frac{8}{\frac{3}{2}} \Rightarrow e_1(x) = -\frac{5}{3} + \frac{8}{3} e^{-\frac{3}{2}x} \quad (e_1(0) = 1, \text{OK})$$

$$e_2(x) = e_1(x) + 5 = \frac{10}{3} + \frac{8}{3} e^{-\frac{3}{2}x}$$

$$J_4 = -\frac{5}{2\Omega^2} \therefore j_4(x) = -\frac{5}{2}x \quad \text{DA 2ª EQUAÇÃO: } j_3(x) = 2e_1(x) + e_2(x) - j_4(x)$$

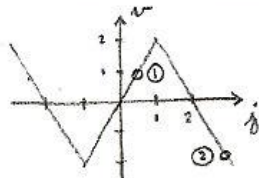
$$j_3(x) = -\frac{10}{3} + \frac{16}{3} e^{-\frac{3}{2}x} + \frac{10}{3} + \frac{8}{3} e^{-\frac{3}{2}x} + \frac{5}{2}x = 8e^{-\frac{3}{2}x} + \frac{5}{2}x \quad (j_3(0) = 8A, \text{OK})$$

2) TEM QUE SER UMA DEVIDO AOS RESISTORES CONTROLADOS A CORRENTE



$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & -1 & 1 & 0 \\ 2 & -1 & 2 & 1 & -2 \\ 3 & -1 & 0 & R_1 & 0 \\ 4 & 0 & -1 & 0 & R_2 \end{bmatrix} \begin{bmatrix} e_{1m+1} \\ e_{2m+1} \\ j_{3m+1} \\ j_{4m+1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -V_1 \\ -V_2 \end{bmatrix}$$

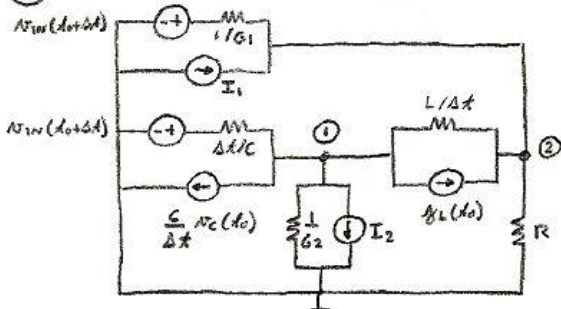
OBSERVANDO OS PONTOS DE OPERAÇÃO DOS RESISTORES



$$R_1 = 2, V_1 = 0$$

$$R_2 = -2, V_2 = 4$$

3) DESLOCA-NDO A FONTE DE TENSÃO:



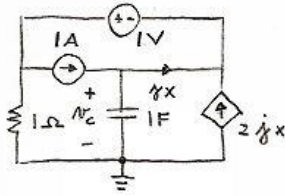
$$\begin{bmatrix} 1 & \frac{C}{\Delta t} + \frac{\Delta t}{L} + G_2 & -\frac{\Delta t}{L} \\ 2 & -\frac{\Delta t}{L} & \frac{1}{R} + \frac{\Delta t}{L} + G_1 \end{bmatrix} \begin{bmatrix} e_{1m+1}(t_0 + \Delta t) \\ e_{2m+1}(t_0 + \Delta t) \end{bmatrix} = \begin{bmatrix} \frac{C}{\Delta t} N_{w}(t_0 + \Delta t) - \frac{C}{\Delta t} N_{c}(t_0) - I_2 - j_L(t_0) \\ G_1 N_{w}(t_0 + \Delta t) + I_1 + j_L(t_0) \end{bmatrix}$$

$$G_1 = 3(N_{w}(t_0 + \Delta t) - e_{2m})^2 \quad I_1 = (N_{w}(t_0 + \Delta t) - e_{2m})^3 - G_1(N_{w}(t_0 + \Delta t) - e_{2m})$$

$$N_c(t_0) = N_w(t_0) - e_1(t_0)$$

CIRCUITOS ELÉTRICOS II - 1º SEMESTRE DE 2011 - 1ª PROVA

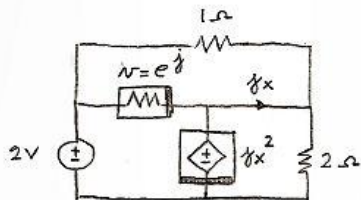
- ① ACHE $N_c(x)$ USANDO UM SISTEMA MODAL SIMPLES EM TRANSFORMADA DE LAPLACE.



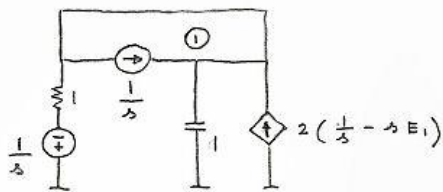
$N_c(0) = 0V$

- ② PARA O MESMO CIRCUITO, ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A SOLUÇÃO EM $t = t_0 + \Delta t$, CONHECIDA A SOLUÇÃO EM $t = t_0$, USANDO O MÉTODO DOS TRAPÉZIOS.

- ③ PARA O CIRCUITO ABAIXO ESCREVA O SISTEMA DAS MALHAS, SIMPLES, QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON



①



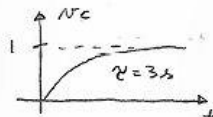
$$[s+1] E_1(s) = \left[-\frac{1}{s} + \frac{2}{s} - 2s E_1(s) \right]$$

$$[3s+1] E_1(s) = \frac{1}{s}$$

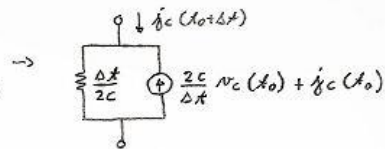
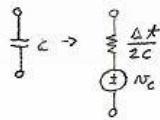
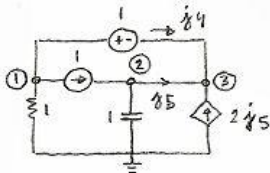
$$E_1(s) = \frac{1}{s(3s+1)} = \frac{\frac{1}{3}}{s^2 + \frac{1}{3}s} = \frac{A}{s} + \frac{B}{s + \frac{1}{3}}$$

$$A = \frac{\frac{1}{3}}{s + \frac{1}{3}} \Big|_{s=0} = 1 \quad B = \frac{\frac{1}{3}}{s} \Big|_{s=-\frac{1}{3}} = -1$$

$$E_1(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{3}} \therefore e_1(t) = 1 - e^{-\frac{1}{3}t}$$



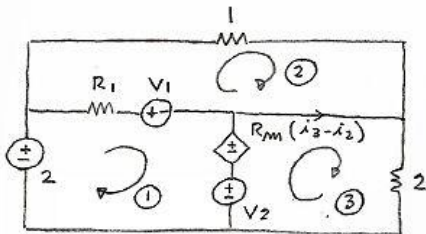
②



$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & \frac{2}{\Delta t} & 0 & 0 \\ 3 & 0 & 0 & -1 & -1 \\ 4 & -1 & 0 & +1 & 0 \\ 5 & 0 & -1 & +1 & 0 \end{bmatrix} \begin{bmatrix} e_1(t_0 + \Delta t) \\ e_2(t_0 + \Delta t) \\ e_3(t_0 + \Delta t) \\ i_4(t_0 + \Delta t) \\ i_5(t_0 + \Delta t) \end{bmatrix} = \begin{bmatrix} -1 \\ +1 + \frac{2}{\Delta t} e_2(t_0) + i_5(t_0) \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$i_5(t_0 + \Delta t)$ DEVE SER CALCULADA COMO
 $i_5(t_0 + \Delta t) = e_2(t_0 + \Delta t) \frac{2}{\Delta t} - \frac{2}{\Delta t} e_2(t_0) - i_5(t_0)$

③



$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & R_1 & -R_1 - R_m & 0 + R_m \\ 2 & -R_1 & R_1 + 1 & 0 \\ 3 & 0 & 0 + R_m & 2 - R_m \end{bmatrix} \begin{bmatrix} i_{1,m+1} \\ i_{2,m+1} \\ i_{3,m+1} \end{bmatrix} = \begin{bmatrix} 2 - V_1 - V_2 \\ V_1 \\ V_2 \end{bmatrix}$$

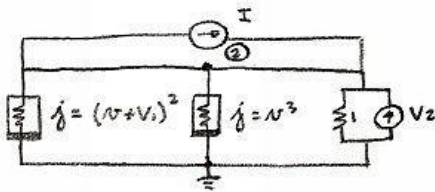
$$R_1 = e^{(i_{1m} - i_{2m})}$$

$$V_1 = e^{(i_{1m} - i_{2m})} - R_1 (i_{1m} - i_{2m})$$

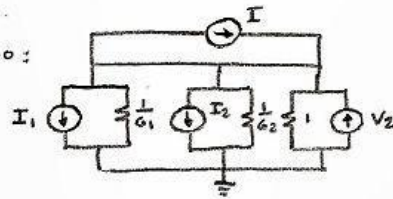
$$R_m = 2 (i_{3m} - i_{2m})$$

$$V_2 = (i_{3m} - i_{2m})^2 - R_m (i_{3m} - i_{2m})$$

DESLOCANDO AS FONTES ANTES DA LINEARIZAÇÃO:



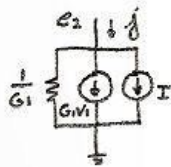
LINEARIZADO:



$$G_1 = 2(e_{2m} + v_1) \quad G_2 = 3e_{2m}^2$$

$$I_1 = (e_{2m} + v_1)^2 - G_1 e_{2m} \quad I_2 = e_{2m}^2 - G_2 e_{2m}$$

NA VERSÃO ANTERIOR, A FONTE DESLOCADA PARA O RESISTOR $j = v^2$ GERAVA:

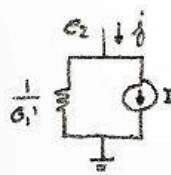


$$j = G_1 e_2 + G_1 v_1 + I_1$$

$$= 2e_{1m} e_2 + 2e_{1m} v_1 + e_{1m}^2 - 2e_{1m}^2$$

$$= 2e_{1m} e_2 + 2e_{1m} v_1 - e_{1m}^2$$

NA NOVA VERSÃO:



$$j = G_1' e_2 + I_1'$$

$$= 2(e_{2m} + v_1) e_2 + (e_{2m} + v_1)^2 - 2(e_{2m} + v_1) e_{2m}$$

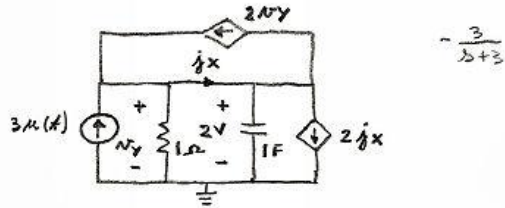
$$= 2e_{1m} e_2 + e_{1m}^2 - 2e_{1m} e_{2m}$$

$$= 2e_{1m} e_2 + e_{1m}^2 - 2e_{1m}(e_{1m} - v_1)$$

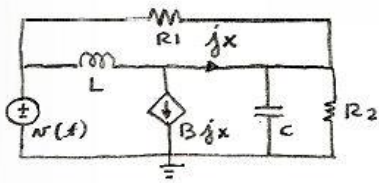
$$= 2e_{1m} e_2 - e_{1m}^2 + 2e_{1m} v_1 \quad \text{O MESMO, COMO TIANA QUE SER.}$$

CIRCUITOS ELÉTRICOS II - 2º SEMESTRE DE 2011 - 1ª PROVA

- ① ACHE $N_0(x)$ USANDO UM SISTEMA NODAL MODIFICADO EM TRANSFORMADA DE LAPLACE

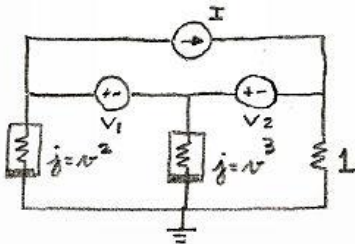


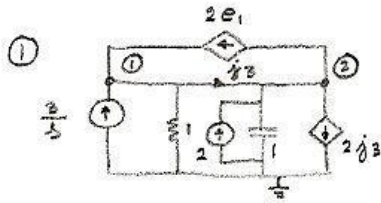
- ② PARA O CIRCUITO ABAIXO ESCREVA O SISTEMA NODAL QUE CALCULA A SOLUÇÃO EM $x = x_0 + \Delta x$, USANDO MODELOS PARA FONTES DE TENSÃO E CURTOS COM AMP. OPS. IDEAIS E O MÉTODO BACKWARD DE EULER, DE FORMA A NÃO AUMENTAR O NÚMERO



DE EQUAÇÕES ALÉM DO NÚMERO DE NÓS

- ③ PARA O CIRCUITO, ESCREVA O SISTEMA QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON. USE UM SISTEMA NODAL SIMPLES.





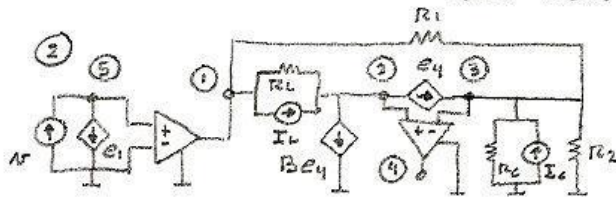
$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1-2 & 0 \\ 2 & 2 & 2 \\ 3 & -1 & +1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} \frac{3}{s} \\ 2 \\ 0 \end{bmatrix}$$

COMO $E_1 = E_2$:

$$\begin{bmatrix} -1 & 1 \\ 2+2 & 1 \end{bmatrix} \begin{bmatrix} E_{1,2} \\ J_3 \end{bmatrix} = \begin{bmatrix} \frac{3}{s} \\ 2 \end{bmatrix}$$

$$E_{1,2} = \frac{\frac{3}{s} - 2}{-1-2-2} = \frac{2 - \frac{3}{s}}{s+3} = \frac{2}{s+3} - \frac{3}{s(s+3)} = \frac{2}{s+3} + \frac{-1}{s} + \frac{+1}{s+3}$$

$$N_c(s) = e_{1,2}(s) = 2e^{-3t} - 1 + e^{-3t} = 3e^{-3t} - 1$$

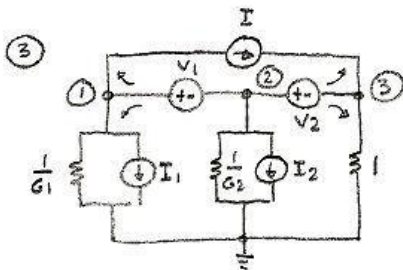


$$R_L = \frac{L}{s} \quad I_L = f_c(t_0)$$

$$R_C = \frac{C}{s} \quad I_C = \frac{C}{s} N_c(t_0)$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ \leftarrow 1 & \frac{1}{R_1} + \frac{1}{R_L} & -\frac{1}{R_L} & -\frac{1}{R_1} & 0 \\ 2 & -\frac{1}{R_L} & \frac{1}{R_L} & 0 & 1+B \\ 3 & -\frac{1}{R_1} & 0 & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_C} & -1 \\ \leftarrow 4 & 0 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1(s) \\ e_2(s) \\ e_3(s) \\ e_4(s) \\ e_5(s) \end{bmatrix} = \begin{bmatrix} -I_L \\ +I_L \\ +I_C \\ 0 \\ N(s) \end{bmatrix}$$

$$\text{REITA:} \quad \begin{bmatrix} -\frac{1}{R_L} & \frac{1}{R_L} & 1+B \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_C} & -1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1(s) \\ e_2(s) \\ e_3(s) \end{bmatrix} = \begin{bmatrix} +I_L \\ +I_C \\ N(s) \end{bmatrix}$$



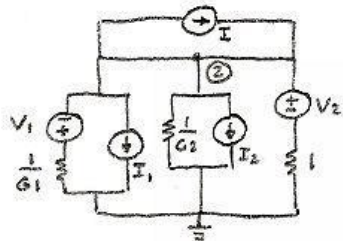
$$G_1 = 2e_{1M}$$

$$I_1 = e_{1M}^2 - G_1 e_{1M}$$

$$G_2 = 3e_{2M}^2$$

$$I_2 = e_{2M}^3 - G_2 e_{2M}$$

DESLOCANDO AS EQUAÇÕES DE TENSÃO:



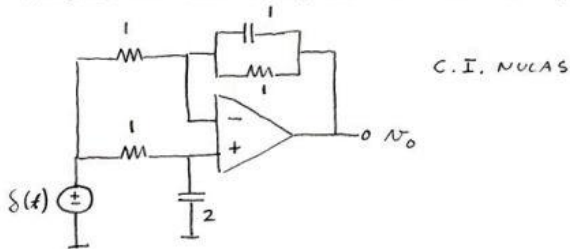
$$\left[G_1 + G_2 + 1 \right] e_{2M+1} = \left[-G_1 V_1 - I_1 - I_2 + V_2 \right]$$

$$e_{1M+1} = e_{2M+1} + V_1$$

$$e_{3M+1} = e_{2M+1} + V_2$$

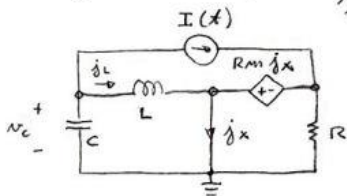
CIRCUITOS ELÉTRICOS II - 1º SEMESTRE DE 2012 - PRIMEIRA PROVA

- 1) RESOLVA O CIRCUITO ABAIXO, ACHANDO $v_o(t)$, USANDO UMA ANÁLISE NODAL COM MÃO MAIS QUE DUAS EQUAÇÕES, EM TRANSFORMADA DE LAPLACE.



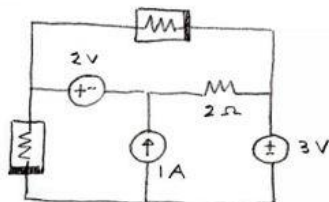
- 2) ESCRIBA O SISTEMA NODAL MODIFICADO QUE CALCULA A SOLUÇÃO EM $t = t_0 + \Delta t$ PARA O CIRCUITO, USANDO O MISTURO DE GRUPO GEMÉRICO:

$$y(t_0 + \Delta t) = y(t_0) + \int_{t_0}^{t_0 + \Delta t} x(t) dt \rightarrow y(t_0 + \Delta t) \approx a_0 y(t_0) + a_1 y(t_0 - \Delta t) + \dots + b \Delta t x(t_0 + \Delta t)$$

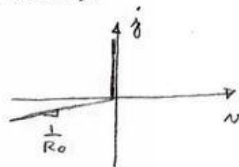


SÃO CONHECIDAS AS SOLUÇÕES ANTERIORES
E DADOS OS COEFICIENTES $a_0 \dots b$

- 3) PARA O CIRCUITO ABAIXO ESCRIBA O SISTEMA DE MALHAS QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON



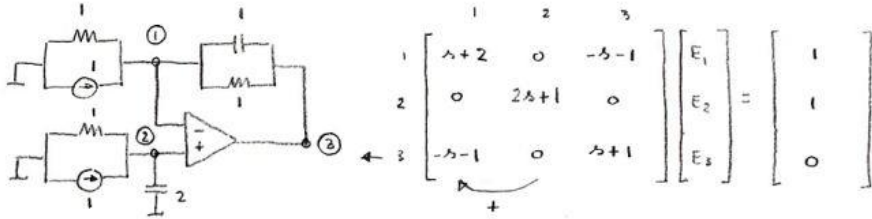
OS RESISTORES NÃO LINEARES
SÃO DIODOS:



SE $j > 0$, $v = 0$

SE $j \leq 0$, $v = R_0 j$

①



$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & s+2 & 0 & -s-1 \\ 2 & 0 & 2s+1 & 0 \\ 3 & -s-1 & 0 & s+1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} s+2 & -s-1 \\ 2s+1 & 0 \end{bmatrix} \begin{bmatrix} E_{12} \\ E_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$v_o(t) = -2e^{-t} + 1.5e^{-0.5t}$$

$$E_3 = \frac{s+2-2s-1}{2s^2+2s+s+1} = \frac{-s+1}{2s^2+3s+1} = \frac{-\frac{1}{2}(s-1)}{s^2+\frac{3}{2}s+\frac{1}{2}}$$

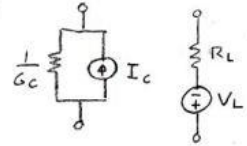
$$= \frac{-\frac{1}{2}(s-1)}{(s+1)(s+\frac{1}{2})} = \frac{-\frac{1}{2}(-1-1)}{s+1} + \frac{-\frac{1}{2}(-0.5-1)}{s+\frac{1}{2}} = \frac{-2}{s+1} + \frac{1.5}{s+0.5}$$

②

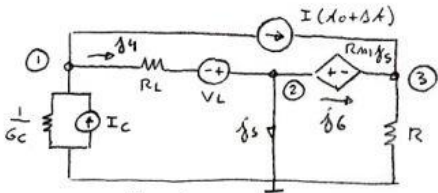
CAPACITOR: $v(t_0+\Delta t) = a_0 v(t_0) + a_1 v(t_0-\Delta t) + \dots + \frac{1}{C} \int_{t_0-\Delta t}^{t_0+\Delta t} i(t) dt$

$$i(t_0+\Delta t) = \frac{C}{t-\Delta t} (v(t_0+\Delta t) - a_0 v(t_0) - a_1 v(t_0-\Delta t) - \dots)$$

INDUTOR: $v(t_0+\Delta t) = \frac{L}{t-\Delta t} (i(t_0+\Delta t) - a_0 i(t_0) - a_1 i(t_0-\Delta t) - \dots)$



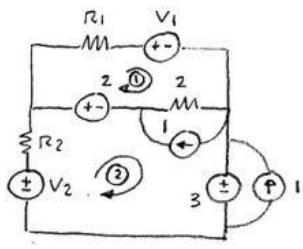
$$G_C = \frac{C}{t-\Delta t}; I_C = \frac{C}{t-\Delta t} (a_0 v(t_0) + a_1 v(t_0-\Delta t) + \dots); R_L = \frac{L}{t-\Delta t}; V_L = \frac{L}{t-\Delta t} (a_0 i(t_0) + a_1 i(t_0-\Delta t) + \dots)$$



$$v_2 = e_1; i_L = i_4$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & G_C & 0 & 0 & +1 & 0 & 0 \\ 2 & 0 & 0 & 0 & -1 & +1 & +1 \\ 3 & 0 & 0 & \frac{1}{R} & 0 & 0 & -1 \\ 4 & -1 & +1 & 0 & R_L & 0 & 0 \\ 5 & 0 & -1 & 0 & 0 & 0 & 0 \\ 6 & 0 & -1 & +1 & 0 & +R_M & 0 \end{bmatrix} \begin{bmatrix} e_1(t_0+\Delta t) \\ e_2(t_0+\Delta t) \\ e_3(t_0+\Delta t) \\ i_4(t_0+\Delta t) \\ i_5(t_0+\Delta t) \\ i_6(t_0+\Delta t) \end{bmatrix} = \begin{bmatrix} I_C - I(t_0+\Delta t) \\ 0 \\ +I(t_0+\Delta t) \\ +V_L \\ 0 \\ 0 \end{bmatrix}$$

③



$$\begin{bmatrix} 1 & 2 \\ 1 & R_1+2 & -2 \\ 2 & -2 & R_2+2 \end{bmatrix} \begin{bmatrix} i_{1m+1} \\ i_{2m+1} \end{bmatrix} = \begin{bmatrix} 2+2-V_1 \\ -2-2-3+V_2 \end{bmatrix}$$

DIODOS:

SE $i_{1m} > 0$: $R_1 = 0, V_1 = 0$

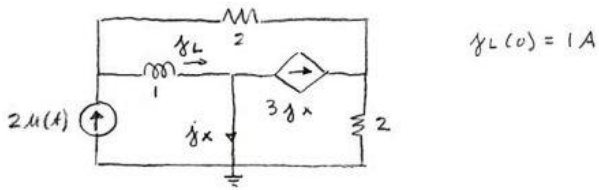
SE $i_{1m} \leq 0$: $R_1 = R_0, V_1 = 0$

SE $i_{2m} > 0$: $R_2 = R_0, V_2 = 0$

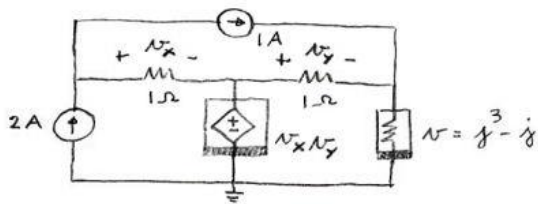
SE $i_{2m} \leq 0$: $R_2 = 0, V_2 = 0$

CIRCUITOS ELÉTRICOS II - 2º SEMESTRE DE 2012 - 1ª PROVA

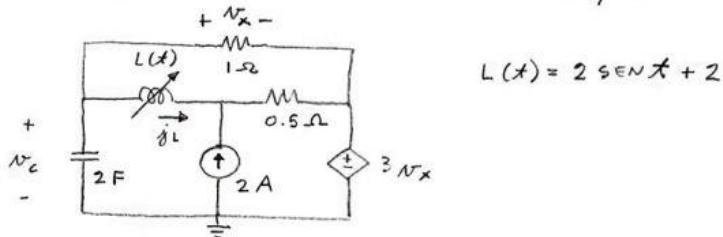
- ① PARA O CIRCUITO ABAIXO, CALCULE $i_L(t)$ USANDO UM SISTEMA NODAL SIMPLES EM TRANSFORMADA DE LAPLACE

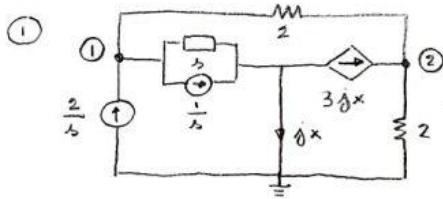


- ② PARA O CIRCUITO ABAIXO ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON



- ③ ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A SOLUÇÃO EM $t = t_0 + \Delta t$ USANDO O MÉTODO BACKWARD DE EULER, CONHECIDA A SOLUÇÃO EM $t = t_0$





MAIS SIMPLES SEMAR CORRENTES NO NÓ DE TERRA
PARA ACHAR j_x

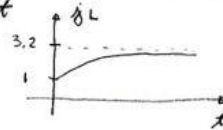
$$j_x = \frac{2}{s} - \frac{E_2}{2} \quad \therefore 3j_x = \frac{6}{s} - \frac{3}{2} E_2$$

$$\begin{bmatrix} \frac{1}{2} + \frac{1}{s} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} + \frac{1}{2} + \frac{3}{2} = \frac{5}{2} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{s} \\ \frac{6}{s} - \frac{3}{2} E_2 \end{bmatrix}$$

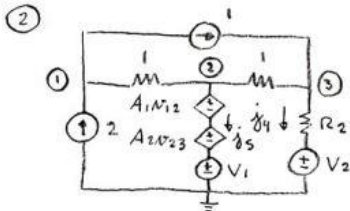
$$E_1 = \frac{\frac{1}{s} \frac{5}{2} + \frac{6}{s} \frac{1}{2}}{\frac{1}{2} \frac{5}{2} + \frac{1}{s} \frac{5}{2} - \frac{1}{4}} = \frac{\frac{1}{s} \frac{11}{2}}{1 + \frac{1}{s} \frac{5}{2}} = \frac{\frac{11}{2}}{s + \frac{5}{2}}$$

$$j_L = \frac{E_1}{s} + \frac{1}{s} = \frac{\frac{11}{2}}{s(s + \frac{5}{2})} + \frac{1}{s} = \frac{\frac{11}{2} \frac{2}{s} - \frac{11}{2} \frac{2}{s + \frac{5}{2}}}{s} + \frac{1}{s}$$

$$j_L = \frac{\frac{11}{s} + 1}{s} - \frac{\frac{11}{s}}{s + \frac{5}{2}} \quad \therefore j_L(x) = \frac{16}{s} - \frac{11}{5} e^{-\frac{5}{2}x}$$



$$j_L(x) = \frac{16}{s} - \frac{11}{s + \frac{5}{2}}$$



$$A_1 = e_{2m} - e_{3m}$$

$$A_2 = e_{1m} - e_{2m}$$

$$V_1 = (e_{1m} - e_{2m})(e_{2m} - e_{3m}) - (e_{2m} - e_{3m})(e_{1m} - e_{2m}) - (e_{1m} - e_{3m})(e_{2m} - e_{3m}) = -(e_{1m} - e_{2m})(e_{2m} - e_{3m})$$

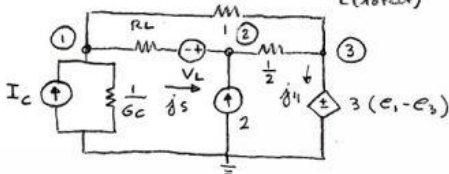
$$R_2 = 3j_{4m}^2 - 1 \quad V_2 = j_{4m}^3 - j_{4m} - (3j_{4m}^2 - 1)j_{4m} = -2j_{4m}^3$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 2 & -1 & 2 & -1 & 0 & 1 \\ 3 & 0 & -1 & 1 & 1 & 0 \\ 4 & 0 & 0 & -1 & R_2 & 0 \\ 5 & A_1 & -A_1 + A_2 & -A_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{1m+1} \\ e_{2m+1} \\ e_{3m+1} \\ j_{4m+1} \\ j_{5m+1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -V_2 \\ -V_1 \end{bmatrix}$$

$$e_3 = R_2 j_4 + V_2$$

$$e_2 = A_1(e_1 - e_2) + A_2(e_2 - e_3) + V_1$$

$$j_L(x_0 + \Delta x) \approx j_L(x_0) \frac{L(x_0)}{L(x_0 + \Delta x)} + \frac{\Delta x}{L(x_0 + \Delta x)} v(x_0 + \Delta x) \quad \therefore v(x_0 + \Delta x) \approx \frac{L(x_0 + \Delta x)}{\Delta x} j_L(x_0 + \Delta x) - \frac{L(x_0)}{\Delta x} j_L(x_0)$$



$$R_L = \frac{L(x_0 + \Delta x)}{\Delta x} = \frac{s \epsilon N(x_0 + \Delta x) + 2}{\Delta x}$$

$$G_C = \frac{C}{\Delta x} = \frac{2}{\Delta x}$$

$$V_L = \frac{L(x_0)}{\Delta x} j_L(x_0) = \frac{s \epsilon N(x_0) + 2}{\Delta x} j_L(x_0)$$

$$I_C = \frac{C}{\Delta x} e_1(x_0) = \frac{2}{\Delta x} e_1(x_0)$$

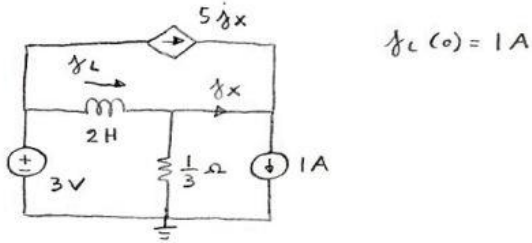
$$\begin{bmatrix} G_C + 1 & 0 & -1 & 0 & 1 \\ 2 & 0 & 2 & -2 & 0 & -1 \\ 3 & -1 & -2 & 3 & 1 & 0 \\ 4 & 3 & 0 & -4 & 0 & 0 \\ 5 & -1 & +1 & 0 & 0 & R_L \end{bmatrix} \begin{bmatrix} e_1(x_0 + \Delta x) \\ e_2(x_0 + \Delta x) \\ e_3(x_0 + \Delta x) \\ j_4(x_0 + \Delta x) \\ j_5(x_0 + \Delta x) \end{bmatrix} = \begin{bmatrix} I_C \\ 2 \\ 0 \\ 0 \\ V_L \end{bmatrix}$$

$$e_3 = 3(e_1 - e_2)$$

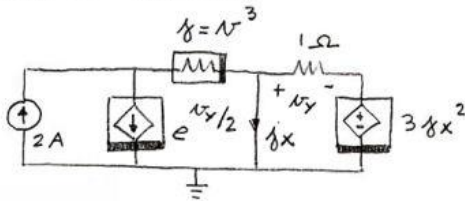
$$e_1 - e_2 = R_L j_5 - V_L$$

CIRCUITOS ELÉTRICOS II - 1º SEMESTRE DE 2013 - 1ª PROVA

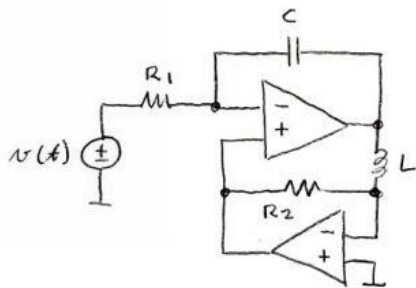
- ① ACHÉ A SOLUÇÃO PARA $i_L(t)$ USANDO UMA ANÁLISE NODAL SIMPLES EM TRANSFORMADA DE LAPLACE

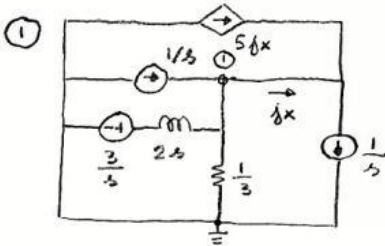


- ② PARA O CIRCUITO ABAIXO, ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON



- ③ ESCREVA O SISTEMA NODAL, ELIMINANDO OS AMPLIFICADORES OPERACIONAIS, QUE CALCULA A SOLUÇÃO EM $t = t_0 + \Delta t$ USANDO O MÉTODO BACKWARD DE EULER, CONHECIDA A SOLUÇÃO EM $t = t_0$





$$jx + 5jx = \frac{1}{s}$$

$$6jx = \frac{1}{s} \therefore jx = \frac{1}{6s}$$

$$5jx = \frac{5}{6} \frac{1}{s} \text{ UMA FONTE CONSTANTE}$$

$$\textcircled{1} \left(3 + \frac{1}{2s}\right) E_1 = \frac{1}{s} - \frac{1}{s} + \frac{5}{6} \frac{1}{s} + \frac{3}{2s^2}$$

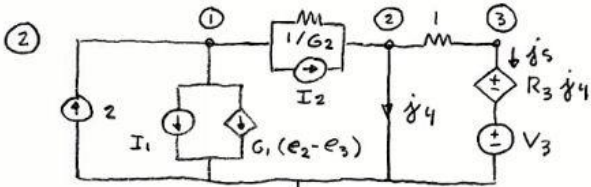
$$E_1 = \frac{\frac{5}{6} \frac{1}{s} + \frac{3}{2s^2}}{3 + \frac{1}{2s}} = \frac{\frac{5}{3} + \frac{3}{s}}{6s + 1} = \frac{\frac{5}{18} + \frac{1}{2s}}{s + 1/6} = \frac{\frac{5}{18} s + \frac{1}{2}}{s(s + 1/6)} = \frac{5}{18} \frac{s + 1.8}{s(s + 1/6)} \quad (\text{OK})$$

$$E_1 = \frac{A}{s} + \frac{B}{s + 1/6} \quad A = \frac{1}{6} = 3 \quad B = \frac{\frac{5}{18}(-1/6) + \frac{1}{2}}{-1/6} = \frac{-\frac{5}{18} + 3}{-1} = \frac{5}{18} - 3 = -\frac{49}{18} = -2.722$$

$$e_1(x) = 3 - 2.722 e^{-\frac{1}{6}x}$$

$$j_L(x) = 3e_1(x) + jx = 3e_1(x) + \frac{1}{6} = 9 - \frac{49}{6} e^{-\frac{1}{6}x} + \frac{1}{6} = \frac{55}{6} - \frac{49}{6} e^{-\frac{1}{6}x}$$

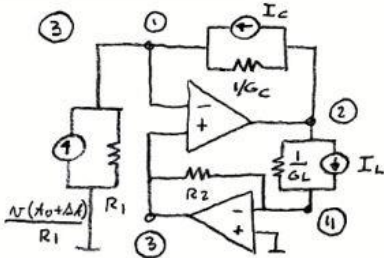
$$\text{VERIFICANDO: } j_L(0) = \frac{55}{6} - \frac{49}{6} = 1, \text{ OK} \quad j_L(\infty) = \frac{55}{6} = 3 \times 3 + \frac{1}{6}, \text{ OK}$$



$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & G_2 - G_2 + G_1 & -G_1 & 0 & 0 \\ 2 & -G_2 & G_2 + 1 & -1 & +1 & 0 \\ 3 & 0 & -1 & 1 & 0 & +1 \\ 4 & 0 & -1 & 0 & 1 & 0 \\ 5 & 0 & 0 & -1 & R_3 & 0 \end{bmatrix} \begin{bmatrix} e_{1,m+1} \\ e_{2,m+1} \\ e_{3,m+1} \\ j^4_{m+1} \\ j^5_{m+1} \end{bmatrix} = \begin{bmatrix} 2 - I_1 - I_2 \\ I_2 \\ 0 \\ 0 \\ -V_3 \end{bmatrix}$$

$$G_1 = \frac{1}{2} e^{(e_{2m} - e_{3m})/2} \quad I_1 = e^{(e_{2m} - e_{3m})/2} - G_1(e_{2m} - e_{3m}) \quad R_3 = 6 j^4_{m+1}$$

$$G_2 = 3(e_{1m} - e_{2m})^2 \quad I_2 = (e_{1m} - e_{2m})^3 - G_2(e_{1m} - e_{2m}) \quad V_3 = 3 j^4_{m+1}^2 - R_3 j^4_{m+1}$$



$$G_c = \frac{C}{\Delta x} \quad I_c = \frac{C}{\Delta x} (e_1(x_0) - e_2(x_0))$$

$$G_L = \frac{\Delta x}{L} \quad I_L = j_L(x_0) \text{ (CALCULADO AO FIM DA ANÁLISE)}$$

SISTEMA SEM OS AMP. OPS.:

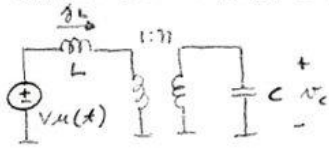
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & \frac{1}{R_1} + G_c & -G_c & 0 & 0 \\ 2 & -G_c & G_c + G_L & 0 & -G_L \\ 3 & 0 & 0 & \frac{1}{R_2} & -\frac{1}{R_2} \\ 4 & 0 & -G_L & -\frac{1}{R_2} & \frac{1}{R_2} + G_L \end{bmatrix} \begin{bmatrix} e_1(x_0 + \Delta x) \\ e_2(x_0 + \Delta x) \\ e_3(x_0 + \Delta x) \\ e_4(x_0 + \Delta x) \end{bmatrix} = \begin{bmatrix} \frac{N(x_0 + \Delta x)}{R_1} + I_c \\ -I_c - I_L \\ 0 \\ +I_L \end{bmatrix}$$

SISTEMA RESTANTE:

$$\begin{bmatrix} 1 & 2 \\ 1 & \frac{1}{R_1} + G_c & -G_c \\ 2 & -\frac{1}{R_2} & -G_L \end{bmatrix} \begin{bmatrix} e_{1,3}(x_0 + \Delta x) \\ e_2(x_0 + \Delta x) \end{bmatrix} = \begin{bmatrix} \frac{N(x_0 + \Delta x)}{R_1} + I_c \\ +I_L \end{bmatrix}$$

CIRCUITOS ELÉTRICOS II - 2ª SEMESTRE DE 2013 - 1ª PROVA

- ① PARA O CIRCUITO, CALCULE $i_C(x)$ E $i_L(x)$ USANDO UMA ANÁLISE MODAL MODIFICADA EM TRANSFORMADA DE LAPLACE. PROCURE USAR APENAS DUAS EQUAÇÕES.



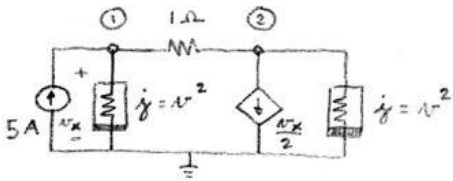
$$i_C(0) = 0$$

PROTE AS RESPOSTAS

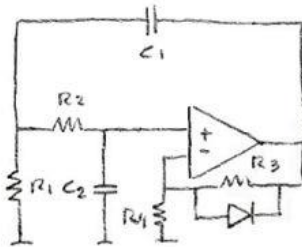
$$i_L(0) = 0$$

UM TRANSFORMADOR NÃO IDEAL MUDARIA O QUE? (NÃO REANALISE)

- ② PARA O CIRCUITO NÃO LINEAR ABAIXO, CALCULE A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO USANDO O MÉTODO DE NEWTON-RAPHSON E UMA ANÁLISE MODAL. A SOLUÇÃO ATUAL VALE $\vec{e}_{n1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

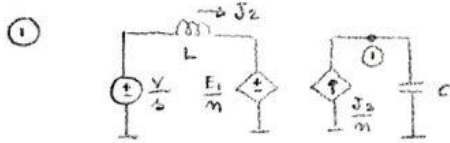


- ③ ESCREVA O SISTEMA MODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO DO CIRCUITO EM $x = x_0 + \Delta x$, USANDO O MÉTODO DE NEWTON-RAPHSON E A INSCRIÇÃO "BACKWARD" DE EVER. A CORRENTE DE SAÍDA DO AMP. OP. DEVE SER CALCULADA.



DIODO: $i = I_s (e^{v/V_T} - 1)$

O CIRCUITO É UM OSCILADOR SENOIDAL COM O DIODO USADO PARA LIMITAR A AMPLITUDE.



BASEIA UMA CORRENTE, JA QUE N'A MESMA NOS 2
RAMOS DA ESQUERDA
A EQUAÇÃO CORRESPONDENTE É $\frac{E_1}{m} + sLJ_2 = \frac{V}{s}$

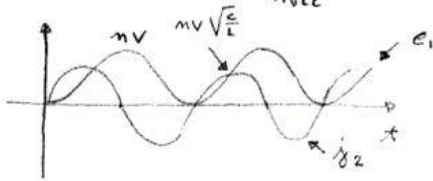
$$\begin{bmatrix} sC & 1 & -\frac{1}{m} \\ -\frac{1}{m} & 1 & sL \\ \frac{1}{m} & 1 & sL \end{bmatrix} \begin{bmatrix} E_1 \\ J_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{V}{s} \end{bmatrix}$$

$$E_1 = \frac{V}{ms} = \frac{V}{mLC} = \frac{mV}{s} - \frac{mVs}{s^2 + \frac{1}{m^2LC}}$$

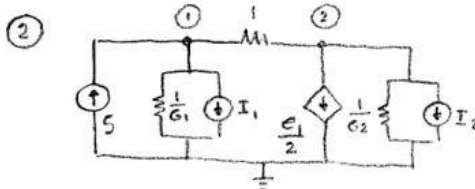
$$J_2 = \frac{CV}{s^2LC + \frac{1}{m^2}} = \frac{V}{s^2 + \frac{1}{m^2LC}} = \frac{mV\sqrt{\frac{C}{L}}}{s^2 + \frac{1}{m^2LC}}$$

$$e_1(t) = mV \left(1 - \cos \frac{1}{m\sqrt{LC}} t \right)$$

$$j_2(t) = mV \sqrt{\frac{C}{L}} \operatorname{sen} \frac{1}{m\sqrt{LC}} t$$



SE O TRANSFORMADOR FOR REAL SURGE UMA
RAMPA SEMELHANTE A j_2 . e_1 CONTINUA COM
NÍVEL CC, POIS j_2 AUMENTA CONTINUAMENTE



$$G_1 = 2e_{1m} = 2$$

$$I_1 = e_{1m}^2 - G_1 e_{1m} = 1 - 2 = -1$$

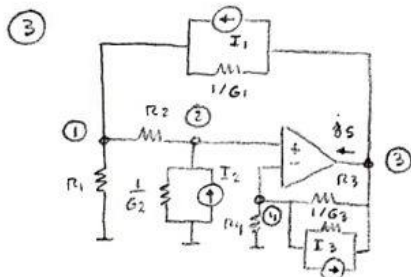
$$G_2 = 2e_{2m} = 2$$

$$I_2 = e_{2m}^2 - G_2 e_{2m} = -1$$

$$\begin{bmatrix} 3 & -1 \\ -1+0.5 & 3 \\ & -0.5 \end{bmatrix} \begin{bmatrix} e_{1m+1} \\ e_{2m+1} \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

$$e_{1m+1} = \frac{6 \times 3 + 1}{3 \times 3 - 0.5} = \frac{19}{8.5} = 2.24$$

$$e_{2m+1} = \frac{3 \times 1 + 0.5 \times 6}{3 \times 3 - 0.5 \times 1} = \frac{6}{8.5} = 0.701$$



$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & \frac{1}{R_1} + \frac{1}{R_2} + G_1 & -\frac{1}{R_2} & -G_1 & 0 & 0 \\ 2 & -\frac{1}{R_2} & \frac{1}{R_2} + G_2 & 0 & 0 & 0 \\ 3 & -G_1 & 0 & \frac{1}{R_3} + G_1 + G_3 & -\frac{1}{R_2} - G_3 & +1 \\ 4 & 0 & 0 & -\frac{1}{R_3} - G_3 & \frac{1}{R_3} + \frac{1}{R_4} + G_3 & 0 \\ 5 & 0 & -1 & 0 & +1 & 0 \end{bmatrix} \begin{bmatrix} e_{1m+1}(t_0 + \Delta t) \\ e_{2m+1}(t_0 + \Delta t) \\ e_{3m+1}(t_0 + \Delta t) \\ e_{4m+1}(t_0 + \Delta t) \\ e_{5m+1}(t_0 + \Delta t) \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ -I_1 + I_3 \\ -I_3 \\ 0 \end{bmatrix}$$

$$G_1 = \frac{\Delta x}{C_1} \quad I_1 = \frac{C_1}{\Delta x} (e_1(t_0) - e_3(t_0))$$

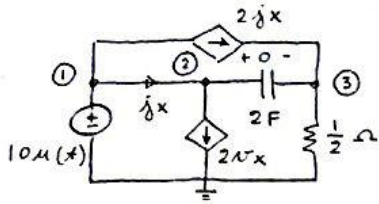
$$G_2 = \frac{\Delta x}{C_2} \quad I_2 = \frac{C_2}{\Delta x} e_2(t_0)$$

$$G_3 = \frac{I_5}{V_T} e^{(e_{1m}(t_0 + \Delta t) - e_{3m}(t_0 + \Delta t)) / V_T}$$

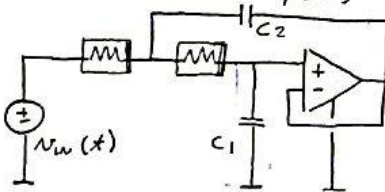
$$I_3 = I_5 \left(e^{(e_{1m}(t_0 + \Delta t) - e_{3m}(t_0 + \Delta t)) / V_T} - 1 \right) - G_3 (e_{4m}(t_0 + \Delta t) - e_{2m}(t_0 + \Delta t))$$

CIRCUITOS ELÉTRICOS II - 1º SEMESTRE DE 2011 - 1ª PROVA

- ① PARA O CIRCUITO ABAIXO, ENCONTRE AS TENSÕES MODAIS $\vec{e}(t)$ USANDO UMA ANÁLISE MODAL SIMPLES EM TRANSFORMADA DE LAPLACE.

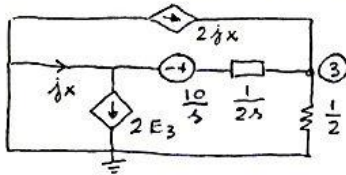


- ② PARA O MESMO CIRCUITO, ESCREVA UM SISTEMA MODAL MODIFICADO NO ESTADO PERMANENTE SENOIDAL, COM A FONTE DE TENSÃO SENDO $10 \text{ SEN } 2t$. ESTE SISTEMA CALCULA A SOLUÇÃO CORRETA? POR QUÊ?
- ③ PARA O CIRCUITO ABAIXO, ESCREVA UM SISTEMA MODAL QUE CALCULE A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO EM $t = t_0 + \Delta t$, USANDO A APROXIMAÇÃO "BACKWARD" DE EULER E O MÉTODO DE NEWTON-RAPHSON, DE FORMA QUE SEJAM DUAS EQUAÇÕES



RESISTORES : $j = K\omega^3$

1) DESLOCANDO A FONTE DE TENSÃO E O CURTO



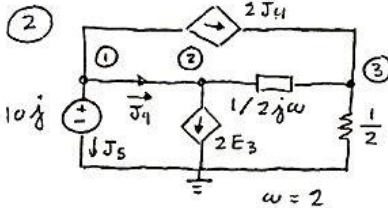
$jx = 2E_3 + \left(\frac{10}{3} - E_3\right) 2A = 2E_3 + 20 - 2A E_3$
 RESOLVA UMA EQUAÇÃO

TAMBÉM PODE:
 $jx + 2jx = 4E_3$

$(2A + 2) E_3 = 20 + 4E_3 + 40 - 4A E_3$

$E_3(A) = \frac{10}{3 - \frac{1}{3}} \therefore e_3(t) = 10 e^{\frac{t}{3}} u(t)$

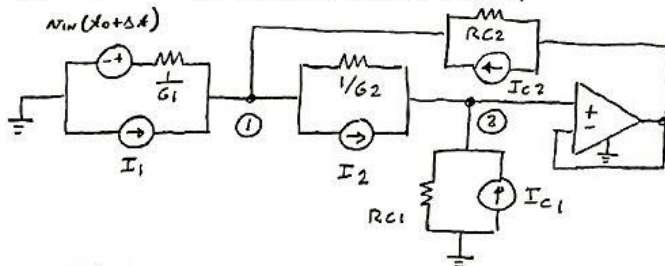
$e_1(t) = e_2(t) = 10 u(t)$



$$\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \begin{bmatrix} 0 & 0 & 0 & 1+2 & 1 \\ 0 & 4j & -4j+2 & -1 & 0 \\ 0 & -4j & 4j+2 & +2 & 0 \\ -1 & +1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ J_4 \\ J_5 \end{bmatrix} & = & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 10j \end{bmatrix} \end{matrix}$$

A SOLUÇÃO NÃO INCLUI O TRANSIENTE INSTÁVEL QUE O CIRCUITO GERA

3) ESCRIVENDO O SISTEMA SEM O AMP. OP., COM A FONTE DESLOCADA PARA O MODELO LINEARIZADO DO RESISTOR LIGADO A ELA:



$G_1 = 3k (v_{in}(t+\Delta t) - e_{1m}(t+\Delta t))^2$
 $I_1 = K (v_{in}(t+\Delta t) - e_{1m}(t+\Delta t))^3 - G_1 (v_{in}(t+\Delta t) - e_{1m}(t+\Delta t))$
 $G_2 = 3k (e_{1m}(t+\Delta t) - e_{2m}(t+\Delta t))^2$
 $I_2 = K (e_{1m}(t+\Delta t) - e_{2m}(t+\Delta t))^3 - G_2 (e_{1m}(t+\Delta t) - e_{2m}(t+\Delta t))$

$R_{C1} = \frac{\Delta x}{C1} \quad I_{C1} = \frac{C1}{\Delta x} e_2(t_0) \quad R_{C2} = \frac{\Delta x}{C2} \quad I_{C2} = \frac{C2}{\Delta x} (e_1(t_0) - e_3(t_0))$

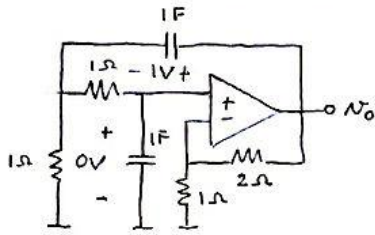
$$\begin{matrix} 1 & 2 & 3 \\ \begin{bmatrix} G_1+G_2+\frac{1}{RC2} & -G_2 & -\frac{1}{RC2} \\ -G_2 & G_2+\frac{1}{RC1} & 0 \\ -\frac{1}{RC2} & 0 & \frac{1}{RC2} \end{bmatrix} & \begin{bmatrix} e_{1m+1}(t+\Delta t) \\ e_2 \\ e_3 \end{bmatrix} & = & \begin{bmatrix} G_1 v_{in}(t+\Delta t) + I_1 - I_2 + I_{C2} \\ I_2 + I_{C1} \\ -I_{C2} \end{bmatrix} \end{matrix}$$

REDUZINDO O SISTEMA:

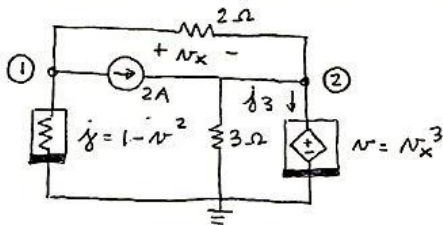
$$\begin{matrix} 1 & 2 \\ \begin{bmatrix} G_1+G_2+\frac{1}{RC2} & -G_2-\frac{1}{RC2} \\ -G_2 & G_2+\frac{1}{RC1} \end{bmatrix} & \begin{bmatrix} e_{1m+1}(t+\Delta t) \\ e_{2,3} \end{bmatrix} & = & \begin{bmatrix} G_1 v_{in}(t+\Delta t) + I_1 - I_2 + I_{C2} \\ I_2 + I_{C1} \end{bmatrix} \end{matrix}$$

CIRCUITOS ELÉTRICOS II - 2º SEMESTRE DE 2014 - PRIMEIRA PROVA

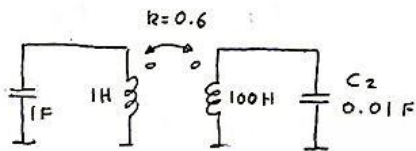
- ① USE UMA ANÁLISE NODAL COM ELIMINAÇÃO DO AMP. OP. EM TRANSFORMADA DE LAPLACE PARA ACHAR $N_0(s)$



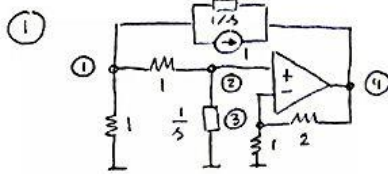
- ② PARA O CIRCUITO ABAIXO, ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON. A ÚLTIMA SOLUÇÃO OBTIDA FOI $e_1 = 1V$, $e_2 = 2V$, $i_3 = 1A$



- ③ ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A SOLUÇÃO EM $t = t_0 + \Delta t$ USANDO O MÉTODO "BACKWARD" DE EULER, DADA A SOLUÇÃO EM $t = t_0$



O QUE MUDA SE C_2 VARIAR NO TEMPO, $C_2(t) = C_0 + a t$?



SISTEMA SEJA O AMP. OP.:

$$\begin{matrix} 1 & 2 & 3 & 4 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} s+2 & -1 & 0 & -s \\ -1 & s+1 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & -\frac{1}{2} \\ -s & 0 & -\frac{1}{2} & s+\frac{1}{2} \end{bmatrix} & \begin{bmatrix} E_1(s) \\ E_2(s) \\ E_3(s) \\ E_4(s) \end{bmatrix} & = & \begin{bmatrix} -1 \\ 0 \\ 0 \\ +1 \end{bmatrix} \end{matrix}$$

SISTEMA REDUZIDO:

$$\begin{bmatrix} s+2 & -1 & -s \\ -1 & s+1 & 0 \\ 0 & \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} E_1(s) \\ E_2(s) \\ E_4(s) \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$E_4(s) = \frac{\frac{3}{2}}{(s^2+3s+2)(-\frac{1}{2}) + \frac{3}{2}s + \frac{1}{2}} = \frac{-3}{s^2+1}$$

$$V_0(t) = -3 \text{ SEN } t$$

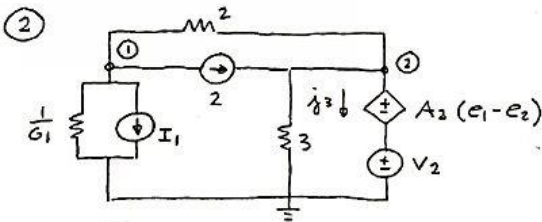
VERIFICAÇÃO SIMPLER: $V_0(0) = 0$, DEVIDO AO CAPACITOR DESCARREGADO NO NÓ 2

$$G_1 = -2e_{1M} = -2$$

$$I_1 = 1 - e_{1M}^2 + 2e_{1M} = 2$$

$$A_2 = 3(e_{1M} - e_{2M})^2 = 3$$

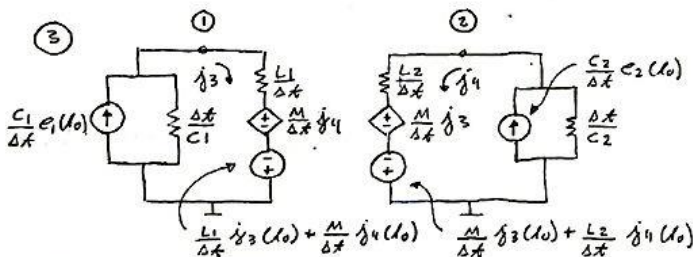
$$V_2 = (e_{1M} - e_{2M})^3 - 3(e_{1M} - e_{2M}) = -1 - 3(-1) = 2$$



SISTEMA:

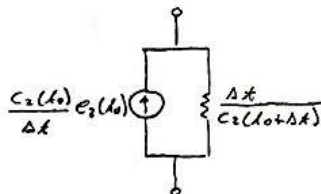
$$\begin{matrix} 1 & 2 & 3 \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} \frac{1}{2} & -2 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} + \frac{1}{3} & 1 \\ 3 & -3 & -1 \end{bmatrix} & \begin{bmatrix} e_{1M+1} \\ e_{2M+1} \\ e_{3M+1} \end{bmatrix} & = & \begin{bmatrix} -2-2 \\ +2 \\ -2 \end{bmatrix} \end{matrix}$$

$$e_{2M+1} = A_2(e_{1M+1} - e_{2M+1}) + V_2$$



$$M = k\sqrt{L_1 L_2} = 6$$

SE C_2 VARIAR NO TEMPO, SEU MODELO PASSA A SER



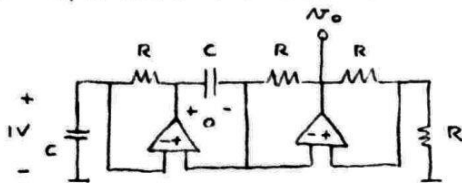
O TERMO * PASSA A SER $\frac{C_0 + a(t_0 + \Delta t)}{\Delta t}$

O TERMO ** PASSA A SER $\frac{C_0 + a t_0}{\Delta t} e_2(t_0)$

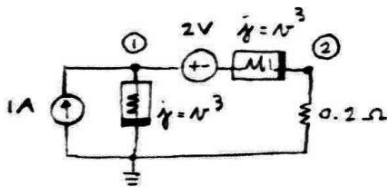
$$\begin{matrix} 1 & 2 & 3 & 4 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} \frac{1}{\Delta t} & 0 & 1 & 0 \\ 0 & \frac{1}{100\Delta t} & 0 & 1 \\ -1 & 0 & \frac{1}{\Delta t} & \frac{6}{\Delta t} \\ 0 & -1 & \frac{6}{\Delta t} & \frac{100}{\Delta t} \end{bmatrix} & \begin{bmatrix} e_1(t_0 + \Delta t) \\ e_2(t_0 + \Delta t) \\ e_3(t_0 + \Delta t) \\ e_4(t_0 + \Delta t) \end{bmatrix} & = & \begin{bmatrix} \frac{1}{\Delta t} e_1(t_0) \\ \frac{1}{100\Delta t} e_2(t_0) ** \\ \frac{1}{\Delta t} e_3(t_0) + \frac{6}{\Delta t} e_4(t_0) \\ \frac{6}{\Delta t} e_3(t_0) + \frac{100}{\Delta t} e_4(t_0) \end{bmatrix} \end{matrix}$$

CIRCUITOS ELÉTRICOS II - 1º SEMESTRE DE 2015 - PRIMEIRA PROVA

- ① ENCONTRE $N_0(A)$, USANDO UMA ANÁLISE NODAL EM TRANSFORMADA DE LAPLACE COM ELIMINAÇÃO DOS AMPLIFICADORES OPERACIONAIS



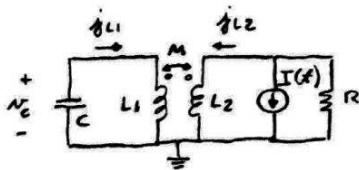
- ② PARA O CIRCUITO RESISTIVO NÃO LINAR ABAIXO, CALCULE A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON



$E_{1M} = 2V$
 $E_{2M} = 0V$

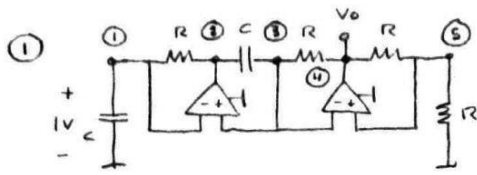
FORNEÇA COM APENAS DUAS EQUAÇÕES

- ③ ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A SOLUÇÃO EM $x = x_0 + \Delta x$ USANDO O MÉTODO "BACKWARD" DE EULER, CALCULADO AS CORREÇÕES NOS INDUTORES



É CONHECIDA A SOLUÇÃO COMPLETA EM $x = x_0$

CIRCUITOS ELÉTRICOS II - 1º SEMESTRO DE 2015 - PIRACIQUERA PROVA - GABARITO

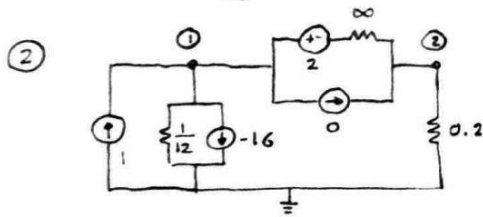


$$\begin{bmatrix} sC + \frac{1}{R} & -\frac{1}{R} & 0 \\ sC + \frac{1}{R} & -sC & -\frac{1}{R} \\ \frac{2}{R} & 0 & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} E_{11s} \\ E_2 \\ E_u \end{bmatrix} = \begin{bmatrix} C \\ 0 \\ 0 \end{bmatrix}$$

$$e_{11}(t) = 2 \cos \frac{t}{RC}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & sC + \frac{1}{R} & -\frac{1}{R} & 0 & 0 \\ 2 & -\frac{1}{R} & sC + \frac{1}{R} & -sC & 0 \\ 3 & 0 & -sC & sC + \frac{1}{R} & -\frac{1}{R} \\ 4 & 0 & 0 & -\frac{1}{R} & \frac{2}{R} \\ 5 & 0 & 0 & 0 & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{bmatrix} = \begin{bmatrix} C \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$E_{11} = \frac{2s \frac{C^2}{R}}{s^2 C^2 \frac{1}{R} + sC \frac{1}{R^2} + \frac{2}{R^3} - \frac{sC}{R^2} - \frac{1}{R^3}} = \frac{2s \frac{C^2}{R}}{\frac{s^2 C^2}{R} + \frac{1}{R^3}} = \frac{2s}{s^2 + \frac{1}{RC^2}}$$

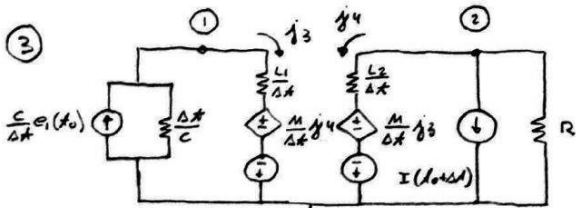


RESISTORES, DA ESQUERDA PARA A DIREITA

$$G_1 = 3e_{1M}^2 = 12 \quad I_1 = e_{1M}^3 - G_1 e_{1M} = 8 - 12 \times 2 = -16$$

$$G_2 = 3(e_{1M} - e_{2M} - 2)^2 = 0 \quad I_2 = (e_{1M} - e_{2M} - 2)^3 - G_3(e_{1M} - e_{2M} - 2) = 0$$

$$\begin{bmatrix} 12 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} e_{1M+1} \\ e_{2M+1} \end{bmatrix} = \begin{bmatrix} 17 \\ 0 \end{bmatrix} \quad \begin{aligned} e_{1M+1} &= \frac{17}{12} = 1.417 \text{ V} \\ e_{2M+1} &= 0 \end{aligned}$$

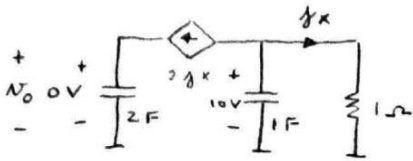


$$\frac{1}{\Delta x} (L_1 j_3(t_0) + M j_4(t_0)) = \frac{1}{\Delta x} (M j_3(t_0) + L_2 j_4(t_0))$$

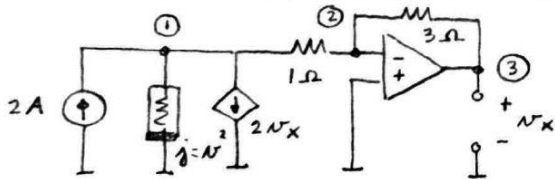
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & \frac{C}{\Delta x} & 0 & 1 & 0 \\ 2 & 0 & \frac{1}{R} & 0 & 1 \\ 3 & -1 & 0 & \frac{L_1}{\Delta x} & \frac{M}{\Delta x} \\ 4 & 0 & -1 & \frac{M}{\Delta x} & \frac{L_2}{\Delta x} \end{bmatrix} \begin{bmatrix} e_1(t_0 + \Delta t) \\ e_2(t_0 + \Delta t) \\ j_3(t_0 + \Delta t) \\ j_4(t_0 + \Delta t) \end{bmatrix} = \begin{bmatrix} \frac{C_1}{\Delta x} e_1(t_0) \\ -I(t_0 + \Delta t) \\ \frac{1}{\Delta x} (L_1 j_3(t_0) + M j_4(t_0)) \\ \frac{1}{\Delta x} (M j_3(t_0) + L_2 j_4(t_0)) \end{bmatrix}$$

CIRCUITOS ELÉTRICOS II - 2º SEMESTRE DE 2015 - PRIMEIRA PROVA

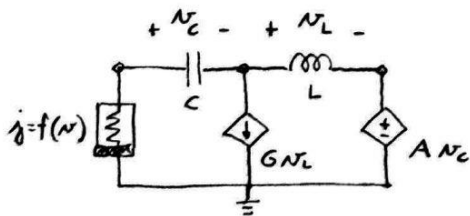
- ① CALCULE $N_0(x)$ USANDO UMA ANÁLISE NODAL MODIFICADA E A TRANSFORMADA DE LAPLACE

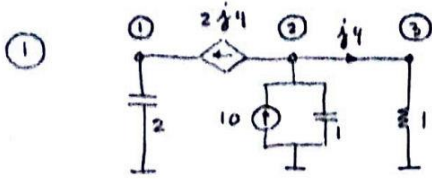


- ② PARA O CIRCUITO ABAIXO, TENHA-SE A ÚLTIMA SOLUÇÃO DEPENDENDO DO MÉTODO DE NEWTON-RAPHSON COMO $e_1 = 1V$, $e_2 = 0V$, $e_3 = -3V$. CALCULE A PRÓXIMA APROXIMAÇÃO USANDO UMA ANÁLISE NODAL COM REDUÇÃO USANDO O AMP. OPERACIONAL



- ③ O CIRCUITO ABAIXO DEVE SER ANALISADO COM O MÉTODO DOS TRAPÉZIOS. ESCRVA O SISTEMA NODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO EM $x = x_0 + \Delta x$, TENDO-SE A SOLUÇÃO EM x_0 E UMA APROXIMAÇÃO DA SOLUÇÃO EM $x_0 + \Delta x$, $\vec{e}(x_0)$ E $\vec{e}_M(x_0 + \Delta x)$

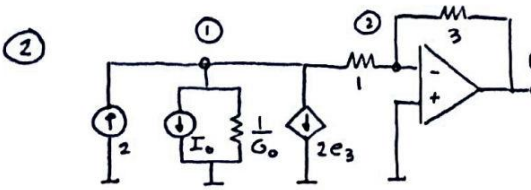




$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2s & 0 & 0 & -2 \\ 2 & 0 & s & 0 & 1+2 \\ 3 & 0 & 0 & 1 & -1 \\ 4 & 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \\ 0 \end{bmatrix}$$

$$E_1 = \frac{\begin{vmatrix} 0 & 0 & 0 & -2 \\ 10 & s & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 2s & 0 & 0 & -2 \\ 0 & s & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \end{vmatrix}} = \frac{2(10)}{2s(3+s)} = \frac{10}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3} = \frac{10}{s} + \frac{-3}{s+3} = \frac{10}{3} \left(\frac{1}{s} - \frac{1}{s+3} \right)$$

$$v_o(t) = e_1(t) = \frac{10}{3} (1 - e^{-3t})$$



$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & G_0+1 & -1 & 2 \\ 2 & -1 & 1+\frac{1}{3} & -\frac{1}{3} \\ 3 & 0 & -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} e_{1M+1} \\ e_{2M+1} \\ e_{3M+1} \end{bmatrix} = \begin{bmatrix} 2-I_0 \\ 0 \\ 0 \end{bmatrix}$$

$$G_0 = 2e_{1M} = 2$$

$$I_0 = e_{1M}^2 - G_0 e_{1M} = 1 - 2 = -1$$

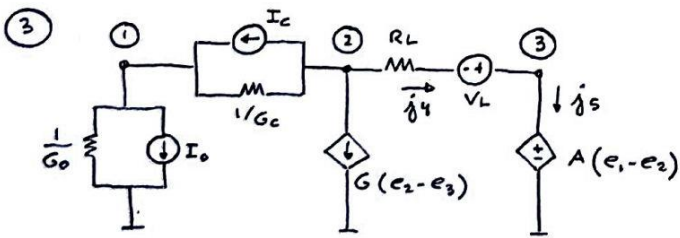
SISTEMA REDUZIDO:

$$\begin{bmatrix} G_0+1 & 2 \\ -1 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} e_{1M+1} \\ e_{3M+1} \end{bmatrix} = \begin{bmatrix} 2-I_0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 \\ -1 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} e_{1M+1} \\ e_{3M+1} \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$e_{1M+1} = \frac{-1}{-1+2} = -1V$$

$$e_{3M+1} = \frac{3}{-1+2} = 3V$$

$$e_{2M+1} = 0V$$



$$G_0 = f'(e_{1M})$$

$$I_0 = f(e_{1M}) - G_0 e_{1M}, \text{ usando } e_{1M}(t_0 + \Delta t)$$

$$G_C = \frac{2C}{\Delta t} \quad I_C = \frac{2C}{\Delta t} (e_1(t_0) - e_2(t_0)) + j_C(t_0)$$

$$R_L = \frac{\Delta X}{2L} \quad V_L = \frac{2L}{\Delta t} j_4(t_0) + (e_2(t_0) - e_3(t_0))$$

$j_C(t_0)$ É CALCULADO AO FIM DA ANÁLISE, DEPOIS DA CONVERGÊNCIA:

$j_C = G_C(e_1 - e_2) - I_C$ O PRIMEIRO VALOR VEM DE UMA ANÁLISE DE P.O. OU DE C.I.

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & G_0+G_C & -G_C & 0 & 0 & 0 \\ 2 & -G_C & G_C+G & -G & +1 & 0 \\ 3 & 0 & 0 & 0 & -1 & +1 \\ 4 & 0 & -1 & +1 & R_L & 0 \\ 5 & +A & -A & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{1M+1}(t_0+\Delta t) \\ e_{2M+1}(t_0+\Delta t) \\ e_{3M+1}(t_0+\Delta t) \\ j_{4M+1}(t_0+\Delta t) \\ j_{5M+1}(t_0+\Delta t) \end{bmatrix} = \begin{bmatrix} -I_0 + I_C \\ -I_C \\ 0 \\ +V_L \\ 0 \end{bmatrix}$$

* E.A. EXTRA: $e_2 - e_3 = R_L j_4 - V_L$
 $e_3 = A(e_1 - e_2)$

CAPACITOR COM TRAPÉZIOS

$$N(t_0+\Delta t) = N(t_0) + \frac{\Delta t}{2C} (j(t_0) + j(t_0+\Delta t))$$

$$j(t_0+\Delta t) = \frac{2C}{\Delta t} (N(t_0+\Delta t) - N(t_0)) - j(t_0)$$

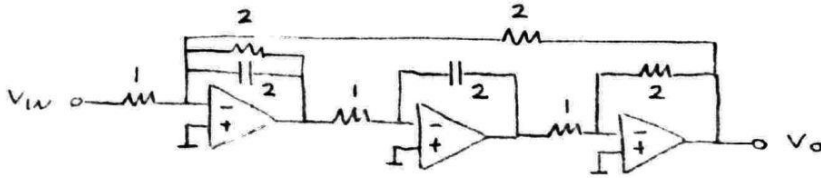
INDUTOR COM TRAPÉZIOS

$$j(t_0+\Delta t) = j(t_0) + \frac{\Delta t}{2L} (N(t_0) + N(t_0+\Delta t))$$

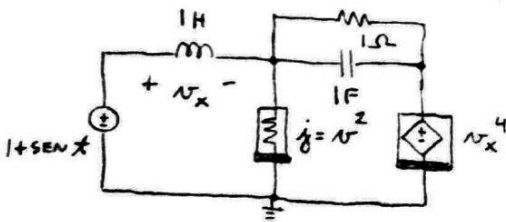
$$N(t_0+\Delta t) = \frac{2L}{\Delta t} (j(t_0+\Delta t) - j(t_0)) + N(t_0)$$

CIRCUITOS ELÉTRICOS II - PRIMEIRO SEMESTRE DE 2016 - PRIMEIRA PROVA

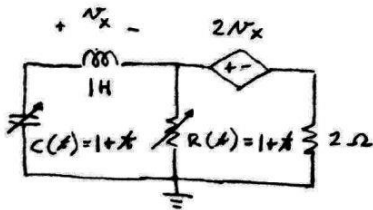
- ① USE UMA ANÁLISE NODAL COM UTILIZAÇÃO DE AMP. OPERACIONAIS PARA CALCULAR $V_o(j\omega)$. PLOTE UM GRÁFICO DE $\left| \frac{V_o}{V_{in}}(j\omega) \right|$.



- ② PARA O CIRCUITO, ESCREVA O SISTEMA NODAL MODIFICADO NECESSÁRIO PARA A OBTENÇÃO DO MODELO DE PEQUENOS SINAIS, E MOSTRE COMO FICA A ESTRUTURA DO MODELO.



- ③ ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA $e(t_0 + \Delta t)$ USANDO O MÉTODO "BACKWARD" DE EULER



CIRCUITOS ELÉTRICOS II - PRIMEIRO SEMESTRE DE 2016 - PRIMEIRA PROVA - GABARITO

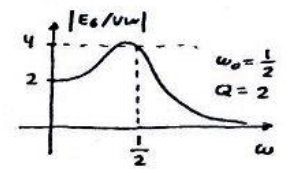
①

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2s+2 & -2s-\frac{1}{2} & 0 & 0 & 0 \\ 2 & -2s-\frac{1}{2} & 2s+\frac{1}{2} & -1 & 0 & 0 \\ 3 & 0 & -1 & 2s+1 & -2s & 0 \\ 4 & 0 & 0 & -2s & 2s+1 & -1 \\ 5 & 0 & 0 & 0 & -1 & \frac{3}{2} \\ 6 & -\frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \end{bmatrix} = \begin{bmatrix} V_{in} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

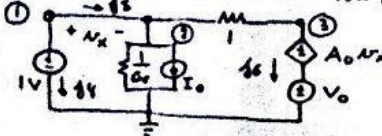
PARANDO COM 5 PARA DEPOIS TROCAR POR $\frac{1}{2}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & -2s-\frac{1}{2} & 0 \\ 2 & -1 & -2s \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} E_2 \\ E_4 \\ E_6 \end{bmatrix} = \begin{bmatrix} V_{in} \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{E_6}{V_{in}} = \frac{1}{(4s^2 + s)(-\frac{1}{2}) - \frac{1}{2}} = \frac{-1}{2s^2 + \frac{1}{2}s + \frac{1}{2}} = \frac{-\frac{1}{2}}{s^2 + \frac{1}{4}s + \frac{1}{4}} \rightarrow \frac{-\frac{1}{2}}{-\omega^2 + \frac{j\omega}{4} + \frac{1}{4}}$$



② É NECESSÁRIO RESOLVER O CIRCUITO COM LABORÉ EM CURTO E CAPACITOR EM ABERTO, COM UNIFORME USANDO NEWTON-RAPHSON. COM UM SISTEMA NORAL MODIFICADO SÃO 6 INCOGNITAS

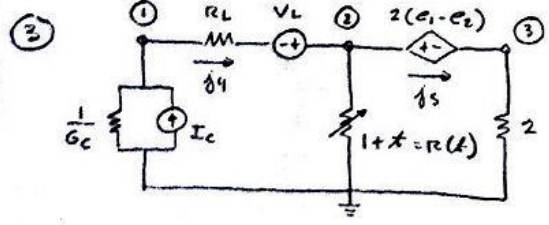
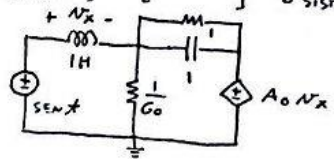


$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 2 & 0 & G_0+1 & -1 & 0 & 0 \\ 3 & 0 & -1 & 1 & 0 & 0 \\ 4 & -1 & 0 & 0 & 1 & 0 \\ 5 & -1 & 0 & 1 & 0 & 0 \\ 6 & A_0 & -A_0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ j_4 \\ j_5 \\ j_6 \end{bmatrix} = \begin{bmatrix} 0 \\ -I_0 \\ 0 \\ -1 \\ 0 \\ -V_0 \end{bmatrix}$$

$e_2 = A_0(e_1 - e_2) + V_0$
 É ÓBVIO QUE VAI DAR $e_1 = 1$ E $e_3 = 0$
 ASSIM $A_0 = 0$
 E $G_0 = 2$, MAS ISSO O SISTEMA VAI CALCULAR

$G_0 = 2 \text{ S}$
 $I_0 = e_{2M}^2 - G_0 e_{2M}$
 $A_0 = 4(e_{1M} - e_{2M})^3$
 $V_0 = (e_{1M} - e_{2M})^4 - A_0(e_{1M} - e_{2M})$

O MODELO DE PEQ. SINAIS USA OS VALORES FINAIS DE G_0 E A_0 APÓS A CONVERGÊNCIA DO NEWTON-RAPHSON



CAPACITOR: $C(\omega+\Delta\omega) \mathcal{N}(\omega+\Delta\omega) = C(\omega)\mathcal{N}(\omega) + \Delta\omega j(\omega+\Delta\omega)$
 INDUCTOR: $L(\omega+\Delta\omega) j(\omega+\Delta\omega) = L(\omega)j(\omega) + \Delta\omega \mathcal{N}(\omega+\Delta\omega)$
 $G_C = \frac{C(\omega+\Delta\omega)}{\Delta\omega} = \frac{1+\omega_0+\Delta\omega}{\Delta\omega}$ $I_C = \frac{C(\omega_0)}{\Delta\omega} \mathcal{N}(\omega_0) = \frac{1+\omega_0}{\Delta\omega} e_1(\omega_0)$

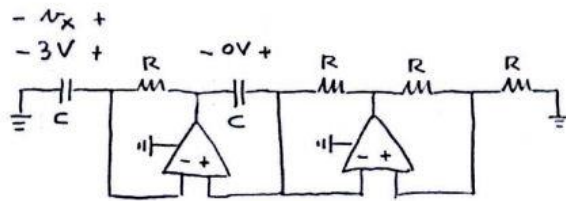
$R_L = \frac{L(\omega_0+\Delta\omega)}{\Delta\omega} = \frac{1}{\Delta\omega}$ $V_L = \frac{L(\omega_0)}{\Delta\omega} j(\omega_0) = \frac{1}{\Delta\omega} j_4(\omega_0)$ (TRATANDO COMO SE PUDÉSSE VARIAR NO TEMPO)

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & G_C & 0 & 1 & 0 \\ 2 & 0 & \frac{1}{R(x)} & -1 & 1 \\ 3 & 0 & 0 & \frac{1}{2} & -1 \\ 4 & -1 & 1 & 0 & R_L \\ 5 & 2 & -3 & 1 & 0 \end{bmatrix} \begin{bmatrix} e_1(\omega_0+\Delta\omega) \\ e_2(\omega_0+\Delta\omega) \\ e_3(\omega_0+\Delta\omega) \\ j_4(\omega_0+\Delta\omega) \\ j_5(\omega_0+\Delta\omega) \end{bmatrix} = \begin{bmatrix} I_C \\ 0 \\ 0 \\ V_L \\ 0 \end{bmatrix}$$

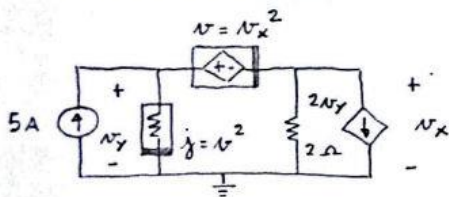
$e_2 - e_3 = 2(e_1 - e_2)$
 $2e_1 - 3e_2 + e_3 = 0$

CIRCUITOS ELÉTRICOS II - 2º SEMESTRE DE 2016 - PRIMEIRA PROVA

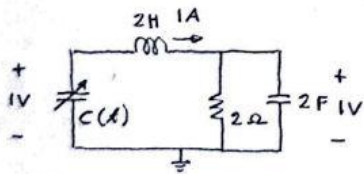
- ① CALCULE $N_x(x)$ USANDO UM SISTEMA MODAL EM TRANSFORMADA DE LAPLACE, COM ELIMINAÇÃO DOS AMPLIFICADORES OPERACIONAIS



- ② ESCRIBA O SISTEMA MODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO DO CIRCUITO, USANDO O MÉTODO DE NEWTON-RAPHSON.



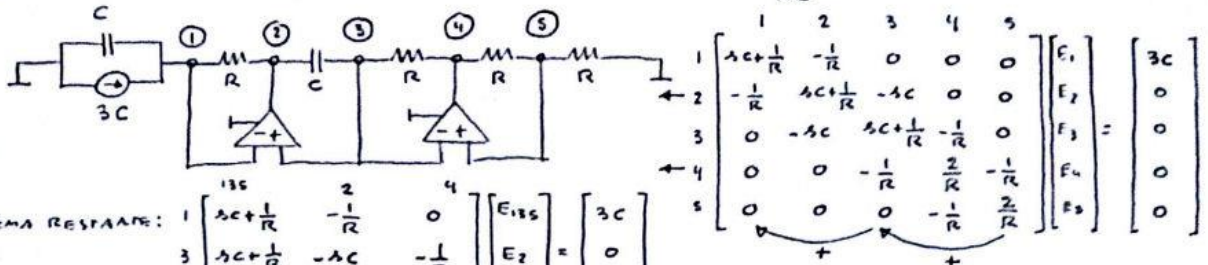
- ③ ESCRIBA O SISTEMA MODAL MODIFICADO QUE CALCULA A SOLUÇÃO EM $t = 0.1s$ USANDO O MÉTODO DOS TRAPÉZIOS.



$$C(x) = 1 + x$$

MOSTRAR O ESTADO EM $t = 0$
LEMBRE DE COMPLETAR A SOLUÇÃO

① É UM TANGENTE LC SIMPLIADO. DEVA DAR $N_x(t) = 3 \cos \frac{1}{RC} t$



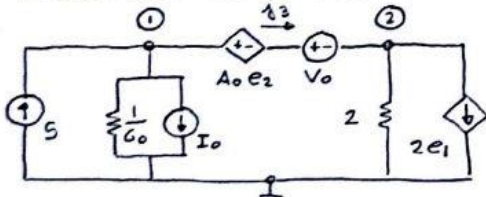
SISTEMA RESPOSTA:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & \Delta C + \frac{1}{R} & -\frac{1}{R} & 0 & 0 & 0 \\ 2 & -\frac{1}{R} & \Delta C + \frac{1}{R} & -\Delta C & 0 & 0 \\ 3 & 0 & -\Delta C & \Delta C + \frac{1}{R} & -\frac{1}{R} & 0 \\ 4 & 0 & 0 & -\frac{1}{R} & \frac{2}{R} & -\frac{1}{R} \\ 5 & 0 & 0 & 0 & -\frac{1}{R} & \frac{2}{R} \end{bmatrix} \begin{bmatrix} E_{135} \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{bmatrix} = \begin{bmatrix} 3C \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$E_{135} = \frac{3C \Delta C}{\left(\Delta C + \frac{1}{R}\right) \frac{\Delta C}{R} + \frac{2}{R^3} - \left(\Delta C + \frac{1}{R}\right) \frac{1}{R^2}} = \frac{3 \Delta C^2}{\frac{\Delta^2 C^2}{R} + \frac{\Delta C}{R^2} + \frac{2}{R^3} - \frac{\Delta C}{R^2} - \frac{1}{R^3}} = \frac{3 \Delta C^2}{\frac{\Delta^2 C^2}{R} + \frac{1}{R^3}}$$

$$= 3 \frac{\Delta}{\Delta^2 + \frac{1}{R^2 C^2}} \rightarrow e_{123}(t) = N_x(t) = 3 \cos \frac{1}{RC} t$$

② MODELO PARA NEWTON-RAPHSON:



$$G_0 = 2e_{1m} \quad I_0 = e_{1m}^2 - 2e_{1m}e_{1m} = -e_{1m}^2$$

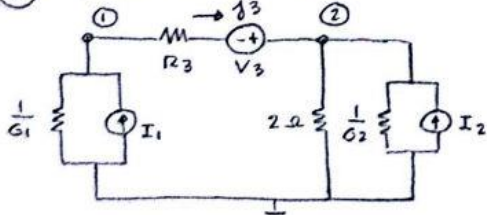
$$A_0 = 2e_{2m} \quad V_0 = e_{2m}^2 - 2e_{2m}e_{2m} = -e_{2m}^2$$

SISTEMA:

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & G_0 & 0 & \vdots & +1 \\ 2 & 2 & \frac{1}{2} & \vdots & -1 \\ 3 & -1 & +1+A_0 & \vdots & 0 \end{bmatrix} \begin{bmatrix} e_{1m+1} \\ e_{2m+1} \\ \beta_{3m+1} \end{bmatrix} = \begin{bmatrix} 5 - I_0 \\ 0 \\ -V_0 \end{bmatrix}$$

$$e_1 - e_2 = A_0 e_2 + V_0$$

③ MODELO PARA TRAPÉZIOS



CAPACITOR VARIÁVEL:

$$q(t_0 + \Delta t) = q(t_0) + \frac{\Delta t}{2} (\dot{q}(t_0) + \dot{q}(t_0 + \Delta t))$$

$$\dot{q}(t_0 + \Delta t) = \frac{2}{\Delta t} (q(t_0 + \Delta t) - q(t_0)) - \dot{q}(t_0)$$

$$\dot{q}(t_0 + \Delta t) = \frac{2}{\Delta t} ((1+t_0 + \Delta t) v(t_0 + \Delta t) - (1+t_0) v(t_0)) - \dot{q}(t_0) \quad \therefore G_1 = \frac{2}{\Delta t} (1+t_0 + \Delta t) = 20 \times 1.1 = 22$$

$$I_1 = \frac{2}{\Delta t} (1+t_0) + \dot{q}(t_0) = 20 \times 1 + (-1) = 19$$

SISTEMA:

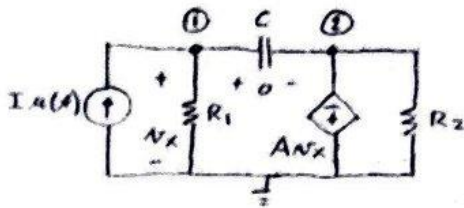
$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 22 & 0 & \vdots & 1 \\ 2 & 0 & 40.5 & \vdots & -1 \\ 3 & -1 & +1 & \vdots & 40 \end{bmatrix} \begin{bmatrix} e_1(0.1) \\ e_2(0.1) \\ \beta_3(0.1) \end{bmatrix} = \begin{bmatrix} 19 \\ 40.5 \\ 40 \end{bmatrix}$$

SOLUÇÃO EM $t=0$

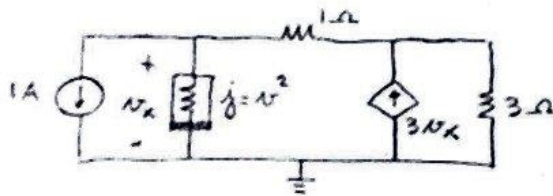


CIRCUITOS ELÉTRICOS II - 1ª SEMESTRE DA 2017 - PRIMEIRA PROVA

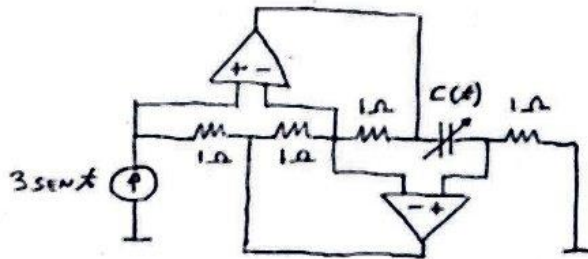
- 1) PARA O CIRCUITO ABAIXO, CALCULE $i_x(t)$ USANDO UMA ANÁLISE NODAL MODIFICADA EM TRANSFORMADA DE LAPLACE.



- 2) PARA O CIRCUITO ABAIXO:
 a) CALCULE A SOLUÇÃO EXATA USANDO UMA ANÁLISE NODAL NÃO LINEAR.
 b) ESCREVA O SISTEMA QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO PELO MÉTODO DE NEWTON-RAPHSON, E TENTE RESOLVER, PARTINDO DA SOLUÇÃO EXATA, EXPLIQUE O RESULTADO.

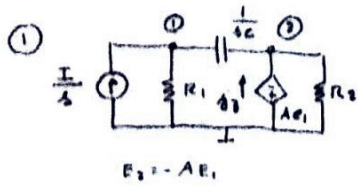


- 3) PARA O CIRCUITO ABAIXO, ESCREVA O SISTEMA NODAL QUE CALCULA A SOLUÇÃO EM $t = t_0 + \Delta t$, CONHECIDA A SOLUÇÃO EM $t = t_0$, ELIMINANDO OS AMP. OPERACIONAIS, E USANDO O MÉTODO DOS TRAPÉZIOS



$C(t) = 2 + \cos t$

CIRCUITOS ELÉTRICOS II - 12 SEMESTRE DE 2017 - PRIMEIRA PROVA - GABARITO



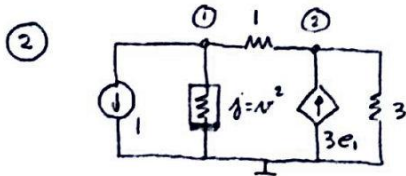
$$\begin{matrix} 1 & 2 & 3 \\ \begin{bmatrix} sC + \frac{1}{R_1} & -sC & 0 \\ -sC & sC + \frac{1}{R_2} & -1 \\ -A & -1 & 0 \end{bmatrix} \end{matrix} \begin{bmatrix} E_1(A) \\ E_2(A) \\ J_3(A) \end{bmatrix} = \begin{bmatrix} \frac{I}{2} \\ 0 \\ 0 \end{bmatrix}$$

$$E_1(A) = \frac{-\frac{I}{2}}{-sCA - sC - \frac{1}{R_1}} = \frac{I}{s \left(sC(1+A) + \frac{1}{R_1} \right)} = \frac{I}{s \left(s + \frac{1}{RC(1+A)} \right)} = \frac{K_1}{s} + \frac{K_2}{s + \frac{1}{RC(1+A)}}$$

$$K_1 = s E_1(s) \Big|_{s=0} = RI \quad K_2 = \left(s + \frac{1}{RC(1+A)} \right) E_1(s) \Big|_{s = -\frac{1}{RC(1+A)}} = -RI$$

$$e_1(t) = RI \left(1 - e^{-\frac{t}{RC(1+A)}} \right) \mu(t)$$

O CAPACITOR FICA MULTIPLICADO POR (1+A) EM // COM O RESISTOR: EFEITO MILLER

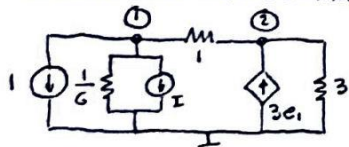


$$\textcircled{1}: e_2 + e_1 - e_3 = -1 \quad \therefore e_2 = e_1^2 + e_1 + 1$$

$$\textcircled{2}: e_2 - e_1 - 3e_1 + \frac{e_2}{3} = 0 \quad \therefore \frac{4}{3}e_2 - 4e_1 = 0$$

$$\frac{4}{3}e_1^2 + \frac{4}{3}e_1 + \frac{4}{3} - 4e_1 = 0 \quad \therefore e_1^2 - 2e_1 + 1 = 0 \quad \therefore e_1 = \frac{2 \pm \sqrt{4-4}}{2} = \begin{cases} 1 & \text{SOLUÇÃO DUPLA} \end{cases}$$

MÓDULO PARA NEWTON-RAPHSON:



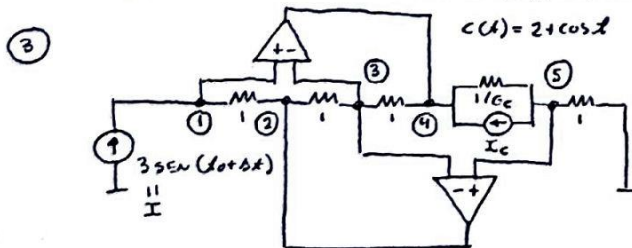
$$G = 2e_{1m} = 2$$

$$I = e_{1m}^2 - 2e_{1m}e_{1m} = -1$$

RETA E PARÁBOLA SÃO TUCADO EM (1,3) ✓

$$\begin{bmatrix} 2+1 & -1 \\ -1-3 & 1+\frac{1}{3} \end{bmatrix} \begin{bmatrix} e_{1m1} \\ e_{2m1} \end{bmatrix} = \begin{bmatrix} -1+1 \\ 0 \end{bmatrix}$$

DETERMINANTE: $3 \times \frac{4}{3} - 4 \times 1 = 0$ AS SOLUÇÕES DÃO 0!? ISSO ACONTECE DEVIDO À SOLUÇÃO DUPLA, O MÉTODO NA VERDADE CONVERGE, MAS CLARAMENTE, E DÁ UMA SINGULARIDADE NA SOLUÇÃO



CAPACITOR VARIANTE NO TEMPO COM PARÂMETROS:

$$q(t_0 + \Delta t) = q(t_0) + \frac{\Delta t}{2} (j_c(t_0 + \Delta t) + j_c(t_0))$$

$$j_c(t_0 + \Delta t) = \frac{2}{\Delta t} (C(t_0 + \Delta t)v_c(t_0 + \Delta t) - C(t_0)v_c(t_0)) - j_c(t_0)$$

$$G_c = \frac{2(2 + \cos(t_0 + \Delta t))}{\Delta t}$$

$$I_c = \frac{2}{\Delta t} (2 + \cos t_0)(e_5(t_0) - e_4(t_0)) + j_c(t_0)$$

SISTEMA REDUZIDO:

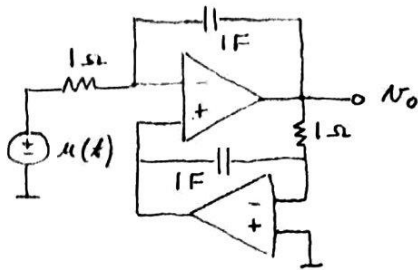
↑
CALCULADO A PARTIR DA ANÁLISE ANTERIOR

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & G_c + 1 & -G_c \\ 0 & 0 & 0 & -G_c & G_c + 1 \end{bmatrix} \end{matrix} \begin{bmatrix} e_1(t_0 + \Delta t) \\ e_2(t_0 + \Delta t) \\ e_3(t_0 + \Delta t) \\ e_4(t_0 + \Delta t) \\ e_5(t_0 + \Delta t) \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ 0 \\ I_c \\ -I_c \end{bmatrix}$$

$$\begin{matrix} 1 & 2 & 4 \\ \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & -1 \\ G_c + 1 & 0 & -G_c \end{bmatrix} \end{matrix} \begin{bmatrix} e_{125}(t_0 + \Delta t) \\ e_2(t_0 + \Delta t) \\ e_4(t_0 + \Delta t) \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ -I_c \end{bmatrix}$$

CIRCUITOS ELÉTRICOS II - 2º SEMESTRE DE 2017 - 1ª PROVA

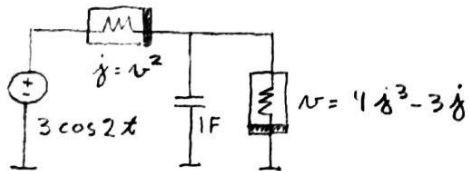
- 1) PARA O CIRCUITO ABAIXO, ENCONTRE $N_0(x)$ USANDO UMA ANÁLISE NODAL COM ELIMINAÇÃO DOS AMPLIFICADORES E FRACIONAIS EM TRANSFORMADA DE LAPLACE.



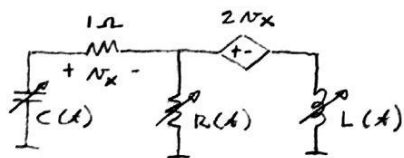
$$\frac{w}{(s+d)^2+w^2} \rightarrow e^{-dx} \text{ sen } wx$$

$$\frac{s+d}{(s+d)^2+w^2} \rightarrow e^{-dx} \text{ cos } wx$$

- 2) PARA O CIRCUITO, ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A PRÓXIMA APROXIMAÇÃO DA SOLUÇÃO, USANDO O MÉTODO DOS TRAPÉZIOS E NEWTON-RAPHSON.



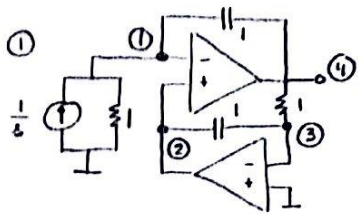
- 3) PARA O CIRCUITO LINEAR VARIANTE NO TEMPO ABAIXO, ESCREVA O SISTEMA NODAL MODIFICADO QUE CALCULA A SOLUÇÃO EM $x = x_0 + \Delta x$ USANDO O MÉTODO DE GEAR DE 2ª ORDEM.



$$\text{SE } y(x_0 + \Delta x) = y(x_0) + \int_{x_0}^{x_0 + \Delta x} x(x) dx$$

ENTÃO:

$$y(x_0 + \Delta x) \approx \frac{4}{3} y(x_0) - \frac{1}{3} y(x_0 - \Delta x) + \frac{2}{3} \Delta x x(x_0 + \Delta x)$$



$$\begin{matrix} & 1 & 2 & 3 & 4 \\ \leftarrow 1 & \begin{bmatrix} s+1 & 0 & 0 & -s \\ 0 & s & -s & 0 \\ 0 & -s & s+1 & -1 \\ -s & 0 & -1 & s+1 \end{bmatrix} & \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} & = & \begin{bmatrix} \frac{1}{s} \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{matrix}$$

$$\omega_0 = 1$$

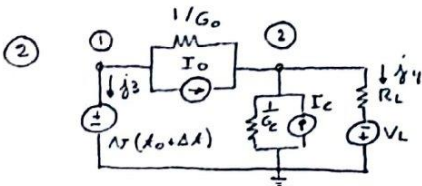
$$Q = 1$$

$$\omega_0 \sqrt{1 - \frac{1}{4Q^2}} = \frac{\sqrt{3}}{2}$$

$$\begin{bmatrix} s+1 & -s \\ -s & -1 \end{bmatrix} \begin{bmatrix} E_{12} \\ E_{41} \end{bmatrix} = \begin{bmatrix} \frac{1}{s} \\ 0 \end{bmatrix}$$

$$E_{41} = \frac{1}{-s-1-s^2} = \frac{-1}{s^2+s+1} = \frac{-\frac{2}{\sqrt{3}} \frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2 + \frac{3}{4}}$$

$$v_o(t) = e_{41}(t) = -\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \text{SEN} \frac{\sqrt{3}}{2} t$$



$$G_0 = 2 (e_{1m}(t_0 + \Delta t) - e_{2m}(t_0 + \Delta t))$$

$$I_0 = (e_{1m}(t_0 + \Delta t) - e_{2m}(t_0 + \Delta t))^2 - G_0 (e_{1m}(t_0 + \Delta t) - e_{2m}(t_0 + \Delta t))$$

$$R_L = 12 j_{4m}(t_0 + \Delta t)^2 - 3$$

$$V_L = 4 j_{4m}(t_0 + \Delta t)^3 - 3 j_{4m}(t_0 + \Delta t) - R_L j_{4m}(t_0 + \Delta t)$$

CAPACITOR COMA DEPENDIZIOS:

$$N(t_0 + \Delta t) = \frac{\Delta t}{2C} (j(t_0) + j(t_0 + \Delta t)) + N(t_0)$$

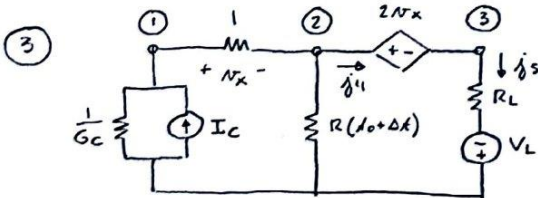
$$\therefore j(t_0 + \Delta t) = \frac{2C}{\Delta t} (N(t_0 + \Delta t) - N(t_0)) - j(t_0)$$

$$G_C = \frac{2}{\Delta t}$$

$$I_C = \frac{2}{\Delta t} e_2(t_0) + j_C(t_0)$$

CALCULO DO FIM DA ANÁLISE ANTERIOR COMO $G_C e_2 - I_C$

$$\begin{matrix} & 1 & 2 & 3 & 4 \\ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} & \begin{bmatrix} G_0 & -G_0 & +1 & 0 \\ -G_0 & G_0 + G_C & 0 & +1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & +R_L \end{bmatrix} & \begin{bmatrix} e_{1m+1}(t_0 + \Delta t) \\ e_{2m+1}(t_0 + \Delta t) \\ j_{3m+1}(t_0 + \Delta t) \\ j_{4m+1}(t_0 + \Delta t) \end{bmatrix} & = & \begin{bmatrix} -I_0 \\ +I_0 + I_C \\ -3 \cos 2(t_0 + \Delta t) \\ +V_L \end{bmatrix} \end{matrix}$$



$$G_C = \frac{3}{2\Delta t} C(t_0 + \Delta t)$$

$$I_C = \frac{3}{2\Delta t} \left(\frac{4}{3} C(t_0) e_1(t_0) - \frac{1}{3} C(t_0 - \Delta t) e_1(t_0 - \Delta t) \right)$$

$$R_L = \frac{3}{2\Delta t} L(t_0 + \Delta t)$$

$$V_L = \frac{3}{2\Delta t} \left(\frac{4}{8} L(t_0) j_5(t_0) - \frac{1}{8} L(t_0 - \Delta t) j_5(t_0 - \Delta t) \right)$$

CAPACITOR:

$$C(t_0 + \Delta t) N(t_0 + \Delta t) = \frac{4}{3} C(t_0) N(t_0) - \frac{1}{3} C(t_0 - \Delta t) N(t_0 - \Delta t) + \frac{2}{3} \Delta t j(t_0 + \Delta t)$$

$$j(t_0 + \Delta t) = \frac{3}{2\Delta t} \left(C(t_0 + \Delta t) N(t_0 + \Delta t) - \frac{4}{3} C(t_0) N(t_0) + \frac{1}{3} C(t_0 - \Delta t) N(t_0 - \Delta t) \right)$$

INDUTOR: ANALOGAMENTE

$$N(t_0 + \Delta t) = \frac{3}{2\Delta t} \left(L(t_0 + \Delta t) j(t_0 + \Delta t) - \frac{4}{3} L(t_0) j(t_0) + \frac{1}{3} L(t_0 - \Delta t) j(t_0 - \Delta t) \right)$$

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 \\ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} & \begin{bmatrix} G_C + 1 & -1 & 0 & 0 & 0 \\ -1 & 1 + \frac{1}{R(t_0 + \Delta t)} & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 & +1 \\ +2 & -1-2 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 & +R_L \end{bmatrix} & \begin{bmatrix} e_1(t_0 + \Delta t) \\ e_2(t_0 + \Delta t) \\ e_3(t_0 + \Delta t) \\ j_4(t_0 + \Delta t) \\ j_5(t_0 + \Delta t) \end{bmatrix} & = & \begin{bmatrix} +I_C \\ 0 \\ 0 \\ 0 \\ +V_L \end{bmatrix} \end{matrix}$$